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ESTABLISHING THE JACOBIAN MATRIX FOR A THREE DEGREES OF FREEDOM SERIAL STRUCTURE

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Abstract: *The algorithms used in the mathematical modeling of mechanical robot structures, are used for establishing, on one hand of the homogeneous transformations in the direct geometry modeling, and on the other hand to determine the Jacobian matrix as well as the kinematics equations expressed by exponential functions. In this paper, is presented on the basis of a few formulations regarding the most important differential matrices in the kinematics of robots, known as the Jacobian matrix, or the matrix of partial derivatives, and in robotics as the velocity transfer matrix.*
Key words: *Jacobian Matrix, algorithm, serial structure, kinematics, robot, matrix exponentials.*

1. INTRODUCTION

The matrix transfer equations in a kinematic chain, for a mechanical structure, can be expressed using matrix exponential functions. By applying the Matrix Exponential Algorithm in kinematics, it is intended to determine the Jacobian matrix (known as the velocity transfer matrix). In this sense, some of the results obtained by applying the mathematical exponential algorithm in direct geometry as well as the exponential functions in the direct kinematics are used as inputs [1].

In the first part of the paper, there will be presented in accordance to [1]-[5], the general expressions in establishing of Jacobian matrix, for any robot structure, which are essential components of the mathematical modeling in the advanced mechanical mechanics of robot systems. In the second part, based on the considerations, for a serial robot structure, the Jacobian matrix will be determined.

2. ESTABLISHING THE GENERAL EXPRESSION OF JACOBIAN MATRIX

In order to determine the general expression of the Jacobian matrix using the matrix exponentials, according to the literature [3], the following are the main steps to be taken in the mathematical modeling of a robotic system, regardless of its type.

The first step is devoted to establish the matrix exponentials, according to [1], as:

$$\underset{(3x3)}{ME}(V_{i1}) = \exp\left\{\sum_{j=0}^{i-1}\left\{\bar{k}_j^0\times\right\}\cdot q_j\cdot\Delta_j\right\}; \tag{1}$$

$$\underset{(3x6)}{ME}(V_{i2}) = \left[I_3 \quad \Delta_i\cdot\left\{\bar{k}_i^{(0)}\times\right\}\right]; \tag{2}$$

$$\underset{\{6x[9+3\cdot(3-i)]\}}{ME}(V_{i3}) = \left[E_1 \mid E_2 \mid E_3\right]$$

$$\text{where:} \qquad E_1 = \begin{pmatrix} I_3 \\ [0] \end{pmatrix};$$

$$E_2 = \begin{pmatrix} [0] \\ \left\{\begin{matrix}\left[\exp\sum_{m=1-1}^{k-1}\left\{\bar{k}_m^{(0)}\times\right\}\cdot q_m\cdot\delta_m\cdot\Delta_m\right] \\ \delta_m = \{\{0; m=i-1\};\{1; m\geq i\}\}\end{matrix}\right\} \end{pmatrix}; \tag{3}$$

$$E_3 = \begin{pmatrix} [0] \\ \exp\left\{\sum_{k=1}^{n}\left\{\bar{k}_k^{(0)}\times\right\}\cdot q_k\cdot\Delta_k\right\} \end{pmatrix},$$

$$\text{for: } k = i \rightarrow 3$$

where $\Delta_{j,m,i} = \{\{1; \mathit{if}\ i=R\};\{0;\ \mathit{if}\ i=T\}\}$, is an operator which highlights the type of joint.

Based on the matrices previously presented in an exponential form, results:

$$\underset{(6x6)}{ME}(J_{i1})=\begin{bmatrix} ME\{V_{i1}\} & [0] \\ [0] & ME\{V_{i1}\} \end{bmatrix} \quad (4)$$

$$\underset{(6x9)}{ME}(J_{i2})=\begin{bmatrix} ME\{V_{i2}\} & [0] \\ [0] & I_3 \end{bmatrix}; \quad (5)$$

$$\underset{\{9x[12+3\cdot(3-i)]\}}{ME}(J_{i3})=\begin{bmatrix} ME\{V_{i3}\} & [0] \\ [0] & I_3 \end{bmatrix} \quad (6)$$

The previous three matrix, (4)-(6), are included in an expression that can be written as:

$$\underset{\{6x[12+3\cdot(3-i)]\}}{ME}\left({}^0J_i\right)=ME\{J_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{J_{i3}\} \quad (7)$$

representing the exponential of the Jacobian matrix relative to the $\{0\}$ reference system.

Further there is determined the column vector $M_{iv\omega}$ whose components are:

$$\underset{\{[12+3\cdot(3-i)]\times 1\}}{ME_{1v\omega}}\left({}^0J_i\right)=\begin{bmatrix} \bar{v}_i^{(0)T} \\ \left[\bar{b}_k;\ k=i\to 3\right]^T \\ \bar{p}_3^{(0)T} \\ \Delta_i\cdot\bar{k}_i^{(0)T} \end{bmatrix}. \quad (8)$$

The column (i) of the Jacobian matrix can be established as:

$$\underset{(6X3)}{{}^0J}(\bar{\theta})\equiv ME\{J_{i1}\}\cdot ME\{J_{i2}\}\cdot ME\{J_{i3}\}\cdot M_{iv\omega}= \\ =ME\left\{{}^0J_i\right\}\cdot M_{iv\omega}. \quad (9)$$

or:

$$\underset{(6X3)}{{}^0J}(\bar{\theta})\equiv\left[\underset{(6X3)}{{}^0J_i}\right]= \\ =\begin{bmatrix} ME\{V_{i1}\} & [0] \\ [0] & ME\{V_{i1}\} \end{bmatrix}\cdot\begin{bmatrix} ME\{V_{i2}\}\cdot ME\{V_{i3}\} & [0] \\ [0] & I_3 \end{bmatrix}\cdot\begin{Bmatrix} \bar{v}_i^{(0)} \\ \left[\bar{b}_k, k=i\to 3\right]^T \\ \bar{p}_3^{(0)} \\ \Delta_i\cdot\bar{k}_i^{(0)} \end{Bmatrix}. \quad (10)$$

In order to determine the (i) column of the Jacobian matrix and obviously a general expression based on the application of the matrix exponentials presented above, it is found that there are several expression variants thereof. One of these takes into account the two transfer matrices, so according to [1] and [3], the previous Jacobian matrix is included in the following generalized expression

$$\left\{\begin{array}{l} \underset{(6\times 3)}{{}^0J}(\bar{\theta})=\left[{}^0J_{iv}^T(\bar{\theta})\quad {}^0J_{i\Omega}^T(\bar{\theta}),\ i=1\to n\right]^T\equiv \\ \equiv\left[\begin{pmatrix} J_{1v} \\ J_{1\Omega}\end{pmatrix}\ \begin{pmatrix} J_{2v} \\ J_{2\Omega}\end{pmatrix}\ \begin{pmatrix} J_{3v} \\ J_{3\Omega}\end{pmatrix}\right]=\left[{}^0J_1\quad {}^0J_2\quad {}^0J_3\right] \end{array}\right\} \quad (11)$$

The previous expression of the Jacobian matrix is used to determine the direct kinematics equations (linear and angular velocities). They complement the equation table, which describes the tool's central point movement of any type of mechanical structure, in order to determine the direct kinematics equations (characteristic effector operating velocities).

3. ESTABLISHING OF THE JACOBIAN MATRIX FOR A SERIAL STRUCTURE

To exemplify the notions presented in the previous paragraphs, it is considered a serial robot structure, which consists of three couplings, one of which is a rotation and two of the translations [6], as resulting from the Fig. 1. As shown in Fig. 1, the three degrees of freedom of the 2TR serial robot consist of a translation along the axis O_{z_0}, one along the axis O_{x_0}, as well as the rotation of the effector around the O_{x_0} axis.

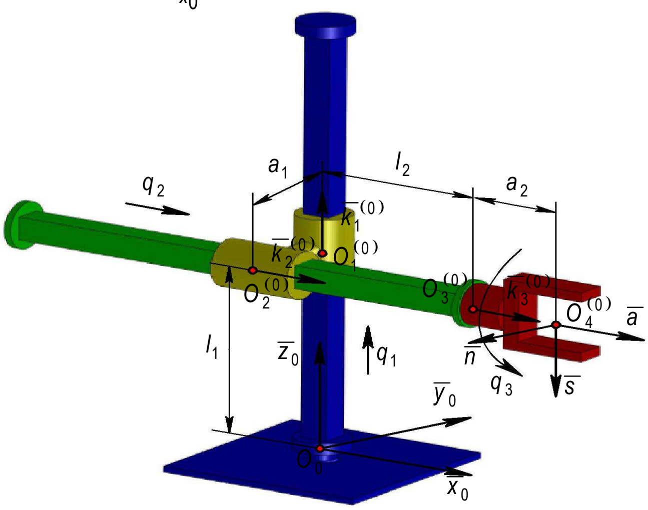

Fig. 1 The TTR serial structure

According to previous considerations, for $i=1$, the following matrix expressions will be determined:

$$\underset{(3x3)}{ME}(V_{11})=\left\{\exp\left\{\sum_{j=0}^{1-1}\left\{\bar{k}_1^{(0)}\times\right\}q_1\cdot\Delta_1\right\}\equiv e^{[0]}\right\}\equiv I_3 \quad (12)$$

$$\underset{(6X6)}{ME}\{J_{11}\}=\left\{\begin{bmatrix} ME\{V_{11}\} & [0] \\ [0] & ME\{V_{11}\}\end{bmatrix}\right\} = \begin{bmatrix} I_3 & [0] \\ [0] & I_3\end{bmatrix}; \quad (13)$$

$$\underset{(3X6)}{ME}(V_{12})=\left[I_3 \quad \Delta_1\cdot\left\{\bar{k}_1^{(0)}\times\right\}\right]= =\left[I_3 \quad [0_3]\right]; \quad (14)$$

$$\underset{(3X9)}{ME}(J_{12})=\left[\begin{array}{c|c} ME\{V_{12}\} & [0] \\ \hline [0] & I_3\end{array}\right]= \left[\begin{array}{c|c} I_3 & \underset{3x6}{[0]} \\ \hline \underset{3x6}{[0]} & I_3\end{array}\right]; \quad (15)$$

$$\underset{(6X15)}{ME}(V_{13})=\left[\begin{array}{c|ccccc} & & & \underset{3x9}{[0]} & & \\ \hline & & & 1 & 0 & 0 \\ I_6 & I_3 & I_3 & 0 & cq_3 & -sq_3 \\ & & & 0 & sq_3 & cq_3 \end{array}\right]; \quad (16)$$

$$\underset{(9X18)}{ME}\{J_{13}\}=\begin{bmatrix} ME\{V_{13}\} & [0] \\ [0] & I_3\end{bmatrix}=$$

$$=\left[\begin{array}{c|ccc|ccc|c} I_3 & & \underset{3x9}{[0]} & & & \underset{3x3}{[0]} & & \underset{3x3}{[0]} \\ \hline & & & & 1 & 0 & 0 & \\ \underset{3x3}{[0]} & I_3 & I_3 & I_3 & 0 & cq_3 & -sq_3 & \underset{3x3}{[0]} \\ & & & & 0 & sq_3 & cq_3 & \\ \hline \underset{3x3}{[0]} & & \underset{3x9}{[0]} & & & \underset{3x3}{[0]} & & I_3 \end{array}\right]; \quad (17)$$

The expressions obtained above, are used to determine the first column of the Jacobian matrix. By performing the product between the expressions (13), (15), and (17), there is obtained the first column of the Jacobian matrix as follows [6]:

$$\underset{(6X18)}{ME}\left\{{}^0J_1\right\}=ME\{J_{11}\}\cdot ME\{J_{12}\}\cdot ME\{J_{13}\}= =\begin{bmatrix} I_3 & \underset{3x15}{[0]} \\ \underset{3x15}{[0]} & I_3\end{bmatrix}. \quad (18)$$

The column vector corresponding to the linear component of the Jacobian matrix is determined by using the expression (8). Hence, there is obtained the following column vector:

$$\underset{\{18\times1\}}{M_{1v\omega}}=\left[\bar{v}_1^{(0)T} \quad \left[\bar{b}_k\,;k=1\to3\right]^T \quad \bar{p}_3^{(0)T} \quad \Delta_1\cdot\bar{k}_1^{(0)T}\right]^T;$$

$$where: \quad \bar{v}_1^{(0)}=[0 \;\; 0 \;\; 1]^T,$$

$$\bar{b}_1^T=\begin{bmatrix}0\\0\\q_1\end{bmatrix}^T; \; \bar{b}_2^T=\begin{bmatrix}q_2\\0\\0\end{bmatrix}^T; \; \bar{b}_3^T=\begin{bmatrix}0\\ l_1\cdot sq_3+a_1\cdot(cq_3-1)\\ a_1\cdot sq_3-l_1(cq_3-1)\end{bmatrix}^T, \quad (19)$$

$$\bar{p}_3^{(0)T}=[l_2 \;\; -a_1 \;\; l_1]; \quad \Delta_1\cdot\bar{k}_1^{(0)T}=[0_3].$$

By performing the matrix product of the matrix functions, (18) and (19), the first Jacobian matrix column, denoted by $\underset{(6\times1)}{{}^0J_1}$, is obtained, as follows:

$$\underset{(6\times1)}{{}^0J_1}=\left[{}^0J_{1v}^T \quad {}^0J_{1\Omega}^T\right]^T=\left\{ME\left\{{}^0J_1\right\}\cdot M_{1v\omega}\right\}=\begin{bmatrix}0\\0\\1\\0\\0\\0\end{bmatrix} \quad (20)$$

The second step $(i=2)$ is devoted to the calculus of the second column of the Jacobian matrix. Similarly to the first, there will be established the following expressions:

$$\underset{(3x3)}{ME}(V_{21})=\exp\left\{\sum_{j=0}^{1}\left\{\bar{k}_j^{(0)}\times\right\}\cdot q_j\cdot\Delta_j\right\}= =\exp\left\{\left\{\bar{k}_2^{(0)}\times\right\}\cdot q_2\cdot\Delta_2\right\}\equiv\left[\,I_3 \quad [0_3]\,\right]. \quad (21)$$

$$\underset{(6\times6)}{ME}\{J_{21}\}=\begin{bmatrix} ME\{V_{21}\} & [0] \\ [0] & ME\{V_{21}\}\end{bmatrix}\equiv$$

$$\equiv\begin{bmatrix} \exp\left\{\left\{\bar{k}_2^{(0)}\times\right\}\cdot q_2\cdot\Delta_2\right\} & [0] \\ [0] & \exp\left\{\left\{\bar{k}_2^{(0)}\times\right\}\cdot q_2\cdot\Delta_2\right\}\end{bmatrix}= \quad (22)$$

$$=\begin{bmatrix} I_3 & [0] \\ [0] & I_3\end{bmatrix}$$

$$\underset{(3\times6)}{ME}(V_{22})\equiv\left[\begin{matrix} I_3 & \Delta_2\cdot\left\{\bar{k}_2^{(0)}\times\right\}\end{matrix}\right]= ; \quad (23)$$
$$=\left[\begin{matrix} I_3 & [0_3]\end{matrix}\right];$$

$$\underset{(6\times9)}{ME}\{J_{22}\}=\begin{bmatrix} ME\{V_{22}\} & [0] \\ [0] & I_3\end{bmatrix}=$$
$$=\begin{bmatrix} I_3 & \underset{3x6}{[0]} \\ \underset{3x6}{[0]} & I_3\end{bmatrix}; \quad (24)$$

$$\underset{(6x12)}{ME}(V_{23})=\left[\begin{array}{c|cc} I_3 & \underset{3x6}{[0]} & \\ \hline [0] & I_3 & {}_3^2[R]\end{array}\right] \quad (25)$$

$$\underset{(9\times15)}{ME}\{J_{23}\}=\begin{bmatrix} ME\{V_{23}\} & [0] \\ [0] & I_3\end{bmatrix}=$$
$$=\left[\begin{array}{c|cc|c} I_6 & \underset{3x3}{[0]} & \underset{3x3}{[0]} & \underset{6x3}{[0]} \\ & I_3 & {}_3^2[R] & \\ \hline \underset{3x6}{[0]} & \underset{3x3}{[0]} & \underset{3x3}{[0]} & I_3\end{array}\right] .(26)$$

By performing the product of (22), (24) and (26) is obtained the transfer matrix included in the second column of the Jacobian matrix. The results are presented below in the form:

$$\underset{(6\times15)}{ME}\left\{{}^0J_2\right\}=ME\{J_{21}\}\cdot ME\{J_{22}\}\cdot ME\{J_{23}\}=$$
$$=\left[\begin{array}{c|c} I_3 & \underset{3x12}{[0]} \\ \hline \underset{3x12}{[0]} & I_3\end{array}\right]; \quad . (27)$$

The column vector corresponding to the linear component of the Jacobian matrix is determined using the expression (8). By performing the calculation, the column vector is obtained as:

$$\underset{\{15\times1\}}{M_{2v\omega}}=\left[\begin{matrix} \bar{v}_2^{(0)T}\left[\bar{b}_k\ ; k=2\rightarrow3\right]^T\bar{p}_3^{(0)T}\ \Delta_2\cdot\bar{k}_2^{(0)T}\end{matrix}\right]^T$$

$$where:\quad \bar{v}_2^{(0)T}=\begin{bmatrix}1\\0\\0\end{bmatrix}^T;\ \bar{b}_2^T=\begin{bmatrix}q_2\\0\\0\end{bmatrix}^T;$$

$$\bar{b}_3^T=\begin{bmatrix}0\\ l_1\cdot sq_3+a_1\cdot(cq_3-1)\\ a_1\cdot sq_3-l_1(cq_3-1)\end{bmatrix}^T; \quad (28)$$

$$\bar{p}_3^{(0)T}=[l_2-a_1\ \ l_1]^T;\quad \Delta_2\cdot\bar{k}_2^{(0)T}=[000]^T$$

By making the matrix product of the last two matrix functions, presented above (27) and (28) the second column of the Jacobian matrix is obtained [6], the expression of which is:

$$\underset{(6\times1)}{{}^0J_2}=\left[\begin{matrix}{}^0J_{2v}^T & {}^0J_{2\Omega}^T\end{matrix}\right]^T=ME\left\{{}^0J_2\right\}\cdot M_{2v\omega}=\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix};(29)$$

For $(i=3)$ is determined the last column of the Jacobian matrix. Similar to the first two steps, are determined the matrices and exponential matrix functions according to the algorithm, as follows:

$$\underset{(3\times3)}{ME}(V_{31})=\left\{\exp\left\{\sum_{j=0}^{2}\left\{\bar{k}_j^{(0)}\times\right\}\ q_j\cdot\Delta_j\right\}=e^{[0]}\right\}\equiv I_3;(30)$$

$$\underset{(6\times6)}{ME}\{J_{31}\}=\begin{bmatrix} ME\{V_{31}\} & [0] \\ [0] & ME\{V_{31}\}\end{bmatrix}= ; \quad (31)$$
$$=\left[\begin{array}{c|c} I_3 & \underset{3x3}{[0]} \\ \hline \underset{3x3}{[0]} & I_3\end{array}\right];$$

$$\underset{(3\times6)}{ME}(V_{32}) \equiv \begin{bmatrix} I_3 & \{\bar{k}_3^{(0)}\times\} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}; \quad (32)$$

$$\underset{(6\times9)}{ME}\{J_{32}\} \equiv \begin{bmatrix} ME\{V_{32}\} & [0] \\ [0] & I_3 \end{bmatrix} = \begin{bmatrix} I_3 & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{matrix} & \underset{3x3}{[0]} \\ \multicolumn{2}{c}{\underset{3x6}{[0]}} & I_3 \end{bmatrix}; \quad (33)$$

$$\underset{(6x9)}{ME}(V_{33}) = \begin{bmatrix} I_3 & \underset{3x3}{[0]} & \underset{3x3}{[0]} \\ \underset{3x3}{[0]} & I_3 & \begin{matrix} 1 & 0 & 0 \\ 0 & cq_3 & -sq_3 \\ 0 & sq_3 & cq_3 \end{matrix} \end{bmatrix}; \quad (34)$$

$$\underset{(9\times12)}{ME}\{J_{33}\} = \begin{bmatrix} ME\{V_{33}\} & [0] \\ [0] & I_3 \end{bmatrix} \equiv \begin{bmatrix} I_6 & \underset{3x3}{[0]} & \\ \underset{3x6}{[0]} & \begin{matrix} 1 & 0 & 0 \\ 0 & cq_3 & -sq_3 \\ 0 & sq_3 & cq_3 \end{matrix} & \underset{6x3}{[0]} \\ \underset{3x6}{[0]} & \underset{3x3}{[0]} & I_3 \end{bmatrix}; \quad (35)$$

By making the product of (31), (33) and (35) the transfer matrix included in the third column of the Jacobian matrix is obtained. The results are presented below in the following form:

$$\underset{(6\times12)}{ME}\{{}^0J_3\} = \begin{bmatrix} I_3 & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{matrix} & \begin{matrix} 1 & 0 & 0 \\ 0 & -sq_3 & -cq_3 \\ 0 & cq_3 & -sq_3 \end{matrix} & \underset{3x3}{[0]} \\ \underset{3x3}{[0]} & \underset{3x3}{[0]} & \underset{3x3}{[0]} & I_3 \end{bmatrix}; \quad (36)$$

The column vector corresponding to the linear component of the Jacobian matrix is determined using the same expression (8). Performing the calculation, the column vector is obtained as:

$$\underset{\{12\times1\}}{M_{3v\omega}} = \begin{bmatrix} \bar{v}_3^{(0)T} & [\bar{b}_k ; k=3]^T & \bar{p}_3^{(0)T} & \Delta_3\cdot\bar{k}_3^{(0)T} \end{bmatrix}^T;$$

$$where: \bar{v}_3^{(0)T} = \begin{bmatrix} 0 \\ l_1 \\ a_1 \end{bmatrix}^T; \bar{b}_3^T = \begin{bmatrix} 0 \\ l_1\cdot sq_3 + a_1\cdot(cq_3-1) \\ a_1\cdot sq_3 - l_1(cq_3-1) \end{bmatrix}^T; \quad (37)$$

$$\bar{p}_3^{(0)T} = \begin{bmatrix} l_2 \\ -a_1 \\ l_1 \end{bmatrix}^T; \ \Delta_3\cdot\bar{k}_3^{(0)T} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T.$$

Making the matrix product between the last two matrix functions, presented above (36) and (37), the third column of the Jacobian matrix is obtained, the expression of which is as follows:

$$\underset{(6x1)}{{}^0J_3} = \begin{bmatrix} {}^0J_{3v}^T & {}^0J_{3\Omega}^T \end{bmatrix}^T = ME\{{}^0J_3\}\cdot M_{3v\omega} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (38)$$

Thus, by replacing the three column matrices with (20), (29) and (38) in the definition expression (11), is obtained:

$$\underset{(6\times3)}{{}^0J(\bar{\theta})} = \begin{bmatrix} {}^0J_1 & {}^0J_2 & {}^0J_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ .... & .... & .... \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (39)$$

and representing the final form of the Jacobian matrix for the serial robot TTR taken into analysis, [6].

4. CONCLUSION

In the paper, there have been established the Jacobian matrix expression, used to determine direct kinematics equations (linear and angular velocities), for a serial robot structure. The expressions obtained, are completing the equations, which describing the movement of the 2TR robot's final effector in Cartesian space. The mathematical model described in this paper, devoted to determine the Jacobian matrix, is also known as the matrix of partial derivatives of position equations, or the velocity transfer matrix. It establishes the connection between the generalized velocities and the operational velocities by which the kinematics equations are defined.

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Stabilirea matricei Jacobiene pentru o structură serială cu trei grade de libertate

Algoritmii utilizați în modelarea matematică a structurilor robotului mecanic sunt utilizați pentru a stabili, pe de o parte, transformările omogene în modelarea geometrică directă și, pe de altă parte, pentru a determina matricea Jacobiană, precum și ecuațiile cinematice exprimate prin funcții exponențiale. În această lucrare, este prezentată pe baza câtorva formulări referitoare la cele mai importante matrici diferențiale din cinematica roboților, matricea Jacobiană sau matricea derivatelor parțiale, cunoscută, de asemenea în robotică ca matricea de transfer a vitezelor.

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