TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue IV, November, 2013

A METHOD FOR SWING-UP OF THE INVERSE PENDULUM ON A CART

Adina BĂLAN, Alin PLEŞA, Radu DONCA, Mihai DRĂGHICI, Vistrian MĂTIEŞ

Abstract: *Inverse pendulum on a cart is a mechanical system with strong nonlinearities. The system is under actuated, with two degrees of freedom and single control signal. It can be highlighted two phases of the process control: swing-up phase and stabilization phase. This paper investigates solutions for the swing-up phase. Usually the control strategy for raising the pendulum is based on energy control. The problem gets more complicated when taking into account that the length of the track on moving cart is finite and the force acting on the cart has limited value. A solution that uses the maximum possible value of the control signal is proposed and the time when the control signal is switched is obtained by means of a search algorithm.*

Key words: *Inverse pendulum on a cart, modeling, simulations, energy control, search algorithms.*

1. INTRODUCTION

Inverse pendulum control problem began to be studied in the second half of the last century, the motivation is the need to develop and test linear control techniques for nonlinear applications [1]. Thereafter, the strong nonlinearities of the system led to the use of new algorithms to test the control of nonlinear systems. In literature are shown different versions of, offering a wide field for the implementation and testing of a wide variety of control algorithms. The main versions are simple pendulum on a cart, double pendulum on a cart, simple rotating pendulum etc.

Control techniques used are numerous, simple conventional control, advanced control techniques based on modern theory of nonlinear control systems. There have been implemented various nonlinear techniques such as stabilizing by reaction [2], passivity-based control [3], nonlinear observers [4], back-stepping [5], friction compensation [6], variable structure control [7] and so on

The system (Fig. 1) is under actuated (i.e. the number of control inputs is less than the number of degrees of freedom) with two degrees of freedom and a certain control signal (force applied to the cart). It can be highlighted two phases of the process control: swing-up phase and stabilization phase. The following sections will investigate solutions for swing-up phase.

System parameters are: m – mass of the pendulum, l – length of the rod, M – mass of the cart. Also, in experimental applications it is necessary to consider the friction forces. Usually it is considered that the system is influenced by the force of friction between the cart and path and the friction force of the rotation joint of the pendulum.

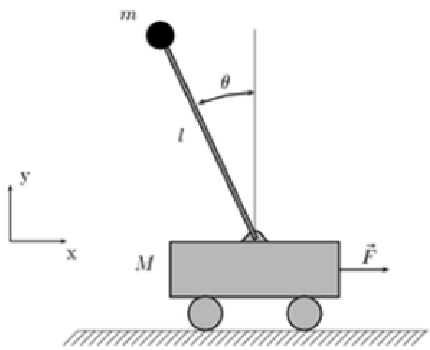

Fig. 1. Inverted pendulum on a cart [8]

Raising the pendulum is achieved by applying a horizontal force variable obtained by using a suitable control algorithm, force that moves the pendulum-cart system along the rail in one direction or the other direction. The areas of application of the problem discussed are numerous: robotics (biped robots), aviation, astronautics, military, etc..

2. THE MODEL

The model used in this paper has the following form:

$$\begin{aligned}
\dot{x}_1 &= \dot{\theta} = x_2 \\
\dot{x}_2 &= \ddot{\theta} = \frac{(M+m)(g\sin x_1 - fx_2) - (lmx_2^2 \sin x_1 + u - bx_4)\cos x_1}{l(M + m\sin^2 x_1)} \\
\dot{x}_3 &= \dot{x} = x_4 \\
\dot{x}_4 &= \ddot{x} = \frac{-mg\sin x_1 \cos x_1 + mlx_2^2 \sin x_1 + fmx_2 \cos x_1 + u - bx_4}{M + m\sin^2 x_1}
\end{aligned} \quad (1)$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$ is the angular position and angular speed of the pendulum, $x_3 = x$ is the cart position, $x_4 = \dot{x}$ is the cart speed, u is the force applied to the cart, b is the friction coefficient of the cart and f is the rotary joint friction coefficient.

If for the case where it is desired to stabilize pendulum in superior vertical position it is possible to apply linear control algorithms (for small values of deviation from the vertical axis), for the raising process a linearization process is not possible. In the case of non-linear systems it is possible to choose more than one operating point and then applying the linearization techniques on intervals but tests have shown that in the case of a strongly non-linear system such techniques do not lead to the desired results.

A radical solution is that, to obtain a proper control algorithm, to forego the benefits of linearization and to consider the designing of a nonlinear model. This will not allow the use of linear control techniques (so it is necessary to implement custom control algorithms for nonlinear application), but on the other hand, it avoids the problems that can occur by performing linearization.

3. METHODS FOR SWING – UP.

In [1] is presented a simple strategy to raise the pendulum, strategy based on energy considerations. It is considered the use of extreme values or the zero value of the control signal. It is shown that raising the pendulum is completely characterized by the ratio *n* between maximum acceleration of the pivot and gravitational force. It is considered the situation in Figure 2, where the pendulum is initially at point A and has the speed 0. Let us consider that the pivot is accelerated to ng maximum acceleration to the right. For an observer located on the pivot, the direction of gravity will be oriented towards OB where $\theta = \arctan n$ and has the module $g\sqrt{1+n^2}$. Therefore the pendulum will swing symmetrically around OB. The speed will be 0 in C where the angle is $\varphi + 2\theta_0$. Therefore the initial angle of the pendulum will increase by an angle $2\theta_0$ every time you change the sign of the speed. This simple strategy allows the introduction of energy in the system.

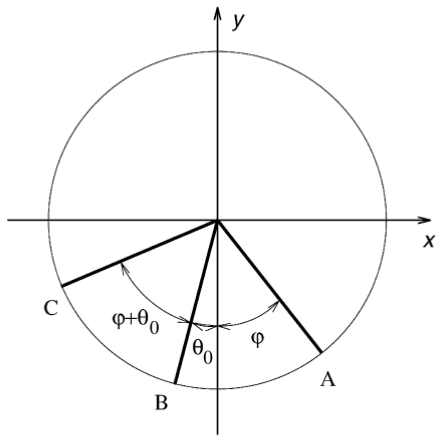

Fig. 2. Swing-Up

For example, it is shown that it is sufficient only one balancing, with two switches of the control signal, where n is greater than 2. In case where 4/3 <*n* <2 it is sufficient only one swing but the number of switching of the control signal is 3. If maximum acceleration is less than 4g/3 to obtain the necessary energy it is required multiple swinging of the pendulum. The paper presents the relationship between acceleration *n* of the pivot and the required number of swings. However, it is not taken into account the fact that the rail length is finite and the force applied to the cart is limited. Moreover, the friction force that occurs is ignored.

The method is repeated in [9] in terms of restrictions on the length of the rail where the cart is moving and limiting the force that controls the movement of the cart. Generalized methods are used to control the energy. The energy is introduced into the system so as not to exceed the limits of travel. When the system accumulates enough energy, it is operated so as it retains its accumulated energy. Finally, near the unstable equilibrium position, a stabilization algorithm is used. The model used does not include friction. The effects of friction forces on the system are studied in [6].

Most of these energetic methods use the switching (changing the direction) of the maximum allowed force. In fact, based on Pontryagin maximum principle it shows that the raise of the pendulum in minimum time the strategies will be "bang-bang" type [1]. It can be shown that there is a close link between these strategies of minimal time and energy control. Basically it will inject energy into the pendulum at a maximum speed possible and also will extract energy at a maximum speed possible in order to reach the equilibrium in minimum time. For small values of n, minimal time strategies generate control signals that initially are identical to the signals generated by the energy strategies. The final part of this rising process differs because the energetic strategy sets the control signal to zero when the necessary energy to reach superior vertical position has been achieved. In fact, the above strategy can be described as strategies without overshoot in terms of the energy of the pendulum. A simple solution that can be applied under restrictions on the movement of the cart, limitation of the maximum force allowed the existence of friction and taking into account the resulting control signal from the previous examples involves the following steps:

Step 1: select the maximum value of the control signal and the number of switches (n_c) of the control signal, with the observation that switching involves changing between the two extreme force (Fmax and -Fmax) and the last switching involves choosing zero force, initially can choose n_c=2;

Step 2: Choose the value of the number of sampling periods between each pair of switches. For each such scenario concerning the control signal, the pendulum evolution is calculated (based on history and pattern of the process) and calculate a performance index of the script.

Step 3: Choose a scenario that ensures considered optimal response.

If such a scenario exists, then the search is over otherwise n_c will be incremented followed by the return to step 2.

Such method involves at first sight a very high computational volume. However, most scenarios will fail quickly. Moreover, one can also consider other indications of future failure scenario. As a result the number of calculations can be reduced.

The following parameters of the process are considered: g=9.81; l=0.11; f=0; M=0.2; m=0.022; b=6; u_{max}=5;. In these conditions, for swing-up phase, the proposed algorithm has generated the signals presented in the fig. 3 (control signal) and fig. 4 (pendulum position, speed).

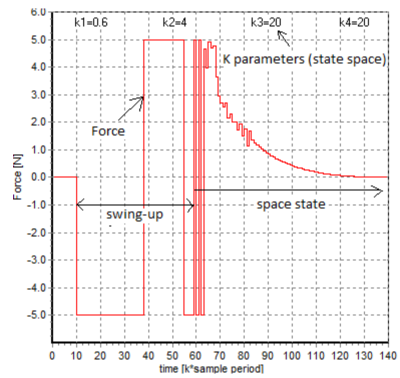

Fig. 3. Control signal

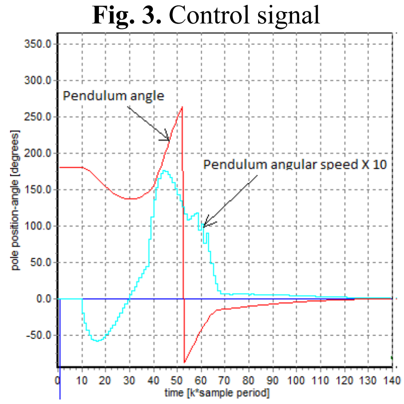

Fig. 4. Cart Position and speed

6. CONCLUSIONS

The paper presents solutions for swing-up phase of the inverse pendulum on a cart. One of the methods more used is based on energy considerations. On the other hand the direct use of nonlinear model in raising pendulum control, on-line simulation and optimum search algorithms leads to efficient solutions to raise the pendulum, even with restrictions regarding limited movement space of the cart, limited force or friction. Even if computation time is significant, results validate the proposed method and provide a basis for experimental implementation.

ACKNOWLEDGEMENTS

This work has been supported by the project "Q-DOC - Creşterea calităţii studiilor doctorale în stiin□e inginereşti pentru sprijinirea dezvoltării societăţii bazate pe cunoaştere" contract no. **POSDRU**/107/1.5/S/78534, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013

7. REFERENCES

[1] K.J. Astrom, K. Furuta, Swinging up a pendulum by energy control, Automatica 36 (2000) 287-295

[2] Kelkar, Atul G., Bo Fang, Warren White, Xin Guo; "Feedback Stabilization of Underactuated Nonlinear Pendulum Cart System Using Matching Conditions," Proceedings of the American Control Conference, v 6, 2002, p 4696-4701.

[3] A. Shiriaev, A. Pogromsky, H. Ludvigsen O. Egeland, On global properties of passivity-based control of an inverted pendulum, Int. J. Robust Nonlinear Control 2000; 10:283}300

[4] Eker, J., & As stroK m, K. J. (1996). A nonlinear observer for the inverted pendulum. In Proceedings of the IEEE conference on control applications (pp. 332-337)

[5] S. Rudra, R. K. Barai, Robust Adaptive Backstepping Control of Inverted Pendulum on Cart System, International Journal of Control and Automation Vol. 5, No. 1, March, 2012

[6] A. Nejadfard, M. J. Yazdanpanah, I. Hassanzadeh, Friction compensation of double inverted pendulum on a cart using locally linear neuro-fuzzy model, Neural Computing and Applications February 2013, Volume 22, Issue 2, pp 337-347

[7] Yao, L.N. and Wang, H. and Zhou, J. and Yue, H. (2004) Variable structure controller for an inverted pendulum. Control Theory and Applications, 21 (5). pp. 724-727.

[8] http://en.wikipedia.org/wiki/Inverted_pendulum

[9] Debasish Chatterjee, Amit Patra, Harish K. Joglekar, Swing-up and stabilization of a cart-pendulum system under restricted cart length, Systems&Control Letters 47 (2002) 355-364

Metodă pentru ridicarea pendulului invers pe un cart

Rezumat: *Pendulul invers pe un cart este un sistem mecanic cu un puternic caracter neliniar. Sistemul este subacţionat, cu două grade de libertate şi cu un sigur semnal de control. Pot fi puse în evidenţă două faze ale controlului acestui proces: faza de ridicare şi faza de stabilizare. În lucrare se investighează soluţii privind faza de ridicare. Uzual strategia de control pentru ridicarea pendulului se bazează pe controlul energiei. Problema se complică dacă se ţine cont de faptul că lungimea şinei pe care se deplasează cartul este finită iar forţa care acţionează asupra cartului are valori limitate. Se propune o soluţie în care se utilizează valorile maxime posibile ale semnalului de control iar momentele în care semnalul de control este comutat sunt obţinute prin intermediul unui algoritm de căutare.*

Adina BĂLAN, drd.prof.ing., Technical University of Cluj-Napoca, Mechatronics Department, E-mail: adinabalan@yahoo.com, Tel.: 0264401756

Alin PLEŞA, dr.ing., Technical University of Cluj-Napoca, Mechatronics Department, E-mail: plesa.alin@gmail.com, Tel.: 0264401756

Radu DONCA, dr.ing., Technical University of Cluj-Napoca, Mechatronics Department, E-mail: doncaradu@yahoo.com , Tel.: 0264401756

Mihai DRĂGHICI, drd.ing., Technical University of Cluj-Napoca, Mechatronics Department, E-mail: mikiduta.dr@gmail.com , Tel.: 0264401756

Vistrian MĂTIEŞ, prof.dr.ing., Technical University of Cluj-Napoca, Mechatronics Department, E-mail: vistrian.maties@yahoo.com , Tel.: 0264401756