TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

**ACTA TECHNICA NAPOCENSIS**

**Series: Applied Mathematics, Mechanics, and Engineering**
**Vol. 61, Issue II, June, 2018**

# KINEMATIC CONTROL FUNCTIONS FOR A SERIAL ROBOT STRUCTURE BASED ON THE TIME DERIVATIVE JACOBIAN MATRIX

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***Abstract:*** *The kinematic modeling of a mechanical system with n degrees of freedom, involves an impressive volume of computational or differential calculus. There are algorithms dedicated to this task developed in the literature. Applying of algorithms allows a detailed, numerical and / or graphical analysis of kinematics for a mechanical structure, regardless of its type and complexity. The results obtained with algorithms are essential in optimal design, dimensional and energetically, but also to simulate the kinematical and dynamic behavior of the mechanical structures of the robots.*

***Key words:*** *robotics, time derivative Jacobian Matrix, algorithm, control functions.*

## 1. INTRODUCTION

The kinematics control for a serial robot, is an important task. According to dedicated literature, there are multiple methods to establish the expressions that modeling the kinematic behavior for any mechanical structure. The paper presents a method for kinematic control, based on matrix exponentials, which are the basis for establishing the time derivative for the Jacobian matrix. Hence in the first part of paper, will be presented the mathematical considerations in regarding the obtaining of first derivative for Jacobian matrix. In the second part will be determined the time derivative of the Jacobian matrix, in the case of a serial robot. The third part, presents the kinematic control functions determined on the basis of the time derivative of the Jacobian matrix for the serial structure.

## 2. THE TIME DERIVATIVE FOR THE JACOBIAN MATRIX

According to the definition expression of the Jacobian matrix [1]-[3], based on the linear and angular transfer matrix for velocity and accelerations, the time derivative of the Jacobian matrix is defined as: [2]

$$\underset{(6xn)}{{}^{0}\dot{J}_i}\left(\overline{\theta}\right)=\left[{}^{0}\dot{J}_i;\ \ i=1\rightarrow n\right]= \\ =\left[\left[{}^{0}\dot{J}_{iv}^{T}\left(\overline{\theta}\right)\ \ {}^{0}\dot{J}_{i\Omega}^{T}\left(\overline{\theta}\right)\right]^{T};\ i=1\rightarrow n\right] \quad (1)$$

where ${}^{0}\dot{J}_{iv}\left(\overline{\theta}\right)$ represents the linear component and ${}^{0}\dot{J}_{i\Omega}\left(\overline{\theta}\right)$ is the angular component. To establish the time derivative for the Jacobian matrix, according to matrix exponential algorithm from the direct kinematics, the symbolic expression of each column from (1) is:

$${}^{0}\dot{J}_i=M\left\{\dot{J}_{i1}\right\}\cdot M\left\{\dot{J}_{i2}\right\}\cdot M\left\{\dot{J}_{i3}\right\}\cdot M\left\{\dot{J}_{i4}\right\} \quad (2)$$

The four matriceal expressions from (2), can be included in a single matrix form as follows:

$$M\left\{\dot{J}_{i1}\right\}=\left[ME\left\{\dot{J}_{i1}\right\}\ \ ME\left\{J_{i1}\right\}\ \ ME\left\{J_{i1}\right\}\right] \quad (3)$$

$$M\left\{\dot{J}_{i2}\right\}=\begin{bmatrix} ME\left\{J_{i2}\right\} & [0] & [0] \\ [0] & ME\left\{\dot{J}_{i2}\right\} & [0] \\ [0] & [0] & ME\left\{J_{i2}\right\} \end{bmatrix} \quad (4)$$

$$M\left\{\dot{J}_{i3}\right\}=\begin{bmatrix} ME\left\{J_{i3}\right\} & [0] & [0] \\ [0] & ME\left\{\dot{J}_{i3}\right\} & [0] \\ [0] & [0] & ME\left\{J_{i3}\right\} \end{bmatrix} \quad (5)$$

$$M\left\{\dot{J}_{i4}\right\}=\left[M_{iv\omega}\; M^{*}_{iv\omega}\; \dot{M}_{iv\omega}\right]^{T}, \; where:$$

$$\begin{aligned} M_{iv\omega} &= \left[\bar{v}_i^{(0)T} \;\; \left[\bar{b}_k; k=i\to n\right]^{T} \; \bar{p}^{(0)T} \left\{\bar{k}_i^{(0)}\cdot\Delta_i\right\}^{T}\right]^{T} \\ M^{*}_{iv\omega} &= \left[\bar{v}_i^{(0)T} \;\; \left[\bar{b}_k; k=i\to n\right]^{T} \;\; \bar{p}^{(0)T} \;\; \bar{0}^{T}\right]^{T}; \\ \dot{M}_{iv\omega} &= \left[\bar{0}^{T} \;\; \left[\dot{\bar{b}}_k; k=i\to n\right]^{T} \;\; \bar{0}^{T} \;\; \bar{0}^{T}\right]^{T}. \end{aligned} \quad (6)$$

where $\Delta_{i,j,k,m}=\left\{\{1; if\ i=R\};\{0;\ if\ i=T\}\right\}$, is an operator which highlights the type of joint.

The time derivative of the expressions (3)-(5), are determined as:

$$ME\left\{\dot{J}_{i1}\right\}=\begin{bmatrix} ME\left\{\dot{V}_{i1}\right\} & [0] \\ [0] & ME\left\{\dot{V}_{i1}\right\} \end{bmatrix}, \quad (7)$$

$$\underset{(3\times3)}{ME}\left\{\dot{V}_{i1}\right\}=\frac{d}{dt}\left\{\exp\left\{\sum_{j=0}^{i-1}\left\{\bar{k}_j^{(0)}x\right\} q_j\Delta_j\right\}\right\} \quad (8)$$

$$\underset{(6x9)}{ME}\left\{\dot{J}_{i2}\right\}=\begin{bmatrix} ME\left\{\dot{V}_{i2}\right\} & [0] \\ [0] & I_3 \end{bmatrix} \quad (9),$$

where $$\underset{(3x6)}{ME}\left\{\dot{V}_{i2}\right\}=\left[I_3\Delta_i\cdot\left\{\bar{k}_i^{(0)}x\right\}\right] \quad (10)$$

$$ME\left\{\dot{J}_{i3}\right\}=\begin{bmatrix} ME\left\{\dot{V}_{i3}\right\} & [0] \\ [0] & I_3 \end{bmatrix},$$

$$ME\left\{\dot{V}_{i3}\right\}=\left[A_1 \quad A_2 \quad A_3\right],$$

$$and: \qquad A_1=\begin{bmatrix} I_3 \\ [0] \end{bmatrix};$$

$$A_2=\begin{bmatrix} [0] \\ \dfrac{d}{dt}\left\{\exp\left\{\sum_{m=i-1}^{k-1}\left\{\bar{k}_m^{(0)}x\right\}q_m\delta_m\Delta_m \atop k=i\to n\right\}\right\} \end{bmatrix}; \quad (11)$$

$$A_3=\begin{bmatrix} [0] \\ \dfrac{d}{dt}\left\{\exp\left\{\sum_{k=1}^{n}\left\{\bar{k}_k^{(0)}x\right\}q_k\Delta_k\right\}\right\} \end{bmatrix}.$$

The time derivative of the column vector $\dot{\bar{b}}_k$, from (6), according to [1], [3] is determined as:

$$\underset{(3\times1)}{\dot{\bar{b}}_k}=I_3+\left\{\bar{k}_k^{(0)}\times\right\}\cdot\sin(q_k\cdot\Delta_k)+ +\left\{\bar{k}_k^{(0)}\times\right\}^2\cdot\left[1-\cos(q_k\cdot\Delta_k)\right]\cdot\bar{v}_k^{(0)}\cdot\dot{q}_k \quad (12)$$

The previous expressions will be applied $(i=1\to n)$, resulting each column of the time derivative for the Jacobian matrix as :

$$\underset{(i=1\to n)}{{}^{0}\dot{J}_i}=Trace\left\{\begin{bmatrix} ME\left\{\dot{J}_{i1}\right\}\cdot ME\left\{J_{i2}\right\}\cdot ME\left\{J_{i3}\right\} \\ ME\left\{J_{i1}\right\}\cdot ME\left\{J_{i2}\right\}\cdot ME\left\{\dot{J}_{i3}\right\} \\ ME\left\{J_{i1}\right\}\cdot ME\left\{J_{i2}\right\}\cdot ME\left\{J_{i3}\right\} \end{bmatrix}\cdot ME_{iv}\right\}, \quad (13)$$

$$where: \quad ME_{iv}=\left[M_{iv\omega} \quad M^{*}_{iv\omega} \quad \dot{M}_{iv\omega}\right]$$

According to literature, the previous expression can be written in a simplified form as results:

$$\underset{(i=1\to n)}{{}^{0}\dot{J}_i}=\begin{bmatrix} ME\left\{\dot{J}_{i1}\right\}\cdot ME\left\{J_{i2}\right\}\cdot ME\left\{J_{i3}\right\}+M_{iv\omega}+ \\ +ME\left\{J_{i1}\right\}\cdot ME\left\{J_{i2}\right\}\cdot ME\left\{\dot{J}_{i3}\right\}+M^{*}_{iv\omega}+ \\ +ME\left\{J_{i1}\right\}\cdot ME\left\{J_{i2}\right\}\cdot ME\left\{J_{i3}\right\}\cdot\dot{M}_{iv\omega} \end{bmatrix} \quad (14)$$

As an important remark, the previous obtained Jacobian matrix, contains on one hand the Coriolis terms and complementary themes, and on the other hand the terms referring to centripetal accelerations.

## 3. DETERMINING THE TIME DERIVATIVE OF JACOBIAN MATRIX FOR THE SERIAL ROBOT 2TR

In keeping with the mathematical expressions presented in the previous paragraph of the paper, further will be applied the steps in establishing of the time derivative of the Jacobian matrix for a serial structure having three degrees of freedom as result from the Figure 1. [5]

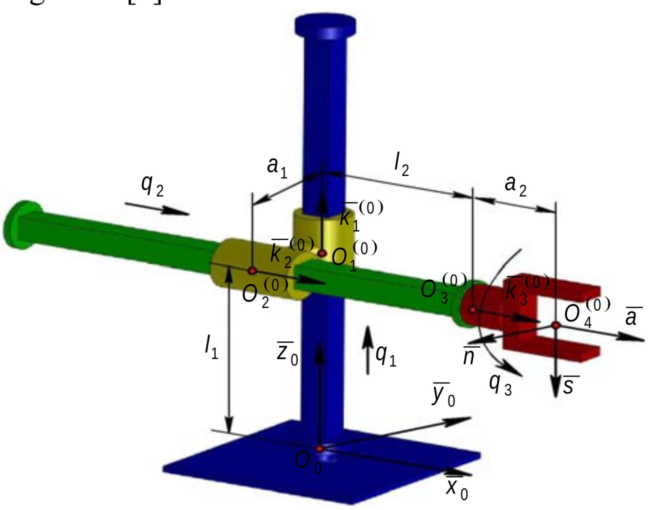

**Fig. 1** The TTR serial structure

Hence, there is opened an external loop for $(i=1\rightarrow 3)$, and on the basis of (8) results:

$$ME\left\{\dot{V}_{11}\right\}=[0_3] \quad (15)$$

$$\underset{(3x3)}{ME}\left\{\dot{V}_{21}\right\}=\left[\left\{\bar{k}_1^{(0)}x\right\}\cdot\dot{q}_1\cdot\Delta_1\cdot\exp\left\{\left\{\bar{k}_1^{(0)}x\right\}\cdot q_1\right\}\right]=[0_3] \quad (16)$$

$$\underset{(3x3)}{ME}\left\{\dot{V}_{31}\right\}=\left[\left\{\bar{k}_2^{(0)}x\right\}\cdot\dot{q}_2\cdot\Delta_2\cdot\exp\left\{\left\{\bar{k}_2^{(0)}x\right\}\cdot q_2\right\}\right]=[0_3] \quad (17)$$

In keeping with (15)-(17), introduced in (7), there are obtained the matrices [5],[6], [7]:

$$\underset{(6\times 6)}{ME}\left\{\dot{J}_{11}\right\}=\begin{bmatrix} ME\left\{\dot{V}_{i1}\right\} & [0] \\ [0] & ME\left\{\dot{V}_{i1}\right\}\end{bmatrix}=[0_6]; \quad (18)$$

$$\underset{(6\times 6)}{ME}\left\{\dot{J}_{21}\right\}=\begin{bmatrix} ME\left\{\dot{V}_{21}\right\} & [0] \\ [0] & ME\left\{\dot{V}_{21}\right\}\end{bmatrix}=[0_6]; \quad (19)$$

$$\underset{(6\times 6)}{ME}\left\{\dot{J}_{31}\right\}=\begin{bmatrix} ME\left\{\dot{V}_{31}\right\} & [0] \\ [0] & ME\left\{\dot{V}_{31}\right\}\end{bmatrix}=[0_6]. \quad (20)$$

According to (6) and (12), for $(i=1\rightarrow 3)$ there are established the following expressions:

$$M^*_{1v\omega}=\left[\bar{v}_1^{(0)T}\left[\bar{b}_k\,;k=1\rightarrow 3\right]^T\bar{p}_3^{(0)T}\,\bar{0}^T\right]^T,\ where:$$

$$\bar{v}_1^{(0)}=[0\ \ 0\ \ 1]^T;$$

$$\left\{\bar{b}_1^T=\begin{bmatrix}0\\0\\q_1\end{bmatrix}^T \bar{b}_2^T=\begin{bmatrix}q_2\\0\\0\end{bmatrix}^T \bar{b}_3^T=\begin{bmatrix}0\\ l_1\cdot sq_3+a_1\cdot(cq_3-1)\\ a_1\cdot sq_3-l_1(cq_3-1)\end{bmatrix}^T\right\}^T \quad (21)$$

$$\left[\ \bar{p}_3^{(0)T}=[l_2\ -a_1\ l_1] \qquad \Delta_1\cdot\bar{k}_1^{(0)T}=[0_3]\ \right]^T$$

$$M^*_{2v\omega}=\left[\bar{v}_2^{(0)T}\ \left[\bar{b}_k;k=2\rightarrow 3\right]^T\ \bar{p}_3^{(0)T}\ \bar{0}^T\right]^T=$$

$$=\begin{Bmatrix} \bar{v}_2^{(0)}=[1\,0\,0]^T \\ \left\{\bar{b}_2^T=\begin{bmatrix}q_2\\0\\0\end{bmatrix}^T \bar{b}_3^T=\begin{bmatrix}0\\ l_1\cdot sq_3+a_1\cdot(cq_3-1)\\ a_1\cdot sq_3-l_1\cdot(cq_3-1)\end{bmatrix}^T\right\}^T \\ \left[\ \bar{p}_3^{(0)T}=[l_2\ -a_1\ l_1] \qquad [0_3]\ \right]^T\end{Bmatrix} \quad (22)$$

$$M^*_{3v\omega}=\left[\bar{v}_3^{(0)T}\quad\left[\bar{b}_k\,;k=3\right]^T\quad\bar{p}_3^{(0)T}\quad\bar{0}^T\right]^T=$$

$$\begin{Bmatrix} \bar{v}_3^{(0)}=[0\quad l_1\quad a_1]^T \\ \left\{\bar{b}_3^T=\begin{bmatrix}0\\ l_1\cdot sq_3+a_1\cdot(cq_3-1)\\ a_1\cdot sq_3-l_1\cdot(cq_3-1)\end{bmatrix}^T\right\}^T \\ \left[\ \bar{p}_3^{(0)T}=[l_2\quad -a_1\quad l_1] \qquad [0_3]\ \right]^T\end{Bmatrix} \quad (23)$$

$$\dot{M}_{1v\omega}=\left[\underset{3x1}{[0]^T}\ \left[\dot{\bar{b}}_k;k=1\rightarrow 3\right]^T\ \underset{3x1}{[0]^T}\ \underset{3x1}{[0]^T}\right]^T=$$

$$=\begin{Bmatrix} \underset{3x1}{[0]^T} \\ \left\{\bar{b}_1^T=\begin{bmatrix}0\\0\\\dot{q}_1\end{bmatrix}^T \bar{b}_2^T=\begin{bmatrix}\dot{q}_2\\0\\0\end{bmatrix}^T \bar{b}_3^T=\begin{bmatrix}0\\ -(a_1\cdot sq_3-l_1\cdot cq_3)\cdot\dot{q}_3\\ (a_1\cdot cq_3+l_1\cdot sq_3)\cdot\dot{q}_3\end{bmatrix}^T\right\}^T \\ \underset{6x1}{[0]^T}\end{Bmatrix} \quad (24)$$

$$\dot{M}_{2v\omega}=\left[\underset{3x1}{[0]^T}\ \left[\dot{\bar{b}}_k;k=2\rightarrow 3\right]^T\ \underset{3x1}{[0]^T}\ \underset{3x1}{[0]^T}\right]^T=$$

$$=\begin{Bmatrix} \underset{3x1}{[0]^T} \\ \left\{\bar{b}_1^T=\begin{bmatrix}\dot{q}_2\\0\\0\end{bmatrix}^T \bar{b}_2^T=\begin{bmatrix}0\\ -(a_1\cdot sq_3-l_1\cdot cq_3)\cdot\dot{q}_3\\ (a_1\cdot cq_3+l_1\cdot sq_3)\cdot\dot{q}_3\end{bmatrix}^T \bar{b}_3^T=\begin{bmatrix}0\\0\\0\end{bmatrix}^T\right\}^T \\ \underset{3x1}{[0]^T}\end{Bmatrix} \quad (25)$$

$$\dot{M}_{3v\omega}=\left[\underset{3x1}{[0]^T}\quad\left[\dot{\bar{b}}_k;\ k=3\right]^T\quad\underset{3x1}{[0]^T}\quad\underset{3x1}{[0]^T}\right]^T=$$

$$=\begin{Bmatrix} \underset{3x1}{[0]^T} \\ \left\{\bar{b}_3^T=\begin{bmatrix}0\\ -(a_1\cdot sq_3-l_1\cdot cq_3)\cdot\dot{q}_3\\ (a_1\cdot cq_3+l_1\cdot sq_3)\cdot\dot{q}_3\end{bmatrix}^T\right\}^T \\ \underset{6x1}{[0]^T}\end{Bmatrix} \quad (26)$$

According to previous expressions, the time derivative for the Jacobian matrix in the case of the serial structure from Figure 1, becomes:

$$ {}^{0}\dot{J}_1 = \begin{Bmatrix} ME\{\dot{J}_{11}\}\cdot ME\{J_{12}\}\cdot ME\{J_{13}\}\cdot M_{1v\omega} + \\ + ME\{J_{11}\}\cdot ME\{J_{12}\}\cdot ME\{\dot{J}_{13}\}\cdot M^{*}_{1v\omega} + \\ + ME\{J_{11}\}\cdot ME\{J_{12}\}\cdot ME\{J_{13}\}\cdot \dot{M}_{1v\omega} \end{Bmatrix} = \underset{6x1}{[0]} \quad (27) $$

$$ {}^{0}\dot{J}_2 = \begin{Bmatrix} ME\{\dot{J}_{21}\}\cdot ME\{J_{22}\}\cdot ME\{J_{23}\}\cdot M_{2v\omega} + \\ + ME\{J_{21}\}\cdot ME\{J_{22}\}\cdot ME\{\dot{J}_{23}\}\cdot M^{*}_{2v\omega} + \\ + ME\{J_{21}\}\cdot ME\{J_{22}\}\cdot ME\{J_{23}\}\cdot \dot{M}_{2v\omega} \end{Bmatrix} = \underset{6x1}{[0]} \quad (28) $$

$$ {}^{0}\dot{J}_3 = \begin{Bmatrix} ME\{\dot{J}_{31}\}\cdot ME\{J_{32}\}\cdot ME\{J_{33}\}\cdot M_{3v\omega} + \\ + ME\{J_{31}\}\cdot ME\{J_{32}\}\cdot ME\{\dot{J}_{33}\} + M^{*}_{3v\omega} + \\ + ME\{J_{31}\}\cdot ME\{J_{32}\}\cdot ME\{J_{33}\}\cdot \dot{M}_{3v\omega} \end{Bmatrix} = \underset{6x1}{[0]} \quad (29) $$

Substituting (27)-(29) in definition expression (1), there is obtained [5],[6] :

$$ {}^{0}\dot{J}(\bar{\theta}) = \underset{6x3}{[0]} \quad (30) $$

The time derivative, for the Jacobian matrix in the case of the structure TTR. According to (30), duet o the fact that is a simple mechanical structure, the time derivative is null.

## 4 DIRECT KINEMATICAL MODELING FOR 2TR SERIAL ROBOT

According to [1], [2], [3] to establish the operational kinematical parameters, for any serial robot structure, there is needed the Jacobian and it time derivative, which are substituted in the following generalized expression:

$$ \begin{bmatrix} {}^{0}\dot{\bar{X}} \\ \ldots\ldots \\ {}^{0}\ddot{\bar{X}} \end{bmatrix} = \begin{bmatrix} \left[{}^{0}\bar{v}_n^{T} \quad {}^{0}\bar{\omega}_n^{T}\right]^{T} \\ \ldots\ldots \\ \left[{}^{0}\dot{\bar{v}}_n^{T} \quad {}^{0}\dot{\bar{\omega}}_n^{T}\right]^{T} \end{bmatrix} = \begin{bmatrix} [0] & {}^{0}J(\bar{\theta}) \\ \ldots\ldots & \\ {}^{0}J(\bar{\theta}) & {}^{0}\dot{J}(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \ldots \\ \dot{\bar{\theta}} \end{bmatrix} \quad (31) $$

representing the expressions of the column vector of operational velocities and accelerations, which form the direct kinematics equations, with respect to fixed reference frame.

In the case of the mechanical structure, presented in the Figure 1, the expression(31), becomes:

$$ \begin{bmatrix} {}^{0}\dot{\bar{X}} \\ \ldots \\ {}^{0}\ddot{\bar{X}} \end{bmatrix} = \begin{bmatrix} \left[{}^{0}\bar{v}_3^{T} \quad {}^{0}\bar{\omega}_3^{T}\right]^{T} \\ \ldots\ldots \\ \left[{}^{0}\dot{\bar{v}}_3^{T} \quad {}^{0}\dot{\bar{\omega}}_3^{T}\right]^{T} \end{bmatrix} = \begin{bmatrix} [0] & {}^{0}J(\bar{\theta}) \\ \ldots\ldots & \\ {}^{0}J(\bar{\theta}) & {}^{0}\dot{J}(\bar{\theta}) \end{bmatrix} \cdot \begin{bmatrix} \ddot{\bar{\theta}} \\ \ldots \\ \dot{\bar{\theta}} \end{bmatrix} \quad (32) $$

where, the Jacobian matrix, according to [5], and [6] is:

$$ \underset{(6\times3)}{{}^{0}J(\bar{\theta})} = \begin{bmatrix} {}^{0}J_1 & {}^{0}J_2 & {}^{0}J_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ \ldots & \ldots & \ldots \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (33) $$

The direct kinematics equations, with respect to fixed reference frame $\{0\}$, are obtained on the basis of (33) and (30), which substituted in (32), are leading to:

$$ {}^{0}\dot{\bar{X}} \equiv \begin{bmatrix} {}^{0}\bar{v}_3 \\ \ldots\ldots \\ {}^{0}\bar{\omega}_3 \end{bmatrix} = {}^{0}J(\bar{\theta})\cdot\dot{\bar{\theta}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} \dot{q}_2 \\ 0 \\ \dot{q}_1 \\ \hline \dot{q}_3 \\ 0 \\ 0 \end{bmatrix}; \quad (34) $$

$$ {}^{0}\ddot{\bar{X}} \equiv \begin{bmatrix} {}^{0}\dot{\bar{v}}_3 \\ \ldots\ldots \\ {}^{0}\dot{\bar{\omega}}_3 \end{bmatrix} = {}^{0}J(\bar{\theta})\cdot\ddot{\bar{\theta}} + {}^{0}\dot{J}(\bar{\theta})\cdot\dot{\bar{\theta}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = \begin{bmatrix} \ddot{q}_2 \\ 0 \\ \ddot{q}_1 \\ \hline \ddot{q}_3 \\ 0 \\ 0 \end{bmatrix} \quad (35) $$

The previous expressions (34) and (35) , are representing the direct kinematics model and characterizing the end-effector`s the velocity and acceleration of the 2TR serial structure in the cartesian space.

## 5 THE EQUATIONS OF INVERSE KINEMATIC MODEL BASED ON JACOBIAN MATRICES

Knowing the direct kinematics equations, based on the Jacobian matrices and its time derivative, according to [1],[2], [4], the inverse kinematics equations can be expressed as:

$$\begin{bmatrix} \dot{\bar{\theta}}(t) \\ .... \\ \ddot{\bar{\theta}}(t) \end{bmatrix} = \left\{ \begin{array}{ccc} [0] & {}^0J\left[\bar{\theta}(t)\right]^{-1} & [0] \\ \multicolumn{3}{c}{\dots\dots\dots\dots} \\ {}^0J\left[\bar{\theta}(t)\right]^{-1} & [0] & -{}^0J\left[\bar{\theta}(t)\right]^{-1} \end{array} \right\} \cdot \left\{ \begin{array}{c} {}^0\ddot{\bar{X}}(t) \\ {}^0\dot{\bar{X}}(t) \\ {}^0\dot{J}\left[\bar{\theta}(t)\right]\cdot\dot{\bar{\theta}} \end{array} \right\} . (36)$$

where ${}^0J\left[\bar{\theta}(t)\right]^{-1}$ represents the inverse of the Jacobian matrix. According to [8], the kinematic singularities are characterized by the null value of the determinant associated to the Jacobian matrix. Therefore, in order to avoid this possible situation, which blocks the operation of the robot, the kinematical command functions can be determined by applying the pseudoinverse method, based on Greville's Algorithm, developed in [9], [10]. The inverse kinematic model according to [1], is established having as starting expression (36) , which is leading to the following:

$$\begin{aligned} &{}^0\dot{\bar{X}}(t) = {}^0J\left[\bar{\theta}(t)\right]\cdot\dot{\bar{\theta}}(t); \\ &\dot{\bar{\theta}}(t) = {}^0J\left[\bar{\theta}(t)\right]^{-1}\cdot{}^0\dot{\bar{X}}(t); \qquad ; (37) \\ &\ddot{\bar{\theta}}(t) = {}^0J\left[\bar{\theta}(t)\right]^{-1}\cdot{}^0\ddot{\bar{X}}(t) - {}^0J\left[\bar{\theta}(t)\right]^{-1}\cdot{}^0\dot{J}\left[\bar{\theta}(t)\right]\cdot\dot{\bar{\theta}}. \end{aligned}$$

On the previous considerations, in the case of 2TR serial structure, in keeping (33), the inverse of the Jacobian matrix is;

$$J^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad (38)$$

Substituting the previous expression in (37), there is obtained:

$$\begin{aligned} &\dot{\bar{\theta}}(t) = {}^0J\left[\bar{\theta}(t)\right]^{-1}\cdot{}^0\dot{\bar{X}}(t) = \\ &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} \dot{q}_2 & 0 & \dot{q}_1 & \dot{q}_3 & 0 & 0 \end{pmatrix}^T = (39) \\ &= \begin{pmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 \end{pmatrix}^T \end{aligned}$$

$$\begin{aligned} &\ddot{\bar{\theta}}(t) = {}^0J\left[\bar{\theta}(t)\right]^{-1}\cdot{}^0\ddot{\bar{X}}(t) - {}^0J\left[\bar{\theta}(t)\right]^{-1}\cdot{}^0\dot{J}\left[\bar{\theta}(t)\right]\cdot\dot{\bar{\theta}} = \\ &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} \ddot{q}_2 & 0 & \ddot{q}_1 & \ddot{q}_3 & 0 & 0 \end{pmatrix}^T = \qquad (40) \\ &= \begin{pmatrix} \ddot{q}_1 & \ddot{q}_2 & \ddot{q}_3 \end{pmatrix}^T \end{aligned}$$

which are expressing the kinematic control functions of the 2TR type serial structure.

## 6. CONCLUSION

The paper, is dedicated to revealing of the inverse kinematical model, known also as kinematic control functions for a serial robot. Having as starting point the matrix exponential functions, on the basis of Jacobian matrix, there have been determined the first time derivative, for the velocity transfer matrices.

According to the study, the kinematic command functions can be established based on matrix exponential. An important remark is that the rigidity hypothesis is maintained, but the static hypothesis is removed, the column vectors of the generalized and operational coordinates becoming time functions. As can be seen from the considerations, for determining the kinematic command functions for a robot, they assume the application of some mathematical methods that help establish the link between the elements that determine the location of the end-effector in the Cartesian space, and the velocities and accelerations of the joints.

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„Funcţiile de control cinematic pentru o structură serială bazate pe derivata în raport cu timpul a matricei Jacobiene”

Pentru modelarea cinematică a unui sistem mecanic cu n grade de libertate, care implică un volum impresionant de calcule fie matriceale fie diferenţiale, în literatura de specialitate există dezvoltaţi o serie de algoritmi dedicaţi acestui domeniu. Aplicarea algoritmilor, permite o analiză detaliată, sub formă numerică şi/sau grafică, cu privire la cinematica structurii analizate, indiferent de tipul şi complexitatea acesteia. Rezultatele obţinute cu ajutorul algoritmilor, sunt esenţiale în proiectarea optimală, sub aspect dimensional şi energetic, dar şi pentru simularea comportamentului cinematic şi dinamic al structurilor mecanice din componenţa roboţilor.



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