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## MECHANICAL MODELING 4-EPPC CORRESPONDING TO LEGS, PELVIS AND BODY, SUPPORTED ON A RIGID SUPPORT, WITH ONE LEG ON EXCITATION

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**Abstract:** This article discusses a mechanical model of human body to treat with four degrees of mobility, it is apparent the simulation model 4-movement variation EPPC with representations as well as the phase diagram for each mass. This pattern is characteristic of performance athletes and persons performing various physical exercises in wards of fitness equipment with vibrators.

**Key words:** vibration, biomechanical model, displacement model, amplitude, vibration, elastic and dumping constants

### 1. INTRODUCTION

In nature there are no isolated phenomena. Always consistent with the phenomenon under investigation is carried out a number of substantial phenomenon with which it has relationships and mutual influences.

Paradoxical is that to model accurately a phenomenon is necessary to know how much more comprehensive, which reduces the need to investigate it. On the other hand, the model must be appropriate for the intended use: an overly complicated model – which aims to take into account all the aspects of the phenomenon – can become costly, unwieldy or even inoperative, and a simplistic or too summary, may not be valid, due to the neglect of some important aspects of the phenomenon.

Any body performs a number of functions in the biochemical, biomechanical biophysics, etc. Some of them may be purely mechanical, others can be mixed. So, for example, the medical centre has the task of bearing the mass of the body and move with a certain speed. However, members do not only mechanical movements but, they take place and the physical and biochemical processes that provide the energy for movement.

Parts of the body and separate bodies have form and corresponding to the dimensions for there accomplished functions. Mecano-mathematical

models of biological phenomena are up on the basis of general principles of the mechanics and physics, and for most of the time are filled with models in other sciences and fields of the art [3].

### 2. MECHANICAL MODEL WITH FOUR MOBILITY DEGREES

Model 4-EPPC (the corresponding with four equations model of legs, pelvisului and the human body), the operator shall be deemed to be in the position of standing on one foot resting on a vibrating platform and the other foot is raised in front of the 90°. Take into account the rotation of the leg  $\theta_4$ , axis  $Oy$ . This pattern is characteristic of athletes' performance or people who do different exercises in fitness, by means of vibrating devices. Mechanical system that simulates the behavior of the human body is segmented into four distinct parts, each of which has mass, spring and damper, whose values and expressions are presented in the following issues:

$$\omega = 2\pi * f = 2 * 3,14 * 23.437 = 147.18 \text{ rad/s}$$

$f = 23.437 \text{ Hz}$  - vibrating frequency (value taken from measurements)

$$L - \text{foot length} = 49\text{cm} = 0.49\text{m}$$

$$g = \text{gravitational acceleration} = 9.81\text{m/s}^2$$

$$\theta_4 - \text{the angle variation of } 90^0$$

$J_D$  – the axial mechanical moment of inertia of foot high, reported to the rotation axis and is considered to be horizontal in coxo-femural articulation.

$$J_D = \frac{m_4 \cdot L^2}{3} = \frac{7.74 \cdot 0.49^2}{3} = \frac{7.74 \cdot 0.24}{3} = \frac{1.8576}{3} = 0.6192 \text{Kgm}^2 \quad (1)$$

The excitations are presented below:

$c_1\dot{u} + k_1u$  – represents the value of the input signal on the basis consists of two harmonic functions;

$k_1u$  și  $k_4u$  - is the elastic force transmitted from sole to leg due to excitation;

$c_1u$  și  $c_4u$  – represents the damping force transmitted from sole to leg due to excitation;

$$k_1u = k_1u_0 \sin \omega t = 25500 \cdot 6 \cdot 10^{-5} \sin \omega t = 15.3 \sin \omega t \quad (2)$$

$$c_1\dot{u} = c_1u_0\omega \cos \omega t = 378 \cdot 6 \cdot 10^{-5} \cdot 147.18 \cos \omega t = 3.38 \cos \omega t \quad (3)$$

### 2.1 System of differential equations

The mathematical model for mechanic model in Figure 1 is write the equations of equilibrium for each of the four masses, of mechanical model and the change of angle with  $90^0$  ( $m_1, m_2, m_3, m_4$  and  $\theta_4$ ). The body shall be considered in the standing position, on a vibrating platform with a single-leg and the other foot raised to  $90^0$  [2].

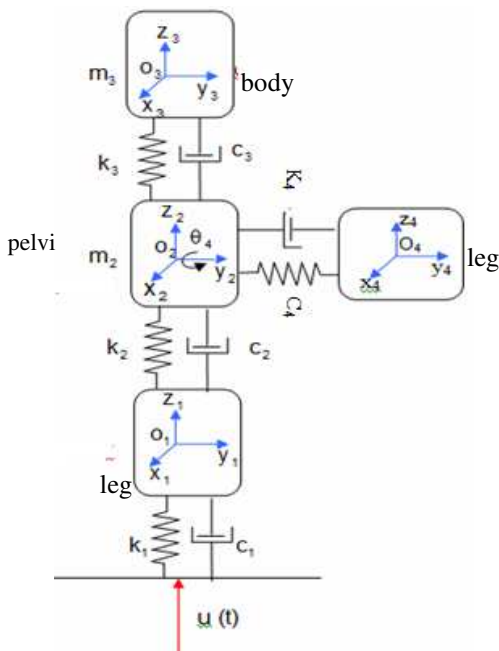


Fig.1 Simplified mechanical Model of the legs, pelvis and human body

Tabelul 1

Characteristics of coefficients for the model 4- EPPC [1]

Characteristics		Unit	Value
Denomination	Symbol	Measure	
Elastic constant	$k_1$	N/m	25500
Elastic constant	$k_2$	N/m	53640
Elastic constant	$k_3$	N/m	8941
Elastic constant of transversal direction	$k_{4t}$	N/m	6200
Dumping constant	$c_1$	Ns/m	378
Dumping constant	$c_2$	Ns/m	3651
Dumping constant	$c_3$	Ns/m	298
Dumping constant	$c_{4t}$	Ns/m	397
Mass of vertical leg	$m_1$	Kg	7.74
Mass of pelvis	$m_2$	kg	16.17
Mass of human body	$m_3$	kg	23.31
Mass of horizontal leg	$m_{4t}$	kg	7.74

Differential equations of mechanical system are

$$\begin{cases} m_1 \ddot{z}_1 + c_1(\dot{z}_1 - \dot{u}) + k_1(z_1 - u) - c_2(\dot{z}_2 - \dot{z}_1) - k_2(z_2 - z_1) = 0 \\ m_2 \ddot{z}_2 + c_2(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_1) - c_3(\dot{z}_3 - \dot{z}_2) - k_3(z_3 - z_2) - c_{4t}(\dot{z}_4 - \dot{z}_2) - k_{4t}(z_4 - z_2) = 0 \\ m_3 \ddot{z}_3 + c_3(\dot{z}_3 - \dot{z}_2) + k_3(z_3 - z_2) = 0 \\ m_4 \ddot{z}_4 + c_{4t}(\dot{z}_4 - \dot{z}_2) + k_{4t}(z_4 - z_2) = 0 \\ J_{\Delta\theta_4} \ddot{\theta}_4 = -C_{4t} \dot{z}_4 - k_{4t} z_4 + L / 2m_4 g \end{cases}$$

But  $z_4 = \frac{1}{2} \sin \theta_4 \quad (4)$

Throu derivation of this expression is obtained:

$$\dot{z}_4 = \frac{1}{2} \dot{\theta}_4 \cos \theta_4 \quad (5)$$

Transverse damping coefficients of elasticity transverse constants can take values between:

$$\begin{aligned} c_{4t} &= \left( \frac{1}{4} \dots \frac{1}{10} \right) c_4 \\ k_{4t} &= \left( \frac{1}{2} \dots \frac{1}{5} \right) k_4 \end{aligned} \quad (6)$$

It details the 2nd derivative of each generalized coordinate system to integrate with the Runge-Kutta order 4 1/2 with using MATLAB SIMULINK.

$$\begin{aligned} \ddot{z}_1 &= \frac{1}{m_1} [-c_1\dot{z}_1 + c_1\dot{u} - k_1z_1 + k_1u + c_2\dot{z}_2 - c_2\dot{z}_1 + k_2z_2 - k_2z_1] \\ \ddot{z}_2 &= \frac{1}{m_2} [-c_2\dot{z}_2 + c_2\dot{z}_1 - k_2z_2 + k_2z_1 + c_3\dot{z}_3 - c_3\dot{z}_2 + k_3z_3 -] \\ \ddot{z}_3 &= \frac{1}{m_3} [-c_3\dot{z}_3 + c_3\dot{z}_2 - k_3z_3 + k_3z_2] \\ \ddot{z}_4 &= \frac{1}{m_4} [-c_{4t}\dot{z}_4 + c_{4t}\dot{z}_2 - k_{4t}z_4 + k_{4t}z_2] \\ \ddot{\theta}_4 &= -\frac{1}{J_{\Delta\theta_4}} [-c_{4t}\dot{z}_4 - k_{4t}z_4 + L/2m_4g] \end{aligned} \quad (7)$$

or

$$\begin{cases} \ddot{z}_1 = \frac{1}{m_1} [-\dot{z}_1(c_2 + c_1) + c_1\dot{u} - z_1(k_1 + k_2) + k_1u + c_2\dot{z}_2 + k_2z_2] \\ \ddot{z}_2 = \frac{1}{m_2} [-\dot{z}_2(c_2 + c_3) + c_2\dot{z}_1 - z_2(k_2 + k_3) + k_2z_1 + c_3\dot{z}_3 + k_3z_3 + c_{4t}\dot{z}_4 + c_{4t}\dot{z}_2 + k_{4t}z_4 - k_{4t}z_2] \\ \ddot{z}_3 = \frac{1}{m_3} [-c_3\dot{z}_3 + c_3\dot{z}_2 - k_3z_3 + k_3z_2] \\ \ddot{z}_4 = \frac{1}{m_4} [-c_{4t}\dot{z}_4 + c_{4t}\dot{z}_2 - k_{4t}z_4 + k_{4t}z_2] \\ \ddot{\theta}_4 = \frac{1}{J_{\Delta}} [-c_{4t}\dot{z}_4 - k_{4t}z_4 + L/2m_4g] \end{cases}$$

(8)

**2.1.1 Program 4-EPPC corresponding of 4-EPPC human system model**

Program for calculating the movements of the system is shown in Figure 7. Time integration was considered to be 10 seconds. The integration was completed with the Runge-Kutta of order four and a half using Simulink software package.

Each equation is modeled separately, Figure 2 for  $z_1$  the generalized coordinate, Figure 3 for  $z_2$  the generalized coordinate, Figure 4 for  $z_3$  the generalized coordinate, Figure 5 for  $z_4$  the generalized coordinate, Figure 6 for  $\theta_4$  the generalized coordinate. But they interact; therefore there are connections between the modules.

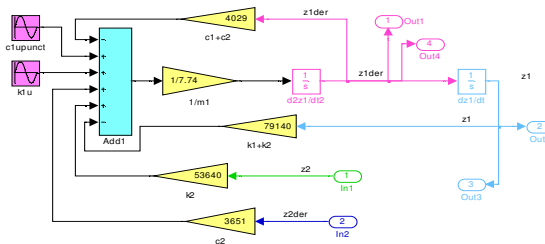


Fig. 2 Subsystem  $z_1$  of program 4-EPPC

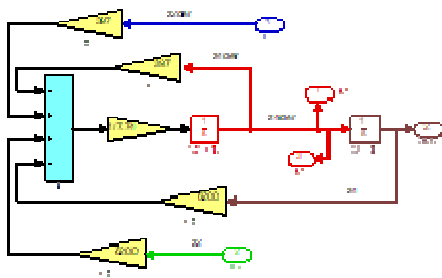


Fig. 3 Subsystem  $z_2$  of program 4-EPPC

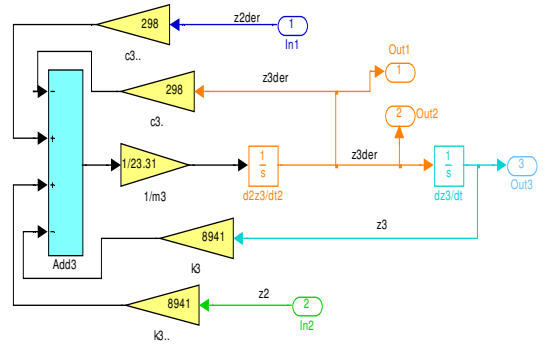


Fig. 4 Subsystem  $z_3$  of program 4-EPPC

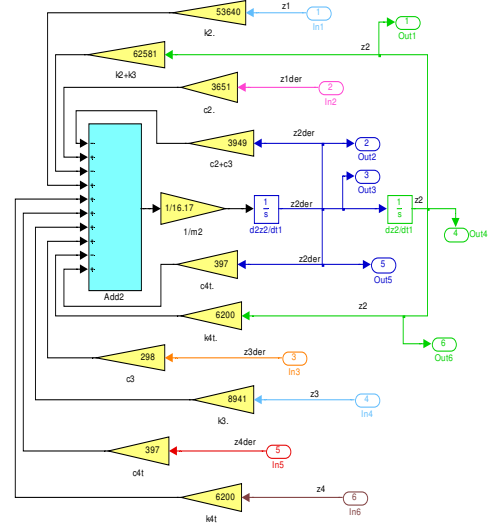


Fig. 5. Subsystem  $z_4$  of program 4-EPPC

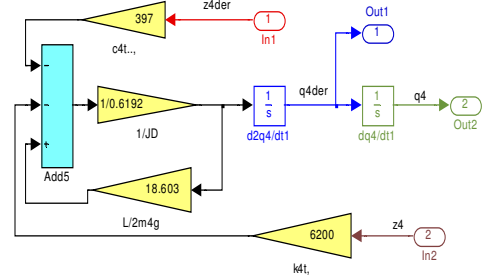


Fig. 6 Subsystem  $\theta_4$  of program 4-EPPC

**2.2 The solution for the integration of the mathematical model of 4-EPPC human operator**

The results of the generalized coordinates, integration of the system 4-EPPC (displacement and stability), are obtained as graphics and are played in the figures below (Fig. 8 and Fig. 9).

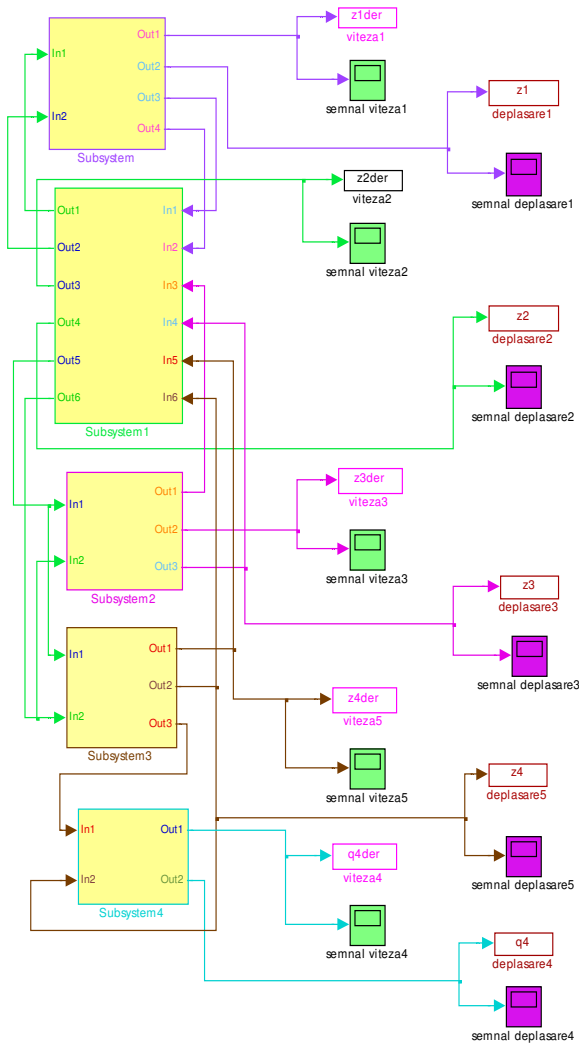


Fig. 7. Program 4-EPPC

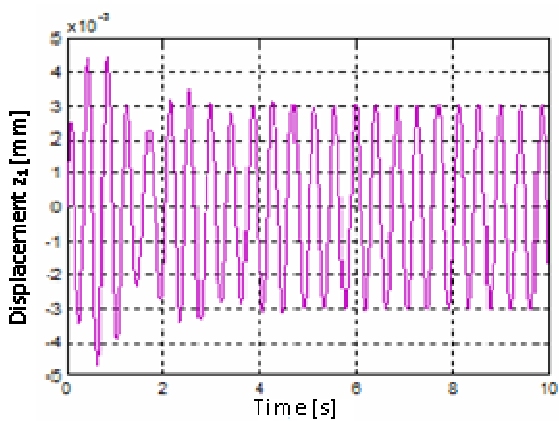


Fig.8 Representation of the movement variation law for  $z_1$  coordinate of 4-EPPC model

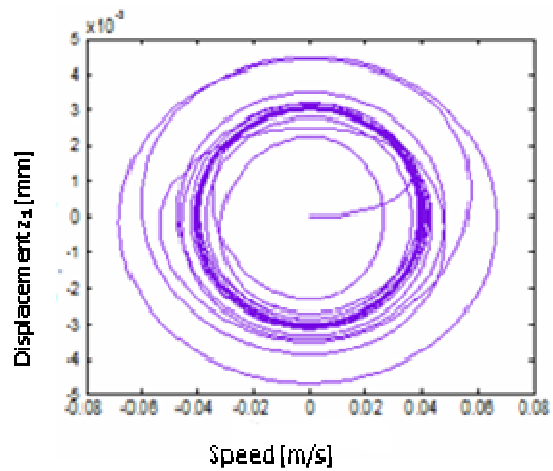


Fig. 9. Stability diagram for coordinate  $z_1$  of a model 4-EPPC

In the regime movement for model biomechanic 4-EPPC at a frequency of 23.437 Hz can be seen that it is stable, so it does not destroy the human body on which acts the maximum displacement,  $4 \times 10^{-3}$  mm [2].

Figure 10 corresponds to the variance of the coordinate  $z_2$  in time, and is represented in Figure 11 the stability of human system for  $z_2$ , 4-EPPC where der is the velocity and displacement is  $z_2$ .

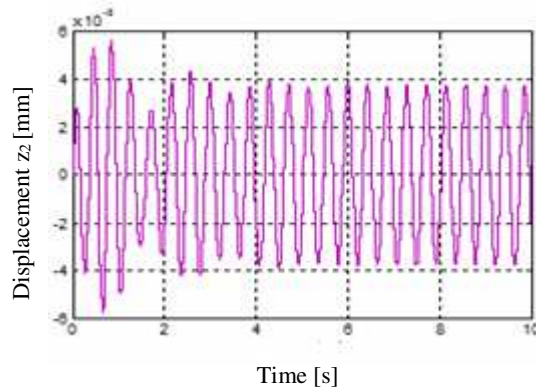


Fig.10 Representation of the movement variation law for  $z_2$  coordinate of 4-EPPC model

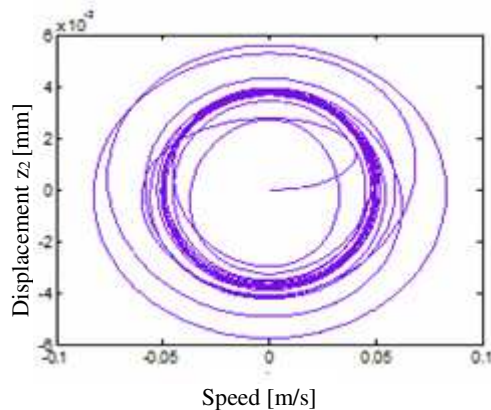


Fig. 11. Stability diagram for coordinate  $z_2$  of a model

As can be seen to coordinate the movement of the  $z_2$  is stable, with a maximum  $5, 5 \times 10^{-3}$  mm. Figure 12 suit variation movement in time, and is represented in Figure 13 human system 4-EPPC for  $z_3$ .

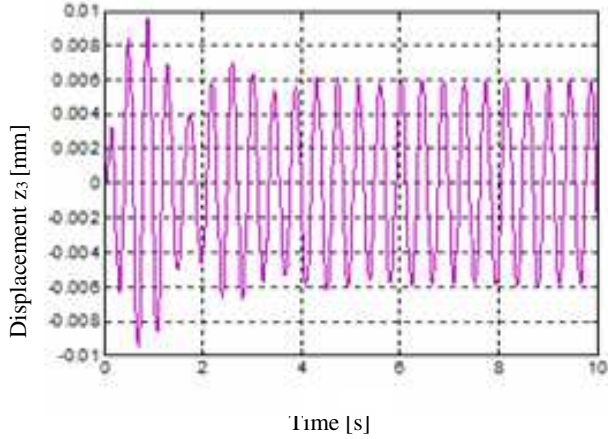


Fig. 12 Representation of the movement variation law for  $Z_3$  coordinate of 4-EPPC model

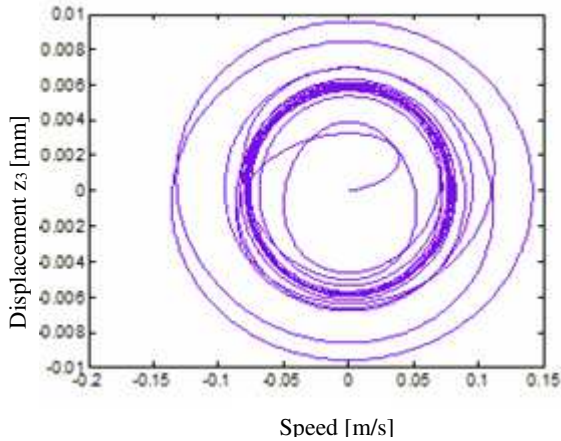


Fig. 13 Stability diagram for coordinate  $z_3$  of a model 4-EPPC

The maximum value of the displacement for the  $z_3$  coordinate is 0.01 mm, and for this variable the movement is stable.

Figure 14 corresponds to the variation in the time of the movement's  $z_4$ , and Figure 15 is the representation of the  $z_4$  in system human noted 4-EPPC.

Regime shift in the movement for model biomechanic 4-EPPC at a frequency of 23.437 Hz can be seen that the movement is stable, which is due to that it does not destroy the body on which is the action with the maximum displacement  $6 \times 10^{-3}$  mm.

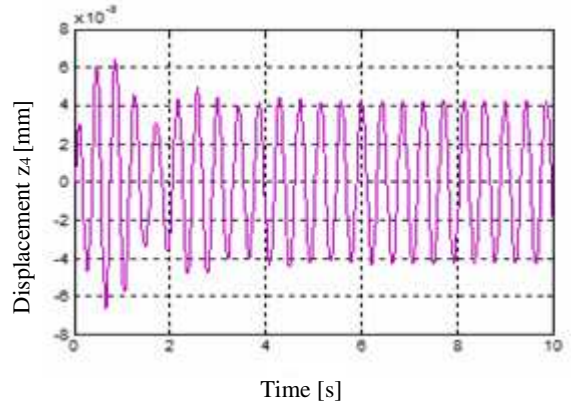


Fig. 14 Representation of the movement variation law for  $Z_4$  coordinate of 4-EPPC model

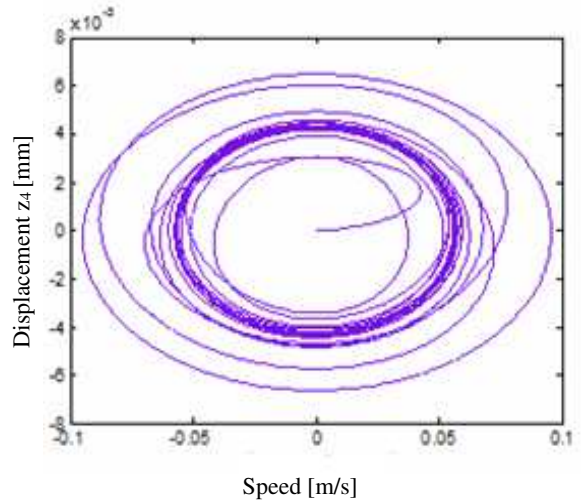


Fig. 15 Stability diagram for coordinate  $z_4$  of a model 4-EPPC

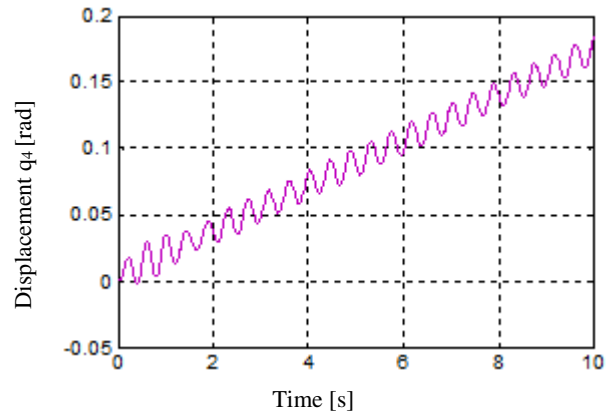


Fig. 16 Representation of the movement variation law for  $\Theta_4$  coordinate of 4-EPPC model

In the case  $\theta_4$  coordinate its amplitude is 0.01 radians, the movement is forced damped and it is stable.

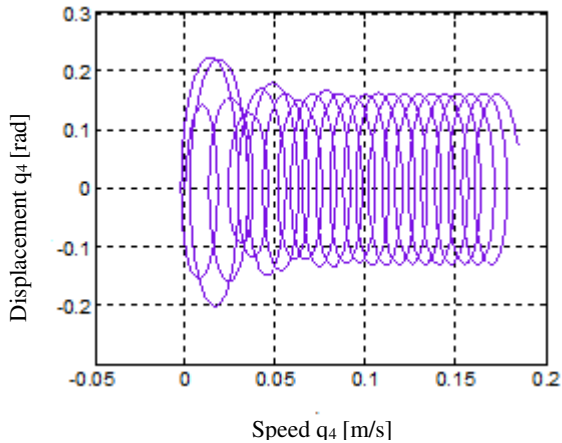


Fig. 17 Stability diagram for coordinate  $\theta_4$  of a model 4-EPPC

### 3. CONCLUSIONS

1. The human body, as a natural system, can be considered as a material system having three mechanical characteristics: mass, elasticity and dumping.

2. The human body can be approximate all or divided in different parts, function of the direction of study, or function of the necessary results.

3. In this paper the idea was to consider the human body formed with five different parts: two legs (with two segments each of them) and pelvis. The name was established as 4- This study was necessary for the comparison with the experiment that follows.

The adapted system for four degrees of mobility 4-EPPC values shifts for foot ( $m_1$ ) is

$2.6 \times 10^{-3}$  mm with a weight of 7.74kg, amount of walkthroughs for pelvis ( $m_2$ ) is  $5.5 \times 10^{-3}$  mm, with a total mass equal of 16.17kg, the body movements ( $m_3$ ) is 0.01 mm with a weight of 23.31kg and the maximum displacement for the leg in horizontal position ( $m_{4t}$ ) is  $6 \times 10^{-3}$  mm, with a mass of 7,74 kg. The study was carried out with frequency excitation of 23.437 Hz, which resulted from measurements performed on a human subject. So you can see that the highest note in the case of lower mass  $m_{4t}$  i.e. amounted to  $90^0$ , this is the current instability in the foot because of the balance.

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### Studiul mobilității corpului uman prin modelare mecanică

**Rezumat:** *Lucrarea prezintă un model mecanic cu patru grade de libertate al corpului uman supus la vibrații exterioare. Se analizează mobilitatea sistemului material asimilat corpului uman. Se prezintă simularea corespunzătoare unui model notat 4-EPPC, care este compus din cele doua picioare, susținut pe un suport rigid, cu un picior în excitație care se sprijină pe o platformă vibratoare, și transmite mișcarea la pelvis. În lucrare se analizează stabilitatea sistemului mecanic asimilat corpului uman, pentru fiecare segment în parte, fiecare picior are două segmente, iar cel de al cincilea segment este pelvisul. Din studiul efectuat, rezultă că mișcarea fiecărui segment este stabilă, deci nu produce disfuncționalități corpului asupra căruia s-a efectuat acest studiu*

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