



TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 61, Issue I, March, 2018

ADJUSTMENT OF THE LINEAR CORRELATION BETWEEN THE SURFACE AREA AND THE VOLUME OF THE LEFT VENTRICLE

José Sérgio DOMINGUES, Marcos DE PAULA VALE, Carlos BARREIRA MARTINEZ

Abstract: This work aims to study the linear correlation between the surface area and the left ventricle volume, considering that this cardiac chamber has the truncated prolate spheroid geometry, and also aims to compare it with a correlation between these two parameters, already described in literature. We used mathematical models to determine the surface area and the left ventricle volume. The volumes that were considered for its real terms and for patients with no cardiac abnormalities, obtained in medical literature. The volumes of this chamber were obtained in the same bibliographic reference. They were determined by MRI and were used to find more realistic correlations.

Key words: Ventricular volume, ventricular surface area, truncated prolate spheroid, left ventricle, mathematical models.

1. INTRODUCTION

Some of the main heart parameters for the left ventricle (LV), which allow us to verify the functioning of the heart, are: ventricular diastolic volume (V_d) and systolic volume (V_s), ejection fraction, shortening fraction, and ventricular mass. Usually, they are based on the fact that this chamber has a similar geometry to the one of a revolution ellipsoid, or still, of a truncated prolate spheroid (TPS) [1-5].

However, the surface area of the LV (A_s) is a little discussed parameter on literature. A possible application of this parameter is on determining the dimensions of the heart portion to be removed of the LV's free wall when performing a Partial Left Ventriculectomy on patients with dilated cardiomyopathy, a surgical procedure which execution was drastically reduced all over the world, but has been successfully conducted, especially on Japan [6-9].

It is also possible that the A_s can be used as a new cardiac normality verification criterion, besides being a determining point of the interruption of growth of the extracellular matrix on the development of artificial organs, in a way that the developed artificial heart has each patient's specific features [10-13].

Feng et al. [14] presented a project where they use a mathematical model on ellipsoidal coordinates, which allowed them to obtain an equation for the collection of the A_s values, and another equation to obtain the V_d values. For that, they used the measures obtained on the LV of ten patients, without informing any other feature of the patients, except the A_s and V_d results.

The results obtained present a strong correlation (practically perfect) between A_s and V_d (Fig. 1).

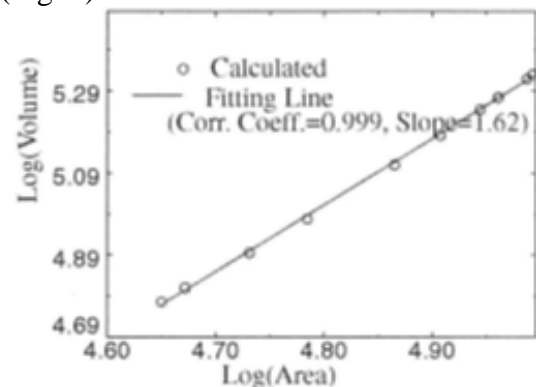


Fig. 1. Log-log correlation between A_s and V_d , from Feng, Sitek, and Gullberg [14].

The equations obtained for A_s and V_d in [17] are highly dependent, because both are based on the same geometrical model and on the same parameters. That way, it is mathematically assured that this strong correlation is obtained.

The main goal of this project is to verify if this strong correlation remains when A_s is correlated with the V_d values, previously presented on literature, and calculated through traditional medical methods. For that, we present a model equivalent to the one used in [14], but built from Cartesian coordinates, to facilitate the acquiring of the necessary measures for simulations, along with the V_d values, described on literature. Then, the results are presented and compared, where we verify that, when considering the same geometry and the same model for the attainment of A_s and V_d the correlation between them is going to be practically perfect, which won't happen when the A_s parameter is correlated to the V_d obtained by medical methods.

2. MATHEMATICAL MODELS

2.1 Mathematical models for the LV's area and volume

The mathematical model we used to calculate the average A_s values is based on Cartesian coordinates, to facilitate the use and attainment of the measures, since it depends only on the LV's linear measures.

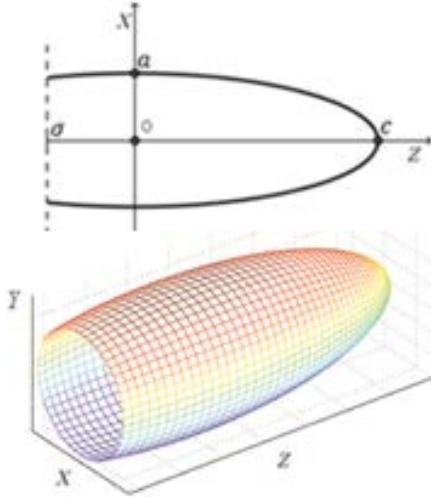


Fig. 2. Representation of the truncated ellipsoid and the TPS that is generated, considered as the LV's geometry.

We considered that the LV has the geometry of a TPS [1–5] with semi-axis a , b and c , where $c > a = b > 0$ and generated by the rotation of the truncated ellipsoid, of axes in the intervals $[\sigma, c]$ and $[-a, a]$ (Fig. 2).

The equation of the curve $x(z) \geq 0$ of the truncated ellipsoid is given on Eq. (1).

$$x(z) = a \cdot \sqrt{1 - \frac{z^2}{c^2}} \quad (1)$$

Therefore, the TPS is given by Eq. (2).

$$A_\sigma(S) = 2\pi \int_\sigma^c x(z) \cdot \sqrt{1 + [x'(z)]^2} dz, \quad (2)$$

which, when solved, results in the Eq. (3),

$$A_\sigma(S) = \pi a \left\{ \frac{c^2 \left[\sin^{-1} \left(\frac{\sqrt{c^2 - a^2}}{c} \right) - \sin^{-1} \left(\frac{\sigma \sqrt{c^2 - a^2}}{c^2} \right) \right]}{\sqrt{c^2 - a^2}} + a - \sigma \sqrt{1 + \frac{(a^2 - c^2)\sigma^2}{c^4}} \right\} \quad (3)$$

already presented and shown on [13].

2.2 Association of the TPS and LV measures

On Eq. (3), the a , c and σ parameters of the TPS correspond, respectively, to the final diastolic diameter of the LV (D), to the coordinate of the LV apex and to the coordinates of the truncating point of the TPS (which represents the coordinate of the average point of the LV base) (Fig. 3). The ways of determining the measures of these parameters are given by Eqs. (4), (5), and (6) [13].

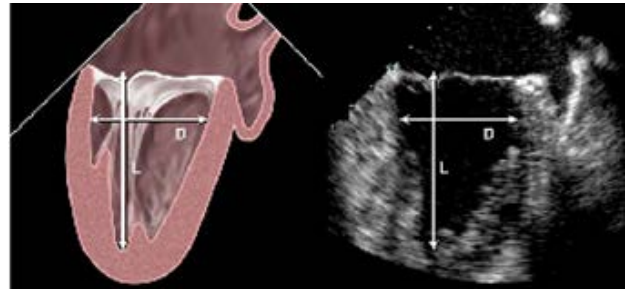


Fig. 3. Representation of the LV's short and long axis, adapted from Lang et al. [15].

$$a = \frac{D}{2} \quad (4)$$

$$c = \frac{2}{3}L \quad (5)$$

$$|\sigma| = \frac{1}{3}L, \quad (6)$$

where $L = c + |\sigma|$ is the dimension of the LV's long axis, on average hearts, respectively taken.

In normal patients, the relation D/L (short axis/long axis) on the LV varies from 0.45 to 0.62 [16]. Considering this relation as a $\delta \in [0.45, 0.62]$ parameter, in which $D/L = \delta$, that is:

$$L = \frac{D}{\delta} \quad (7)$$

So, taking Eq. (7) to the Eqs. (5) and (6), it is possible to obtain the $|\sigma|$ and c values according to D , obtaining:

$$|\sigma| = \frac{D}{3\delta} \quad \text{and} \quad c = \frac{2D}{3\delta} \quad (8)$$

Based on these measures and using the Eq. (3), it is possible to estimate the $A_\sigma(S)$ parameter, relative to the LV's surface area on average patients.

2.3 TPS volume and the perfect correlation between $A_\sigma(S)$ and V_d

The model presented on [14] calculates the V_d and the $A_\sigma(S)$ based on the same geometrical model on ellipsoidal coordinates. With that, there is a self-dependence between the equations that were used, which mathematically assures that there will be a strong correlation between those two parameters ($r = 0,999$). In order to illustrate that this dependence will always generate a practically perfect correlation between the parameters, we obtained an equation for the TPS volume, Eq. (9), based on the revolution of the curve on Eq. (1) around the Z axis and we obtain the correlation between $A_\sigma(S)$ and V_d .

$$V_d = \pi \int_{\sigma}^c [x(z)]^2 dz$$

$$\therefore V_d = \frac{2}{3} \pi a^2 (c - \sigma) \quad (9)$$

3. REAL MEDICAL DATA AND STATISTICAL ANALYSIS

On a study that aimed to obtain reference measures regarding the left and right ventricles, through magnetic resonance, on average Brazilian patients, Macedo *et al.* [17] particularly presented the average D and V_d values for 107 individuals (asymptomatic and with no heart diseases), in which 54 of them were men, and 53 were women. Images in orthogonal planes, long vertical and horizontal axes, and heart short axes were performed in the four-chamber plane (4ch) to describe the short axes, covering all the LV, from the basis up to the end, using the Steady State Free Precession (SSFP) technique. The D and V_d measures, by

age group and sex, and the calculated values for a , by Eq. (4), are described on Table 1.

Table 1
Average D , V_d and a values, by sex and age group.

Sex	Age (years)	D (cm)	V_d (mL)	a (cm)
Men	20 – 29	5.00	165.3	2.50
	30 – 39	5.10	149.3	2.55
	40 – 49	5.10	141.3	2.55
	50 – 59	4.80	131.9	2.40
	≥ 60	4.60	121.6	2.30
Women	20 – 29	4.70	127.0	2.30
	30 – 39	4.90	117.6	2.45
	40 – 49	4.40	110.0	2.20
	50 – 59	4.20	101.5	2.10
	≥ 60	4.50	110.2	2.25

The D values are used by Eqs. (4) and (8), and when substituted on Eqs. (3) and (9), determined the average $A_\sigma(S)$ and V_d values.

The calculated values of $A_\sigma(S)$ were correlated to the calculated values of V_d through Equation (9), and also, with the V_d values, obtained on [17]. These correlations were analyzed by Pearson's coefficient, r , by coefficient of determination, R^2 , and the statistical significance was determined by the p -value, calculated based on Student's t test, with three degrees of freedom.

From the real medical D values obtained on [17], and considering that $\delta \in [0.45, 0.62]$, we performed simulations using three values for δ : 0.45, 0.62, and also 0.50, which is one of the most used relations on methods that calculate the ventricular mass [5]. So, the correlations between $A_\sigma(S)$ and V_d were obtained.

4. RESULTS AND DISCUSSION

For simulations with self-dependent equations, we obtained $r = 0.9999$ with $p = 0.000002$ for men, with $\delta = 0.45$. Still for men, but when $\delta = 0.50$ or $\delta = 0.62$, we obtained $r = 0.9999$ with $p = 0.000009$ (Fig. 4).

For women, we obtained $r = 0.9997$ with $p = 0.000005$ for the three values used for parameter δ (Fig. 5).

Therefore, considering the models with self-dependent equations, the results were identical to those of Feng, Sitek, and Gullberg [14]. This

indicates that the models with ellipsoidal and Cartesian coordinates are equivalent, and that, consequently, using Cartesian coordinates in clinical practice is more viable, because it only depends on linear measures which are easily obtained on echocardiography or magnetic resonance exams.

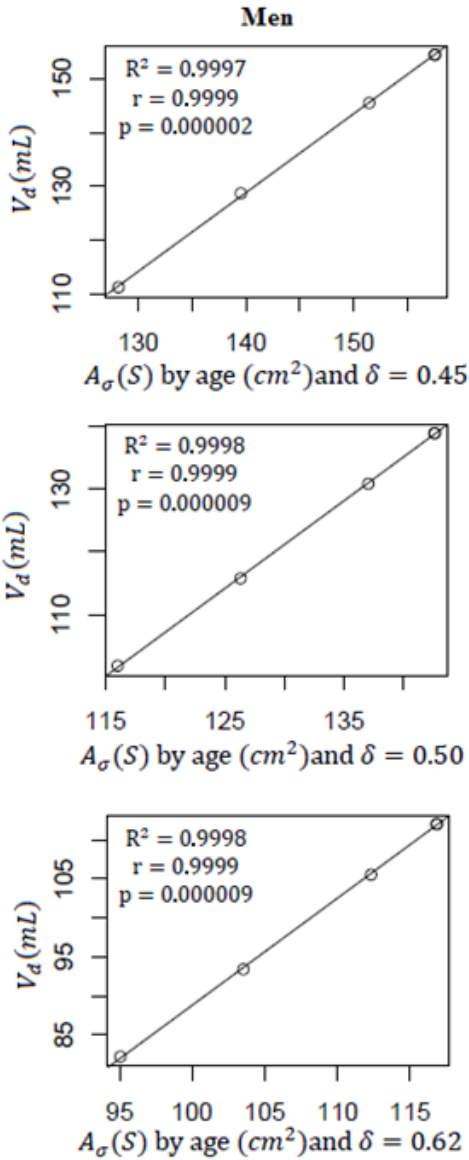


Fig. 4. Correlations obtained between $A_\sigma(S)$ and V_d , for men, through dependent models.

The simulations with the equations without self-dependence, and that, therefore, are more realistic, since they use real values of the LV's diastolic volume, already discussed in scientific literature, were also performed. We obtained $r = 0.7413$ with $p = 0.1517$ on all three values of parameter δ for men (Fig. 6).

For women, the values obtained were $r = 0.7983$ with $p = 0.1054$, also for all three values of δ (Fig. 7).

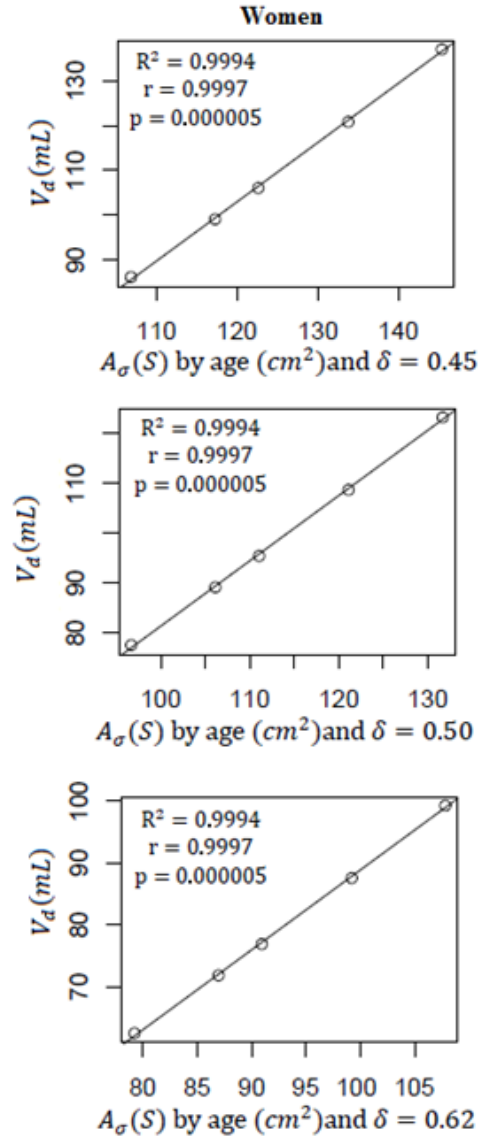


Fig. 5. Correlations obtained between $A_\sigma(S)$ and V_d , for women, through dependent models.

The correlations for the equations model without self-dependence, although strong, displayed values that were inferior (and less significant) to the ones obtained by Feng, Sitek, and Gullberg [14].

5. CONCLUSION

In conclusion, the mathematical models to determine the LV's surface area and its diastolic volume used by Feng, Sitek, and Gullbert [14] are self-dependent and, therefore, practically perfect correlations are mathematically expected. More realistic correlations were obtained by taking the LV's surface area as a base, calculated through the model described on [13], and the values of the diastolic volume

already described in literature. These values were determined by Simpson's rule, which is the most efficient method to calculate the ventricular volume [5, 18–20].

These correlations, although also strong, have little statistical significance and are very inferior to the ones described by Feng, Sitek, and Gullbert [14]. That way, as opposed to what is presented in that study [14], it is not possible to precisely determine the diastolic volume from the left ventricle's surface area indirectly.

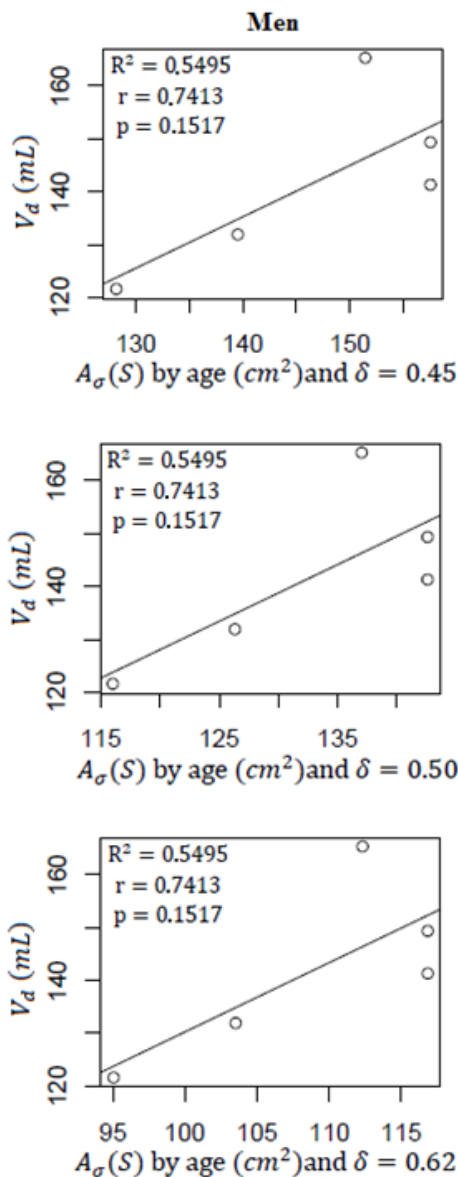
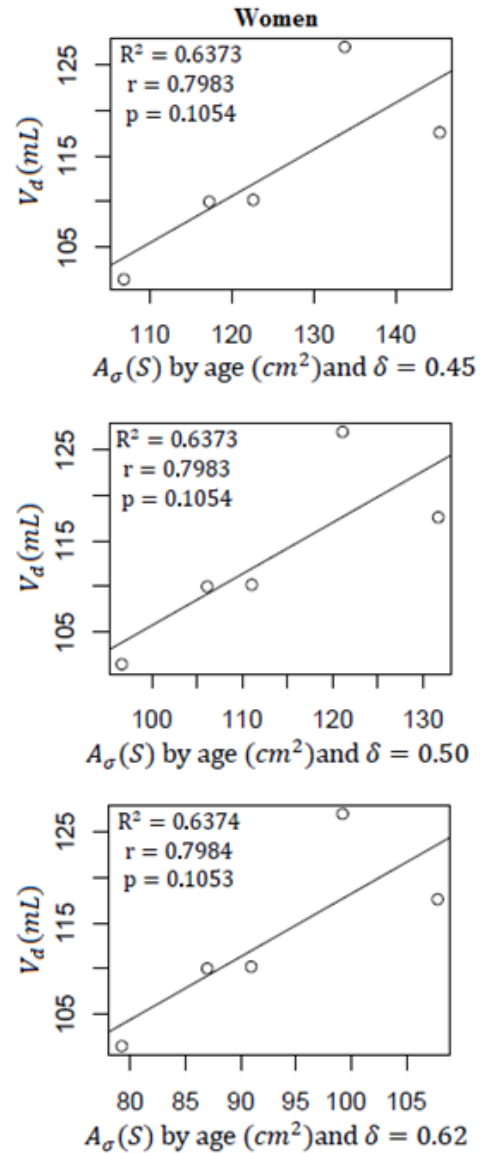


Fig. 6. Correlations obtained between $A_\sigma(S)$ and V_d calculated by Equation (9), for men.



6. REFERENCES

1. Yeh G.C.K., Martinek J., *Comparison of surface potentials due to several singularity representations of the human heart*, Bulletin of Mathematical Biophysics. 1957, 19: 293–308.
2. Baccani B., Domenichini F., Pedrizzetti G., *Model and influence of mitral valve opening during the left ventricular filling*, Journal of Biomechanics. 2003, 36:355–361.
3. Nielsen P.M.F., Le Grice I.J., Smaill B.H., Hunter P.J., *Mathematical model of geometry and fibrous structure of the heart*, American Journal of Physiology. 1991, 260(4-Pt 2), H1365–78.
4. Katz T.L., Mattos C.F.G., Kowatsch I., Vieira M.L.C., *Critical Analysis of Echocardiographic Measurements of Left Ventricular Mass*, Revista Brasileira de ecocardiografia e Imagem Cardiovascular. 2010, 23(4): 95–100.

5. Ferreira Filho P.R.P., *Padrões de Hipertrofia e Geometria do Ventrículo Esquerdo pela Ecocardiografia Transtorácica*, Revista Brasileira de ecocardiografia e Imagem Cardiovascular. 2012, 25(2):103–115.
6. Domingues J.S., Barbosa, M.P., Vale M.P., *Modelo Matemático para Fatia Cardíaca Removida na Ventriculectomia Parcial*, In: 4th National Meeting of Biomechanical Engineering (ENEBI), 2013, Vitória-ES, Brasil. Anais do IV ENEBI, 2013.
7. Domingues J.S., Barbosa, M.P., Vale M.P., *Mathematical and Computational Aspects in Partial Ventriculectomy*, In: 8th International Conference on Advanced Computational Engineering and Experimenting (ACEX 2014), 2014, Paris, France.
8. Domingues J.S., Barbosa, M.P., Vale M.P., *Mathematical and Computational Aspects in Partial Ventriculectomy*, In: *Applications of Computational Tools in Biosciences and Medical Engineering*, Advanced Structured Materials. Öchsner A, Altenbach H. Eds. Springer International, 2015, cap. 3, 37–41.
9. Domingues J.S., Vale M.P., Barbosa M.P., *Partial left ventriculectomy: have well-succeeded cases and innovations in the procedure been observed in the last 10 years?*, Brazilian Journal of Cardiovascular Surgery. 2015, 30(5):579 – 585.
10. Angelo B., Santanu R., Michael S. *et al.*, *Arbitrary Self-Assembly of Peptide Extracellular Microscopic Matrices*, Angewandte Chemie International. 2012, 51:428 – 431.
11. Paul S.M., *Are Artificial Organs Still Needed?*, Artificial Organs, 38(10):827 – 828.
12. Michael E., *Honey, I shrunk the lungs: Miniature versions of hearts, lungs and other organs are heralding a bright future for drug research and discovery*, Nature. 2015, 519: S16 – S18.
13. Domingues J.S., Vale M.P., Martinez C.B., *New mathematical model for the surface area of the left ventricle by the truncated prolate spheroid*, The Scientific World Journal. 2017, 2017:1 – 9.
14. Feng B., Sitek A., Gullberg T., *The prolate Spheroidal Transform for Gated SPECT*, IEEE Transactions on Nuclear Science. 2001, 48(3):872 – 875.
15. Lang R.M., Bierig M., Devereux R.B. *et al.*, *Recommendations for chamber quantification*, European Journal of Echocardiography. 2006, 4:79 – 108.
16. Graziosi P., *Análise ecocardiográfica da estrutura e da função sistólica ventricular esquerda na hipertensão arterial*, HiperAtivo, 1998, 5(3):161 – 174.
17. Macedo R., Fernandes J.L., Andrade S.S. *et al.*, *Morphological and Functional Measurements of the Heart Obtained by Magnetic Resonance Imaging in Brazilians*, Arquivos Brasileiros de Cardiologia, 2013, 101(1):68 – 77.
18. Burkhoff D., Weisfeldt M.L., *Cardiac function and circulatory control*. In: L. Goldman and I. Andrew (Eds). Goldman-Cecil Medicine, vol. 2, 21th Edition, Elsevier, 2000.
19. Silva A.A., Silva A.F., Ferreira C.B.N.D. *et al.*, *Ecocardiograma Transesofágico Intraoperatório*, In: A. Bagatini, O. C. Pires, M. F. S. Filho *et al.* (Eds). *ETI Ecocardiografia Transesofágica no Intraoperatório*. Sociedade Brasileira de Anestesiologia, 2013.
20. Otto, C.M., *Textbook of Clinical Echocardiography*, 5th ed., Saunders: Elsevier, 2013.

Ajustarea corelării liniare între suprafața și volumul ventriculului stâng

Rezumat: Această lucrare își propune să studieze corelație liniară între suprafața și volumul ventriculului stâng, având în vedere că această cameră cardiacă are trunchiate izabella prolate geometrie, și, de asemenea, își propune să compareți-l cu o corelație între acești doi parametri, deja descrise în literatura de specialitate. Am folosit modele matematice pentru a determina suprafața și volumul ventriculului stâng. Volumele care au fost luate în considerare pentru statutul său real și pentru pacienții cu nu anomalii cardiace, obținute în literatura medicală. Volumele din această cameră au fost obținute în aceeași referință bibliografică. Acestea au fost determinate de MRI și au fost folosite pentru a găsi corelațiile mai realist.

José Sérgio DOMINGUES, Prof. PhD, Federal Institute of Minas Gerais - Campus Formiga, Mathematics department, Address: Rua São Luiz Gonzaga, s/nº, São Luiz, Formiga - MG, Brazil, 35570-000, Email: sergio.domingues@ifmg.edu.br

Marcos DE PAULA VALE, MD, MSc, Nossa Senhora das Dores Hospital, *Chief of Cardiac Surgery*, Address: Av. João Soares Silva, 135 - Penha, Itabira - MG, Brazil, 35900-062, Email: marcosvale@globo.com

Carlos BARREIRA MARTINEZ, Prof. PhD, Federal University of Minas Gerais, Department of Hydraulic Engineering and Hydric Resources, Address: Av. Presidente Antônio Carlos, 6627 - Pampulha, Belo Horizonte - MG, Brazil, 31270-901, Email: martinez@cce.ufmg.br