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STUDIES ABOUT BENDING BEAMS WITH COMPLEX LOADS ON ELASTIC ENVIRONMENT BY TRANSFER-MATRIX METHOD (TMM)

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Abstract: *A The analytical calculus for bending beams on elastic environment is very special because the elastic environment sends in all points a reaction force proportional to deformation with same constants of proportionality in all points. This calculus based on the Transfer-Matrix Method (TMM) is presented in this paper. The state vectors associated at origin section and at end section of beam is determined putting the conditions on beam extremities and, after, we can calculate all state vectors for all beam sections. We can solve this problem very easy on an appropriate soft.*

Key words: *complex load, density function, density of charge, state vector, Transfer-Matrix, embedded beam, elastic environment.*

1. INTRODUCTION

The practical applications in industry and life domains of bending beams on elastic environment makes its an important problem. Classical beam calculus is presented in [1] and classical beam on elastic environment calculus for bending solicitation is presented in [3]. In [4] is developed a recursive matrix approach in kinematics and dynamics modeling of parallel robots. We deal an analytical calculus for bending beams on elastic environment, for state vectors associated at origin section and at end beam section, by Transfer-Matrix Method, based of theory of Dirac's and Heaviside's functions and operators [2]. This approach is applied to a beam embedded at its two ends, on elastic environment, with an uniform load on all length.

2. PREMISES OF CALCULUS FOR BENDING BEAM WITH COMPLEX LOAD ON ELASTIC ENVIRONMENT

We can consider, generally, the elastic environment as homogeneous. That is the first work assumption. Second, we can formulate the hypothesis that in all points of the beam, the elastic environment exerts of the beam a reaction force per unit of the beam length [1], [5]:

$$F'(x) = -k \bullet v(x) \qquad (1)$$

$F'(x)$ is a density function proportional to $-v(x)$.

We consider a bending beam charged with a complex load: a concentrated load in the middle and uniform load at all the beam length (Fig. 1.).

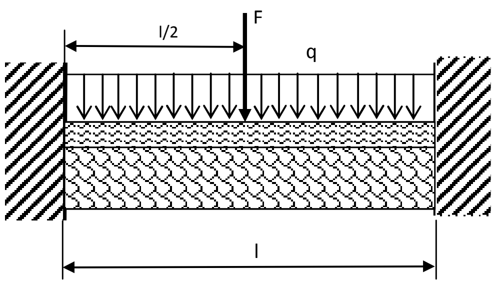

Fig. 1. Beam charged with complex load on elastic environment.

The charge density for a complex load with Dirac's and Heaviside's functions and operators is:

$$q'(x) = -F\delta\left(x - \frac{l}{2}\right) - q(x) - kv(x) \qquad (2)$$

The concentrated load is at $x=l/2$ and the uniform load is of all the beam length.

3. TRANSFER-MATRIX FOR BENDING BEAM ON ELASTIC ENVIRONMENT

In a section x, we have the state vector $\{U\}_x$ with four elements:

$$\{U\}_x = \{M(x),\ T(x),\ \omega(x),\ v(x)\}^{-1} \qquad (3)$$

The section 0, the origin section have the state vector:

$$\{U\}_0 = \{M_0(x),\ T_0(x),\ \omega_0(x),\ v_0(x)\}^{-1} \qquad (4)$$

Matrix relation between the state vector of the section x and the state vector for the origin section 0, $\{U\}_x$, is:

$$\{U\}_x = [T]_x\{U\}_0 + \{U_e\}_x \qquad (5)$$

where: $[T]_x$ is the Transfer-Matrix between the section 0 and the section x and $\{U_e\}_x$ is the vector for the free term at the section x. The matrix $[T]_x$ is (6), with h_i, $i=1,4$ after [1] and [5]:

$$[T]_x = \begin{bmatrix} h_1 & -\frac{h_2+h_3}{2\alpha} & \alpha EI(h_3-h_2) & -2\alpha^2 EIh_4 \\ -\alpha(h_3-h_2) & h_1 & 2\alpha^2 EIh_4 & 2\alpha^3 EI(h_2+h_3) \\ \frac{h_2+h_3}{2\alpha EI} & -\frac{h_4}{2\alpha^2 EI} & h_1 & \alpha(h_3-h_2) \\ \frac{h_4}{2\alpha^2 EI} & -\frac{h_2-h_3}{4\alpha^3 EI} & \frac{h_2+h_3}{2\alpha} & h_1 \end{bmatrix} \qquad (6)$$

and the vector $\{U_e\}_x$ is (7):

$$\{U_e\}_x = \begin{Bmatrix} -\frac{F}{2\alpha}(h_2+h_3)\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)-\frac{q}{2\alpha^2}h_4 Y(x) \\ Fh_1\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)+\frac{q}{2\alpha}(h_2+h_3)Y(x) \\ -\frac{F}{2\alpha^2 EI}h_4\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)-\frac{q}{4\alpha^3 EI}(h_2-h_3)Y(x) \\ -\frac{F}{4\alpha^3 EI}(h_2-h_3)\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)+\frac{q}{4\alpha^4 EI}(h_1-1)Y(x) \end{Bmatrix} \qquad (7)$$

In the right end of beam we have in the section $x=l$ and the matrix relation (5) gives (8):

$$\{U\}_l = [T]_l\{U\}_0 + \{U_e\}_l \qquad (8)$$

We have the state vector of right end function the origin state vector. We can put now the conditions on beam extremities - the conditions of the supports and we obtain a linear system of two equations with two unknowns, very easy to solve. We can calculate all state vectors in all beam sections.

For $x=l$, (7) gives (9):

$$\{U_e\}_l = \begin{Bmatrix} -\frac{F}{2\alpha}[h_2(l)+h_3(l)]-\frac{q}{2\alpha^2}h_4(l) \\ Fh_1(l)+\frac{q}{2\alpha}[h_2(l)+h_3(l)] \\ -\frac{F}{2\alpha^2 EI}h_4(l)-\frac{q}{4\alpha^3 EI}[h_2(l)-h_3(l)] \\ -\frac{F}{4\alpha^3 EI}[h_2(l)-h_3(l)]+\frac{q}{4\alpha^4 EI}[h_1(l)-1] \end{Bmatrix} \qquad (9)$$

Relation (5) with (7) gives expression (10):

$$\begin{Bmatrix} M(x) \\ T(x) \\ \omega(x) \\ v(x) \end{Bmatrix}_x = \begin{bmatrix} h_1 & -\frac{h_2+h_3}{2\alpha} & \alpha EI(h_3-h_2) & -2\alpha^2 EIh_4 \\ -\alpha(h_3-h_2) & h_1 & 2\alpha^2 EIh_4 & 2\alpha^3 EI(h_2+h_3) \\ \frac{h_2+h_3}{2\alpha EI} & -\frac{h_4}{2\alpha^2 EI} & h_1 & \alpha(h_3-h_2) \\ \frac{h_4}{2\alpha^2 EI} & -\frac{h_2-h_3}{4\alpha^3 EI} & \frac{h_2+h_3}{2\alpha} & h_1 \end{bmatrix} \begin{Bmatrix} M(0) \\ T(0) \\ \omega(0) \\ v(0) \end{Bmatrix}_0 + \begin{Bmatrix} -\frac{F}{2\alpha}(h_2+h_3)\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)-\frac{q}{2\alpha^2}h_4 Y(x) \\ Fh_1\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)+\frac{q}{2\alpha}(h_2+h_3)Y(x) \\ -\frac{F}{2\alpha^2 EI}h_4\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)-\frac{q}{4\alpha^3 EI}(h_2-h_3)Y(x) \\ -\frac{F}{4\alpha^3 EI}(h_2-h_3)\left[x-\frac{l}{2}\right]Y\left(x-\frac{l}{2}\right)+\frac{q}{4\alpha^4 EI}(h_1-1)Y(x) \end{Bmatrix} \qquad (10)$$

$$\begin{Bmatrix} M_l \\ T_l \\ \omega_l \\ v_l \end{Bmatrix}_l = \begin{bmatrix} h_1 & -\frac{h_2+h_3}{2\alpha} & \alpha EI(h_3-h_2) & -2\alpha^2 EIh_4 \\ -\alpha(h_3-h_2) & h_1 & 2\alpha^2 EIh_4 & 2\alpha^3 EI(h_2+h_3) \\ \frac{h_2+h_3}{2\alpha EI} & -\frac{h_4}{2\alpha^2 EI} & h_1 & \alpha(h_3-h_2) \\ \frac{h_4}{2\alpha^2 EI} & -\frac{h_2-h_3}{4\alpha^3 EI} & \frac{h_2+h_3}{2\alpha} & h_1 \end{bmatrix} \begin{Bmatrix} M_0 \\ T_0 \\ \omega_0 \\ v_0 \end{Bmatrix}_0 + \begin{Bmatrix} -\frac{F}{2\alpha}[h_2(l)+h_3(l)]-\frac{q}{2\alpha^2}h_4(l) \\ Fh_1(l)+\frac{q}{2\alpha}[h_2(l)+h_3(l)] \\ -\frac{F}{2\alpha^2 EI}h_4(l)-\frac{q}{4\alpha^3 EI}[h_2(l)-h_3(l)] \\ -\frac{F}{4\alpha^3 EI}[h_2(l)-h_3(l)]+\frac{q}{4\alpha^4 EI}[h_1(l)-1] \end{Bmatrix} \qquad (11)$$

For the embedded origin, the end conditions are (12):

$$\begin{cases}\omega(0)=\omega_0=0\\ v(0)=v_0=0\end{cases} \tag{12}$$

For the right embedded end conditions are (13):

$$\begin{cases}\omega(l)=\omega_l=0\\ v(l)=v_l=0\end{cases} \tag{13}$$

With (12) and (13), relation (11) gives (14):

$$\begin{Bmatrix}M_l\\T_l\\0\\0\end{Bmatrix}_l=\begin{bmatrix}h_1 & -\frac{h_2+h_3}{2\alpha} & \alpha EI(h_3-h_2) & -2\alpha^2EIh_4\\ -\alpha(h_3-h_2) & h_1 & 2\alpha^2EIh_4 & 2\alpha^3EI(h_2+h_3)\\ \frac{h_2+h_3}{2\alpha EI} & -\frac{h_4}{2\alpha^2EI} & h_1 & \alpha(h_3-h_2)\\ \frac{h_4}{2\alpha^2EI} & -\frac{h_2-h_3}{4\alpha^3EI} & \frac{h_2+h_3}{2\alpha} & h_1\end{bmatrix}\begin{Bmatrix}M_0\\T_0\\0\\0\end{Bmatrix}_0+\begin{Bmatrix}-\frac{F}{2\alpha}[h_2(l)+h_3(l)]-\frac{q}{2\alpha^2}h_4(l)\\ Fh_1(l)+\frac{q}{2\alpha}[h_2(l)+h_3(l)]\\ -\frac{F}{2\alpha^2EI}h_4(l)-\frac{q}{4\alpha^3EI}[h_2(l)-h_3(l)]\\ -\frac{F}{4\alpha^3EI}[h_2(l)-h_3(l)]+\frac{q}{4\alpha^4EI}[h_1(l)-1]\end{Bmatrix} \tag{14}$$

The linear system is:

$$\begin{cases}h_1M_0-\frac{h_2+h_3}{2\alpha}T_0-M_l=\frac{F}{2\alpha}[h_2(l)+h_3(l)]+\frac{q}{2\alpha^2}h_4(l)\\ -\alpha(h_3-h_2)M_0+h_1T_0-T_l=-Fh_1(l)-\frac{q}{2\alpha}[h_2(l)+h_3(l)]\\ \frac{h_2+h_3}{2\alpha EI}M_0-\frac{h_4}{2\alpha^2EI}T_0=\frac{F}{2\alpha^2EI}h_4(l)-\frac{q}{4\alpha^3EI}[h_2(l)-h_3(l)]\\ \frac{h_4}{2\alpha^2EI}M_0-\frac{h_2-h_3}{4\alpha^3EI}T_0=\frac{F}{2\alpha}[h_2(l)+h_3(l)]+\frac{q}{4\alpha^3EI}[h_1(l)-1]\end{cases} \tag{15}$$

From two last equations can be calculated M_0 and T_0. Solution for this system (15) is (16):

$$\begin{cases}M_0=a_1F+b_1q\\ T_0=a_2F+b_2q\\ M_l=a_3F+b_3q\\ T_l=a_4F+b_4q\end{cases} \tag{16}$$

where a_i, $i=1,4$ and b_i, $i=1,4$ are coefficients obtain after calculus. We return to expression (11) to calculate all state vectors for all beam sections.

4. CONCLUSION

Elastic environment is very necessary in several domains especially in construction for help to alleviate vibration, shocks, for foundations to machinery that produces vibrations during operation, in seismology, in transports, for high speed trains, metro, tram.

This work exposed an analytical calculus for beams, embedded at its two ends, on elastic environment, with complex load, using the Transfer-Matrix Method.

This approach can be applied for beams with different loads, we hope to present in future works other examples. Beam is meshing into elements, for each element is associated a state vector and a state vector for external efforts too. Original contribution is the presented example, a beam on elastic environment with complex load. Algorithm can by program, we hope that will be presented in future works and validate theoretical results with other numerical methods and by experimental tests. Rapidly, we can obtain values for state vector elements of origin and end sections. Now, we can calculate all state vector elements, displacements and stresses in all beam sections on the elastic environment. We hope, in the future, to validate theoretical results with numerical methods and by experimental tests.

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Studii asupra grinzilor solicitate la încovoiere cu sarcini complexe pe mediu elastic prin Metoda Matricelor de Transfer (MMT)

Rezumat: Calculul analitic al grinzilor solicitate la încovoire ce se află pe un mediu elastic este foarte special pentru că mediul elastic dă naştere, în toate punctele, la o forţă de reacţiune proporţională cu deformaţia cu aceleaşi constante de proporţionalitate în toate punctele. *Acest calcul, bazat pe Metoda Matriceor de Transfer (MMT) este prezentat în această lucrare. Vectorii de stare asociaţi secţiunii din origine şi secţiunii de la cealaltă extremitate a grinzii sunt determinaţi punând condiţiile pe extremităţile grinzii şi, apoi, putem calcula toţi vectorii de stare pentru toate secţiunile grinzii. Putem rezolva această problemă foarte uşor cu un soft adecvat.*

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