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THE STATE OF STRESSES IN THE PROXIMAL FEMORAL BONE IN BIPODAL AND UNIPODAL SUPPORT

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Abstract: *In this study, we are trying to determine the stresses state of the proximal femoral bone in the case of bipodal and unipodal support. Apply the section method by evaluating axial force N, shear force T and bending moment* M_i*, and plotting the variation diagrams. By drawing the force and moment variation diagrams, the maximum loaded transversal secţion is highlighted, corresponding to the intertrochanteric area for both cases of support. For unipodal support, in the intertrochanterian area, the stresses produced by the axial force, shear force and bending moment are calculated, their variation diagrams are drawn and the maximum loaded fiber is identified.*

Key words: *proximal femoral bone, bipodal support, unipodal support, intertrochanterian area, axial force, shear force, bending moment, normal stresses, shear stresses*

1. INTRODUCTION

In the position of orthostatism, with bipodal support, the human body has a symmetrical loading on the two ends of the femoral bones. In order to ensure the equilibrium position in the front plane, no action of the muscular forces in required. Braune W. And Fischer O. [1] consider the weight center, denoted with S4, according to Figure 1, as being positioned on the upper right vertical line in the middle of the line joining the centers of the two femoral joints.

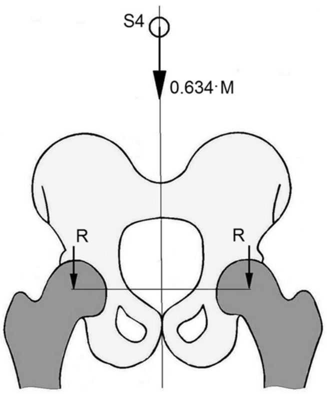

Fig.1 Proximal femoral bone in bipodal support [2, 3].

The center of gravity is the geometric place where the mass of the two upper limbs, the body mass and the mass of the head are concentrated.

If the mass of the lower limbs is considered to be 2x0.185 [4] of the total mass M of the body, it results theat in the center of gravity, positioned in the S4 node, it acts as a 0.63xM mass. Each proximal femoral bone will take up half of this mass.

In the orthostatic position with unipodal support, having the center of gravity in the S5 node, according to Figure 2, the biomechanical joint conditions are changed.

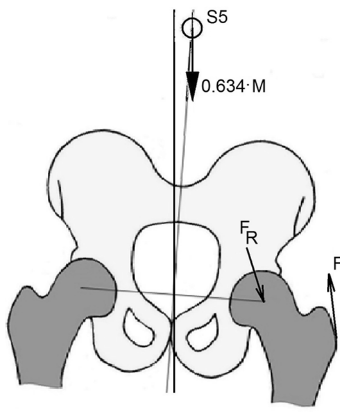

Fig.2 Proximal femoral bone in unipodal support [2, 3].

Thus, the entire weight of the body (upper limbs, tors, head and lower oscillating limb) is taken over by a single joint. Eccentric support of the basin on a single joint causes an action of balancing the muscular forces, preventing the basin from swinging towards the side of the lower oscillating member. Total movements during walking cause the emergence of dynamic forces that increase articular stress [5]. It is assumed that the joint surfaces are perfectly congruent, articulat cartilage behaves as an elastic material [6]. Thus, forces are normally transmitted from one surface to the next.

The femoral bone is stabilized in the cotiloid cavity by the abductor muscles (formed by the small, medium and large fesier muscles), represented as a single resultant of the F_m forces. The angle of this force is about 71 degrees relative to the horizontal plane. For the calculation of the muscle force F_m and the F_R force acting of the hip joint when a person is standing vertically, resting on one foot in a slow movement, it is considered that the size of these forces on the body weight and the geometric relationships between the center of the mass and the proximal femur.

2. METHODS

Thus, the present study will take into account a body mass of 74 kg. For the two support cases, by applying the section method, the axial force N, the shear force T and the bending moment M_i in the lower limb are determined and the variation digrams of these efforts are drawn to highlight the maximum loaded section.

The case of bipodal support. In Figure 3 is presented the lower limb in bipodal support. It has the following dimensional characteristics: the length on the tibia (range 1 – 2) is 400 mm, the length of the femoral bone diaphysis (range 2 – 3) is 450 mm, the angle formed by the anatomical axis of the tibia and the anatomical axis of the femoral bone is 174 degrees, the angle formed by the mechanical axis of the pelvic limb (the center of the femoral head – center of the ankle) with the anatomical axis of the femur is 6 degrees, the femoral neck length (range 3 – 4) is 70 mm, the angle formed by the anatomical axis of femoral bone and the anatomical axis of the femoral neck is 126.22 degrees. Force R has the value of 228.671 Newtons and the G_{mi} force is 134.198 Newtons.

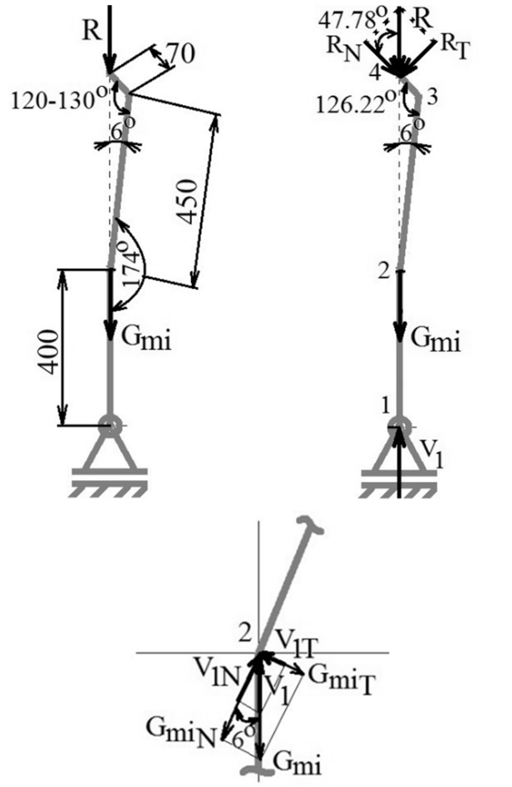

Fig.3 The pelvic limb in bipodal support.

On the range 1 – 2, corresponding to the tibia, the efforts are expressed through the following relations:

$$N_{12} = -V_1 = -\frac{G}{2};\ T_{12} = 0;\ M_{12} = 0$$

On the range 2 – 3, corresponding to the femoral bone diaphysis, the efforts are expressed through the following relations:

$$N_{23} = G_{miN} - V_{1N};\ T_{23} = V_{1T} - G_{miT};$$
$$M_{23} = G_{miT} \cdot x - V_{1T} \cdot x$$

where the variable x has two extreme points representing the limits of the range 2 to 3.

On the range 4 – 3, corresponding to the femoral neck, the efforts are expressed through the following relations:

$$N_{43} = -R_N;\ T_{43} = -R_T;\ M_{23} = -R_T \cdot x$$

where the variable x has two extreme points representing the limits of the range 4 to 3.

In Figure 4 are plotted the axial force N, the shear force T and the bending moment M_i diagrams.

Table 1 gives the effort values corresponding to the three intervals.

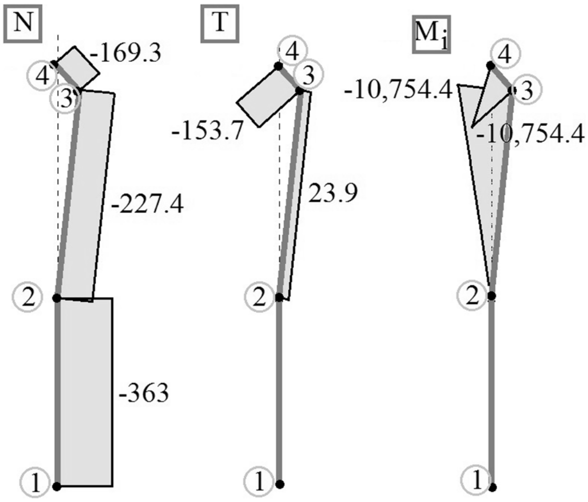

Fig.4 The efforts diagrams of pelvic limb in bipodal support.

Table 1

The effort values of pelvic limb in bipodal support.

Range	Axial force, N [N]	Shear force, T [N]	Bending moment, M_i [N·mm]	
1 - 2	-363	0	0	
2 - 3	-227.4	23.9	Nod 2	Nod 3
			0	-10,754.4
4 - 3	-169.3	-153.7	Nod 4	Nod 3
			0	-10,754.4

The case of unipodal support. Figure 5 shows the lower limb in unipodal support for the case where the mechanical axis relative to the anatomical axis of the tibia forms an angle of 10.24 degrees.

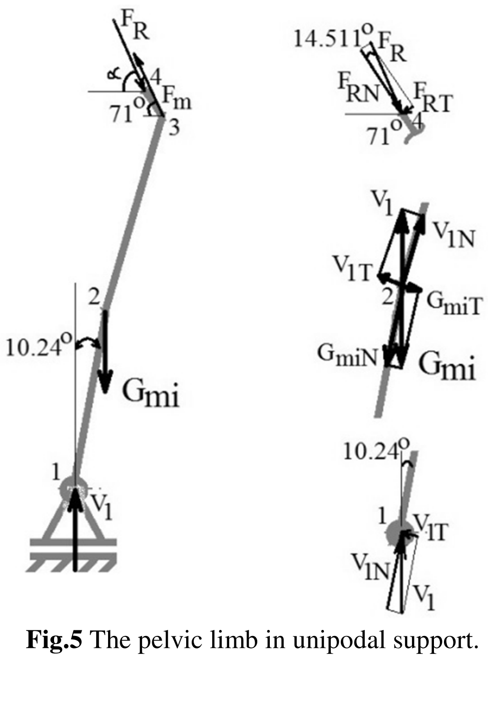

Fig.5 The pelvic limb in unipodal support.

The static equations are written as follows:

1) $\sum F_x = 0 \Rightarrow -F_m \cdot cos71 + F_R \cdot cos\alpha = 0$

2) $\sum F_y = 0 \Rightarrow F_m \cdot sin71 - F_R \cdot sin\alpha - G_{mi} + V_1 = 0$

3) $\sum M_{i3} = 0 \Rightarrow F_m \cdot sin\alpha \cdot 37.067 - -F_R \cdot cos\alpha \cdot 59.365 + G_{mi} \cdot 125.839 - V_1 \cdot 196.948 = 0$

4) $\sum M_{i4} = 0 \Rightarrow F_m \cdot sin71 \cdot 37.067 - -F_m \cdot cos71 \cdot 59.365 + G_{mi} \cdot 87.772 - V_1 \cdot 159.881 = 0$

where: the term 37.067 represents the distance (on mm) from node 3 to node 4 in the horizontal plane; the term 59.365 represents the distance (on mm) from node 3 to node 4 in the vertical plane; the term 196.948 represents the distance (on mm) from node 1 to node 3 in the horizontal plane; 87.772 represents the distance (on mm) from node 2 to node 4 in the horizontal plane; the term 159.881 represents the distance (on mm) from node 1 to node 4 in the horizontal plane.

From the equilibrium relations, the magnitude of the forces F_m=6,633.276 Newtons and F_R=7,193.66 Newtons results for an angle value of α=72.53 degrees.

By applying the section method, the effort is calculated on the three intervals as follows:

On the range 1 – 2, corresponding to the tibia, the efforts are expressed through the following relations:

$$N_{12} = -V_{1N};\ T_{12} = V_{1T};\ M_{12} = -V_{1T} \cdot x$$

where the variable x has two extreme points representing the limits of the range 1 to 2.

On the range 2 – 3, corresponding to the femoral bone diaphysis, the efforts are expressed through the following relations:

$$N_{23} = G_{miN} - V_{1N};\ T_{23} = V_{1T} - G_{miT};$$
$$M_{23} = G_{miT} \cdot x - V_{1T} \cdot x$$

where the variable x has two extreme points representing the limits of the range 2 to 3.

On the range 4 – 3, corresponding to the femoral neck, the efforts are expressed through the following relations:

$$N_{43} = -F_{RN};\ T_{43} = -F_{RT};\ M_{23} = -F_{RT} \cdot x$$

where the variable x has two extreme points representing the limits of the range 4 to 3.

Table 2 gives the effort values corresponding to the three intervals.

Table 2

The effort values of pelvic limb in unipodal support.

Range	**Axial force, N [N]**	**Shear force, T [N]**	**Bending moment, M_i [N·mm]**	
1 - 2	-712.6	1,802	Nod 1	Nod 2
			0	-51,680.6
2 - 3	-566.3	-165.4	Nod 2	Nod 3
			-51,680.6	-126,117.4
4 - 3	-6,964.4	-129.2	Nod 4	Nod 3
			0	-126,117.4

In Figure 6 are plotted the axial force N, the shear force T and the bending moment M_i diagrams.

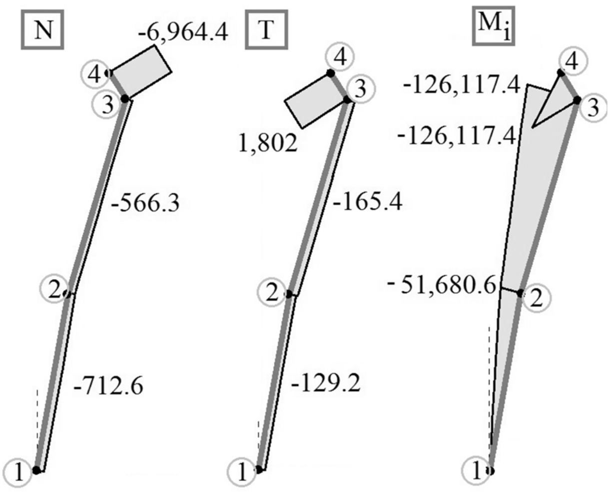

Fig.6 The efforts diagrams of pelvic limb in unipodal support.

3. RESULTS

According to Figure 4 and Figure 6 the maximum cross section loaded is in node 3, corresponding to the intertrochanteric area. In this case, the intertrochanteric area is subjected to a complex compound stress: compressive stress caused by the axial force N, shear stress caused by the shear force T and the bending stress produced by the bending moment M_i. In order to assess the stresses produced by these efforts, it is necessary to know the geometric characteristics of the cross – section: the position of the center of gravity, the area, the axial moment of inertia, section modulus and the static moment.

For the femoral bone studied in this paper, the geometric characteristics of the cross section of the femoral head, femoral neck, diaphysis and the intertrochanteric area were determined in the paper [7]. Thus, for the case where the trabecular tissue does not present a degradation of the architecture, the cross section shows a number of 1,712 trabeculae, the area is 1,512.1 mm^2, the axial moment of inertia is $63.5 x 10^4$ mm^4, the static moment of the half – plane (in the vertical plane) is 13,411.5 mm^3, in relation to the center of gravity of the cross – section, in the vertical plane, the distance to the bottom fiber is 39.7 mm and the distance to the upper fiber is 36.2 mm, the section modulus is $15.99 x 10^3$ mm^3.

The axial effort N will produce in the cross – section a normal compressive stress σ_c, uniformly distributed, and which is calculated as the ratio of this axial force to the cross – sectional area according to the relation:

$$\sigma_c = \frac{N}{A}$$

The bending moment M_i will produce a normall stress σ_b, which presents, in the cross – section, a law of linear variation woth extreme values (compressive and tension stresses) in the lower and higher fibers according to the relation (Naviers's formula):

$$\sigma_b = \frac{M_i}{W_z} = \frac{M_i}{I_z} \cdot y$$

By overlapping the efects, by summing the two normal stresses (σ_c and σ_b), the maximum value of the normal stress σ_r can be calculated.

The shear effort T produced by the bending load will produce a tangential stresses τ which in the cross – section has a parabolic variation law with the maximum value in the neutral axis (for which σ=0) and null value in the upper and lower extreme fibers according to the relation (Juravski's formula):

$$\tau = \frac{T \cdot S_z}{b \cdot I_z}$$

Table 3 summarisez the values of the normal stresses given by the axial effort N and the bending moment M_i, the resulting normal stresses as well as the tangential stresses for the two analyzed cases (bipodal support and unipodal support) and the variation diagrams of these stresses are plotted in Figure 7.

Table 3

The values of the stresses in the intertrochanteric area in the case of bipodal and unipodal support.

		bipodal support	unipodal support
Axial load – compression σ_c [N/mm^2]		-0.15	-4.605
Bending load σ_b [N/mm^2]	Upper fiber	0.613	7.189
	Lower fiber	-0.672	-7.884
Resulting normal stress σ_r [N/mm^2]	Upper fiber	0.463	2.584
	Lower fiber	-0.822	-12.489
Bending load τ_b [N/mm^2]		0.085	1.001

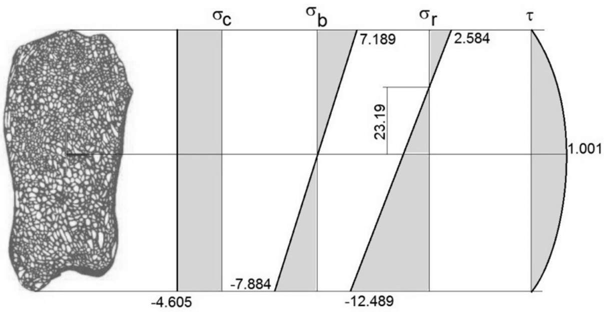

Fig.7 The diagrams of variation of the normal stresses σ and tangential stresses τ in cross – section of intertrochanterian area for the unipodal support.

4. CONCLUSIONS

From the variation diagrams of the bending moments plotted in Figure 4 and Figure 6 it is emphasized thet the maximum loaded cross – section is in the intertrochanteric area. The state of stresse depends, to a great extent, on the trabecular tissue architecture [7, 8]. The challenge is to determine the integrity of the trabecular tissue architecture [9] and to determine the geometric characteristics of the analyzed cross – section.

From Figure 7 the following observations can be highlighted:

-the axial effort N being negative produces an axial compressive load so that in the cross – section the normal stress σ_c is uniformly distributed;

-the bending moment M_i produces in the cross – section a normal stress σ_b which presents a law of linear variation indicating that the area between the lower fiber and the center of gravity of the cross – section is compressed and the area between the center of gravity of the cross – section and the upper fiber is stretched out;

-applying the principle of overlapping effects produced by the two normal stresses (σ_c and σ_b) it can be seen that the neutral axis (for which σ=0) moves from the center of gravity of the cross – section to the upper fiber where the neutral axis position is 23.19 mm from the center of gravity of the cross – section;

-in the lower fiber, subjected to compression, the resulting normal stress has the maximum value.

The strength condition in that this maximum normal stress is less than or equal to the allowable stress of the trabecular and compact tissue, values that are highlighted in the literature.

Compared to normal stresses σ the effect of tangential stress τ may be negligible.

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Starea de tensiuni în osul femural proximal în sprijin bipodal şi unipodal

Rezumat: *În prezentul studiu se urmăreşte determinarea stării de tensiuni din osul femural proximal pentru cazul sprijinului bipodal şi unipodal. Se aplică metoda secţiunilor evaluându-se efortul axial, efortul tăietor şi momentul încovoietor şi se trasează diagrama de variaţie a acestora. Prin trasarea diagramelor de variaţie a eforturilor se evidenţiază secţiunea maxim solicitată, aceasta fiind corespunzătoare zonei intertrohanteriene pentru ambele cazuri de rezemare. Pentru sprijinul unipodal, în zona intertrohanteriană, se calculează tensiuniunile produse de eforturi, se trasează diagramele de variaţie ale acestora şi se identifică fibra maxim solicitată.*

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