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USING OF POLYNOMIAL FUNCTIONS IN MODELING OF THE WORKING PROCESS OF MOBILE ROBOT RmITA

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Abstract: *In the paper, for the mobile structure, RmITA, on the basis of the dynamics equations, developed using new concepts in advanced mechanics, as acceleration energy, will be determined the moving differential equations, which are used to illustrate the modeling of a working process, consisting in the periodical inspection of the cars. The mathematical model will be developed in keeping the restrictions imposed by the particularities of the mechanical structure of the robot, and the car inspection process where the structure is implemented.*
Key words: *advanced mechanics, control, acceleration energy, polynomial interpolation functions.*

1. INTRODUCTION

The mobile robots are parts belonging to a flexible manufacturing system, able to perform complex operations in order to achieve an imposed task. In keeping with the state-of-the art, in the last years, there have been used complex robotic systems for different tasks as inspection or working in dangerous environments, presently they being implemented in the most of human activities due to their benefits. The main task of a mobile robot is to describe certain motion trajectories, based on control functions, which is to displace from a point, to a programmed position.

As a result, the dynamic control functions for the structure of the mobile robots are fundamental for accomplishing these types of tasks. This section of the paper presents the physical properties and mathematical coordinate for a mobile robot, called RmITA [1] that have two independently driven wheels, known as "differentially-driven" robot.

The common characteristic of RmITA is that cannot autonomously produce a velocity which is transversal to the axle of it's wheels, this constrain being called nonholonomic constraint. The nonholonomic constraints reduce the mobile robot's instantaneous velocity degrees of freedom, and hence most robots have only two actuated joints, and one or two driven wheels in the case of a differentially-driven robot. The movement of the mobile robot RmITA, according to Figure 1, is achieved due to the two driving wheels $M_{rf1,2}$, with the centers in the points $O_{1,2}$ executing rotary motion about the axis y_R of its own system, and by the free wheel (driven wheel) M_{rs3}, with the center in the point O_3, which is executing two rotations, one around y_R axis, and another one around z_R axis of its own reference system.

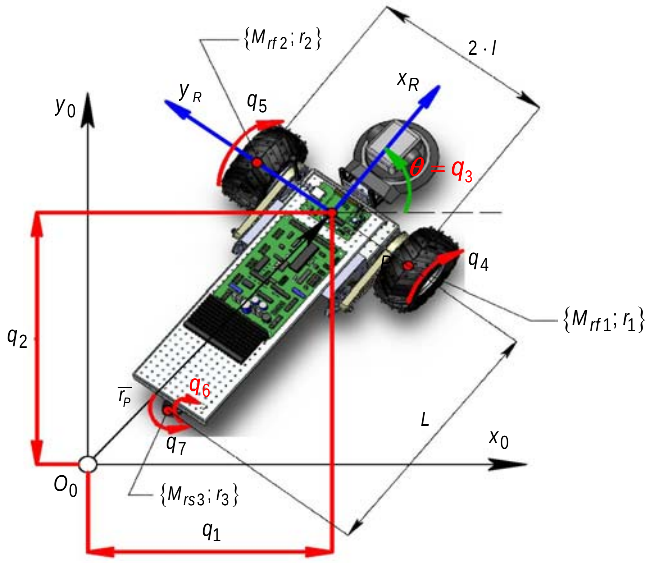

Fig. 1 The independent parameters of RmITA robot

2. THE DYNAMICS OF MOBILE PLATFORM RmITA

In keeping with the Figure 1, the kinematical differential restrictions of the platform are presented in accordance with the Table 1.

Table 1

$-\mathrm{s}\,q_3 \cdot dq_1 + \mathrm{c}\,q_3 \cdot dq_2 = 0$
$\mathrm{c}\,q_3 \cdot dq_1 + \mathrm{s}\,q_3 \cdot dq_2 + l \cdot dq_3 - r_{rf} \cdot dq_4 = 0$
$\mathrm{c}\,q_3 \cdot dq_1 + \mathrm{s}\,q_3 \cdot dq_2 - l \cdot dq_3 - r_{rf} \cdot dq_5 = 0$
$-\mathrm{s}(q_3 + q_7) \cdot dq_1 + \mathrm{c}(q_3 + q_7) \cdot dq_2 - L \cdot \mathrm{c}\,q_7 \cdot dq_3 = 0$
$\mathrm{c}(q_3 + q_7) \cdot dq_1 + \mathrm{s}(q_3 + q_7) \cdot dq_2 - -L \cdot \mathrm{s}\,q_7 \cdot dq_3 - r_{rs} \cdot dq_6 = 0$

In the previous expressions from Table 1, there have been introduced the following notations: $\{\mathrm{c}\,q_i = \cos\mathrm{c}\,q_i;\, sq_i = \sin\mathrm{c}\,q_i, i = 1 \to n\}$. To determine the dynamic equations of mobile structure RmITA are used the same fundamental notions of advanced mechanics-acceleration energy, which is integrated in the following expressions, specific to nonholonomic mechanical systems [2], [3], [4], [5]:

$$\frac{\partial E_A^{tot}}{\partial \ddot{q}_j} + Q_f^j + Q_g^j = Q_m^j + \sum_{i=1}^{7} \lambda_i \cdot a_{ij}, \quad j = 1 \to 7 \quad (1)$$

where, Q_f^j, Q_g^j and Q_m^j are the inertial, gravitational and driving forces; λ_i are the Lagrange`s parameters, and a_{ij} are the displacement coefficients.

In keeping with the kinematical restrictions [6] according to Figure 1 in order to express the dynamic control functions of the mobile structure, on an established trajectory, the actuating motors must overlap the generalized forces , as inertial forces, or gravitational forces. The moving of the structure has to be analyzed in keeping with the fact that the robot RmITA, due to its structure in order to achieve the goal point can`t realize simultaneous a translation and orientation, hence the two displacements will be analyzed independently. Hence, to realize a straight line motion, according to its design, an essential condition is that the driving moments of the wheels must be equal $\{Q_m^4 = Q_m^5\}$. In this case, the expressions (1) of the driving moments of mobile robot, for realize a translation becomes:

$$Q_m^4 = \frac{r_{rf}}{2} \cdot \Big[(M_{pl} + 2 \cdot M_{rf} + M_{rs}) \cdot (\ddot{q}_1 \cdot \mathrm{c}\,q_3 + \ddot{q}_2 \cdot \mathrm{s}\,q_3) + \frac{1}{r_{rs}} \left(I_{\Delta rs} \cdot \ddot{q}_6 + Q_f^{rs} \cdot \mathrm{sgn}(\dot{q}_6) \right) \Big] + I_{\Delta rf} \cdot \ddot{q}_4 + Q_f^{rf} \cdot \mathrm{sgn}(\dot{q}_4) = Q_m^5. \quad (2)$$

To realize the orientation, the following condition has to be accomplished $\{Q_m^4 = -Q_m^5\}$. After a few transformations in (1) there are obtained the expressions of the driving moments for the two wheels in the case of orientation, as:

$$Q_m^4 = \frac{r_{rf}}{2 \cdot l} \cdot \Big[\ddot{q}_3 \cdot \left(I_{\Delta rf} + I_{\Delta pl} + L^2 \cdot M_{rs} + 2 \cdot M_{rf} \cdot l^2 \right) + \frac{L}{r_{rs}} \left(I_{\Delta rs} \cdot \ddot{q}_6 + Q_f^{rs} \cdot \mathrm{sgn}(\dot{q}_6) \right) \Big] + I_{\Delta rf} \cdot \frac{l}{r_{rf}} \cdot \ddot{q}_3 + Q_f^{rf} \cdot \mathrm{sgn}(\dot{q}_4) = -Q_m^5 \quad (3)$$

In (2) and (3) the terms are as follows: $M = M_A + 2 \cdot M_{rf} + M_{rs}$ the total mass of the mobile system, M_A the mass of the platform (without wheels), M_{rs} and M_{rf} is the mass of the back respectively of the front wheels, r_{rs} and r_{rf} the radius of previous wheels, $I_{\Delta Pl}$ is the inertia moment with respect to O_z axis, $I_{\Delta rf}$ and $I_{\Delta rs}$ are the inertia moments of the forward and respectively backward wheels, ${}^R x_C$ the position of the mass center, $\lambda_j, j = 1 \to 7$ Lagrange parameters, μ the friction coefficient; $Q_m^{4,5}$ wheels driving forces, $\mathrm{sgn}(\dot{q}_i)$, $i = 4, 6, 7$ represents the signs of the velocities. [3].

The relations (2) and (3) are represent the expressions of the driving moments of the mobile robot wheels.

3. POLYNOMIAL FUNCTIONS OF $(3n)$ TYPE WITH RESTRICTIONS

In order to present graphically the variation of the driving moments, further will be presented sequences from a working process of the mobile structure RmITA. To study the dynamic behavior of mobile structure, between two intermediary points, of a working sequence, there will be used interpolating functions. The interpolating functions are consisting in generation of linear functions with respect to time for generalized accelerations from every driving joint belonging to the robot. According to [7] it is generated a linear function with respect to time as:

$$\ddot{q}_{ji}(\tau) = \frac{\tau_i - \tau}{t_i} \cdot \ddot{q}_{ji}(\tau_{i-1}) + \frac{\tau - \tau_{i-1}}{t_i} \cdot \ddot{q}_{ji}(\tau_i) \quad (4)$$

where $t_i = \tau_i - \tau_{i-1}$ represents the duration of each $(i = 1 \rightarrow 3)$ segment of the trajectory. The unknowns are the generalized accelerations at τ_{i-1} and τ_i, defined as:

$$\ddot{q}_{ji}(\tau_{i-1}) = \ddot{q}_{ji-1} \; ; \quad \ddot{q}_{ji}(\tau_i) = \ddot{q}_{ji} \, . \quad (5)$$

After a few transformations, are obtained the functions for generalized velocities and coordination as shown:

$$\dot{q}_{ji}(\tau) = -\frac{(\tau_i - \tau)^2}{2 \cdot t_i} \cdot \ddot{q}_{ji-1} + \frac{(\tau - \tau_{i-1})^2}{2 \cdot t_i} \cdot \ddot{q}_{ji} + \left(\frac{q_{ji}}{t_i} - \frac{t_i}{6} \cdot \ddot{q}_{ji}\right) - \left(\frac{q_{ji-1}}{t_i} - \frac{t_i}{6} \cdot \ddot{q}_{ji-1}\right) \quad (6)$$

$$q_{ji}(\tau) = \frac{(\tau_i - \tau)^3}{6 \cdot t_i} \cdot \ddot{q}_{ji-1} + \frac{(\tau - \tau_{i-1})^3}{6 \cdot t_i} \cdot \ddot{q}_{ji} + \left(\frac{q_{ji}}{t_i} - \frac{t_i}{6} \cdot \ddot{q}_{ji}\right) \cdot (\tau - \tau_{i-1}) + \left(\frac{q_{ji-1}}{t_i} - \frac{t_i}{6} \cdot \ddot{q}_{ji-1}\right) \cdot (\tau_i - \tau) \quad (7)$$

Further is considered a working process, where is integrated the RmITA structure, presented in Figure 2, which consists in the periodical inspection of a car. One cycle of car inspection process consists of $j = 1 \rightarrow 25$ phases, so called work sequence, corresponding to $k = 0 \rightarrow 75$ configurations.

To illustrate the mathematical modeling of working process, there will be used polynomial function of $(3n)$ type with restrictions, applied on each sequence of the working process described by Fig. 2. Hence, there will be analyzed the sequences $(j = 4,5)$, which are representing a rotation (orientation) (q_3), and an linear translation (q_1). The fourth sequence, $j = 4$ and $k = 9^* \rightarrow 12$, represents orientation of the robotic system with $-\pi/2$, around Oz_0 axis, by the generalized coordinate (q_3), followed by the sequence $j = 5$ and $k = 12^* \rightarrow 15$ concretized by the linear translation (q_1) along Ox_0 axis, on a $1,6\ m$ distance.

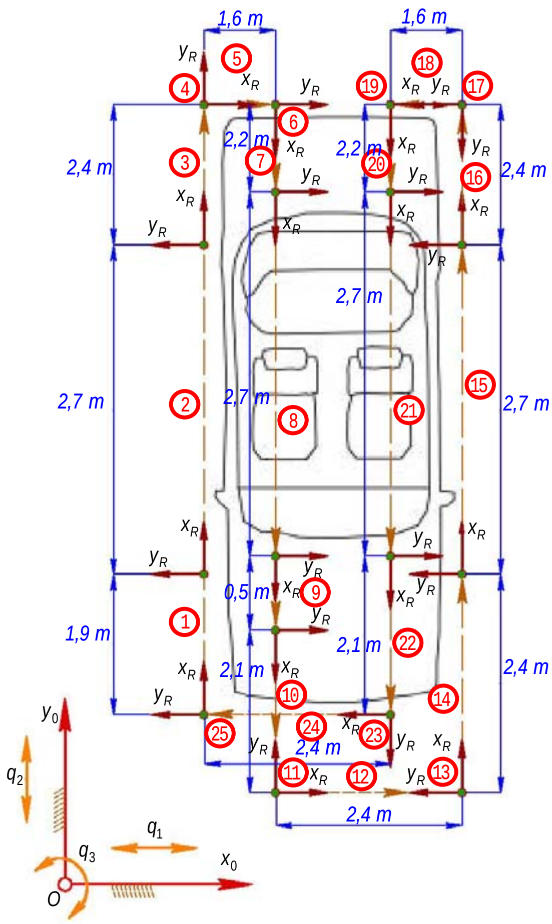

Fig. 2 The working process of RmITA structure

In order to express the driving moments of the mobile structure, the input parameters for study are presented as follows in the Table 2, considering that the first motion is a rotation, and the second is a linear translation.

Table 2.

Input parameters for working process

Seq.	Config. k=0→75	Time $\tau_{jk}\langle s\rangle$	Duration $t_i\langle s\rangle$	Coordinates values $q_{jk}\langle m,rad\rangle$
$4\rightarrow(q_3)$ Rotation	9*	115	3	$\pi/2$
	10	116	1	
	11	117	1	
	12	118	1	0
$5\rightarrow(q_1)$ Translation	12*	118	-	0
	13	122	4	
	14	127	5	
	15	131	4	1.6

On the basis of the parameters presented in Table 2 in keeping with (5)-(7), are determined the expressions for coordinates, velocities and accelerations as in Table 3.

Table 3.

Expressions for generalized kinematic parameters

Secq.	***Interval i=1→3***	***Expressions for generalized coordinates, velocities and accelerations*** $q_{jk}\langle m,rad\rangle$	$\dot{q}_{jk}\langle m/s,\ rad/s\rangle$	$\ddot{q}_{jk}\left\langle \frac{m/s^2}{rad/s^2}\right\rangle$
4 Translation	***1***	$-0{,}27\cdot\tau^3+90{,}3\cdot\tau^2-$ $-10386{,}9\cdot\tau+$ $+398165{,}8$	$-0{,}8\cdot\tau^2+180{,}7\cdot\tau-$ $-10386{,}9$	$-1{,}6\cdot\tau+$ $+180{,}6$
	2	$0{,}6\cdot\tau^3-182{,}9\cdot\tau^2+$ $+21318{,}1\cdot\tau-$ $-827759{,}2$	$1{,}6\cdot\tau^2-365{,}9\tau+$ $+21318{,}1$	$3{,}2\cdot\tau-$ $-365{,}9$
	3	$-0{,}7\cdot\tau^3+92{,}7\cdot\tau^2-$ $-10935{,}9\cdot\tau+$ $+430144{,}8$	$-0{,}8\cdot\tau^2+$ $+185{,}4\cdot\tau-$ $-10935{,}9$	$-1{,}6\cdot\tau+$ $+185{,}4$
5 Rotation	***1***	$0{,}1\cdot\tau^3-1{,}2\cdot\tau^2+$ $+142{,}8\cdot\tau-$ $-5617{,}2$	$0{,}1\cdot\tau^2-$ $-2{,}4\cdot\tau+$ $+142{,}8$	$0{,}1\cdot\tau-$ $-2{,}4$
	2	$-0{,}1\cdot\tau^3+2{,}1\cdot\tau^2-$ $-254{,}1\cdot\tau+$ $+10523{,}7$	$-0{,}1\cdot\tau^2+$ $+4{,}1\cdot\tau-$ $-254{,}1$	$-0{,}1\cdot\tau+$ $+4{,}1$
	3	$0{,}1\cdot\tau^3-1{,}4\cdot\tau^2+$ $+176{,}1\cdot\tau-$ $-7684{,}2$	$0{,}1\cdot\tau^2-2{,}7\cdot\tau+$ $+176{,}1$	$0{,}1\cdot\tau-$ $-2{,}7$

On the basis of Table 2 and Table 3, are represented the time variation of generalized coordinates, velocities and accelerations.

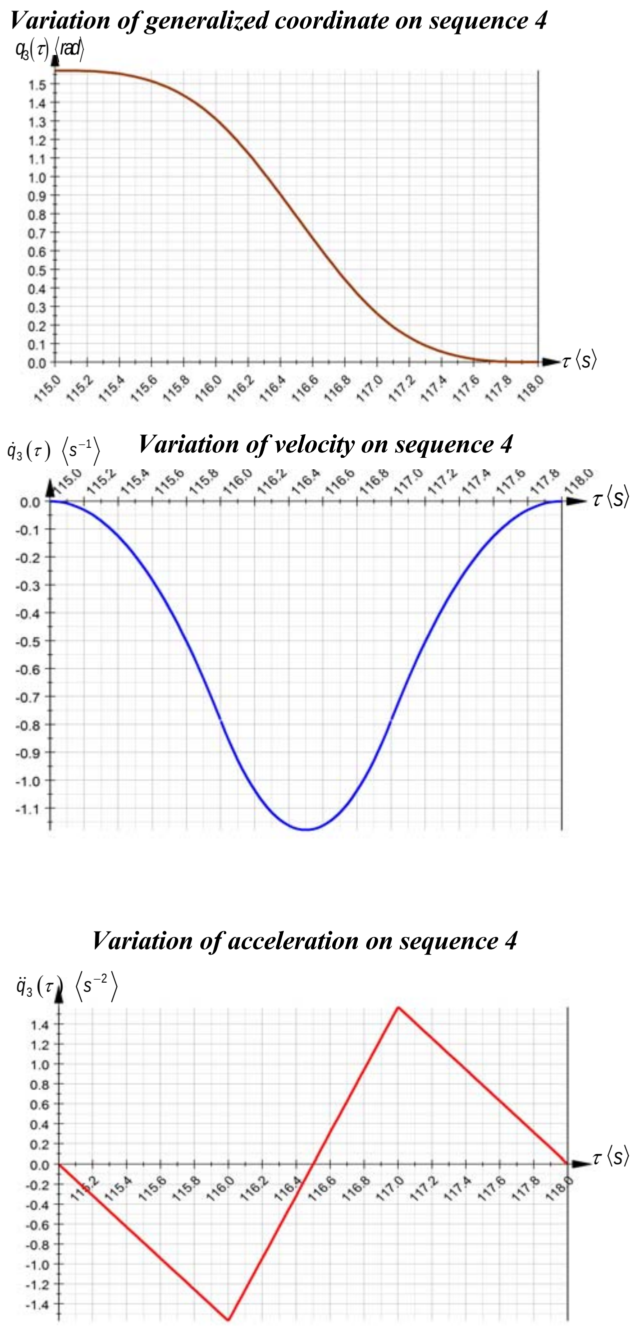

Figure 3 Kinematical parameters on sequence j=4

These representations fully comply with the restrictive conditions imposed by the working process and also ensure continuity in positions velocities and accelerations at passing through intermediary points belonging to sequences of the processes. The same representations highlight analytically and graphically the starting from zero, transitional and stopping at different points belonging to imposed task.

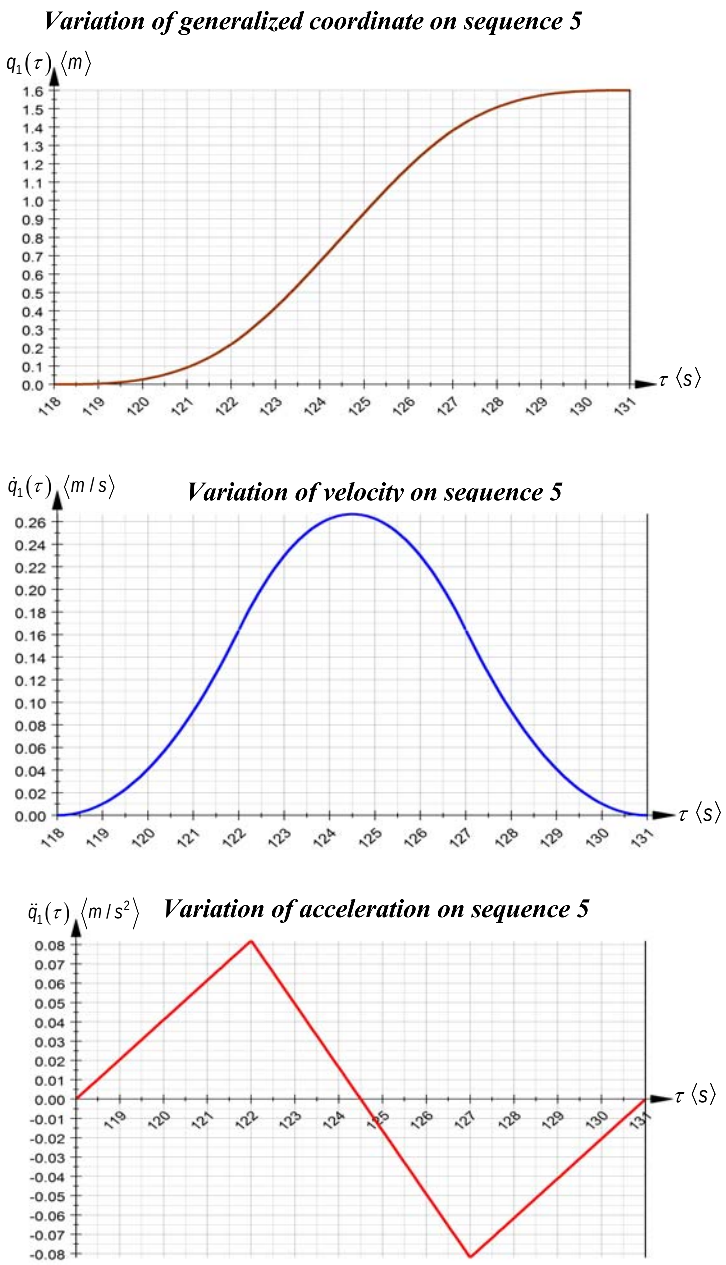

Figure 4 Kinematical parameters on sequence j=5

On the basis of expression (3) and (2), are represented graphically the variation for the driving moments, on the entire working process, where is implemented the mobile robot RmITA. As an important remark, in keeping with Table3, the working process for the sequences$(j=4,5)$, is highlighted with red color, as results from Figure 5.

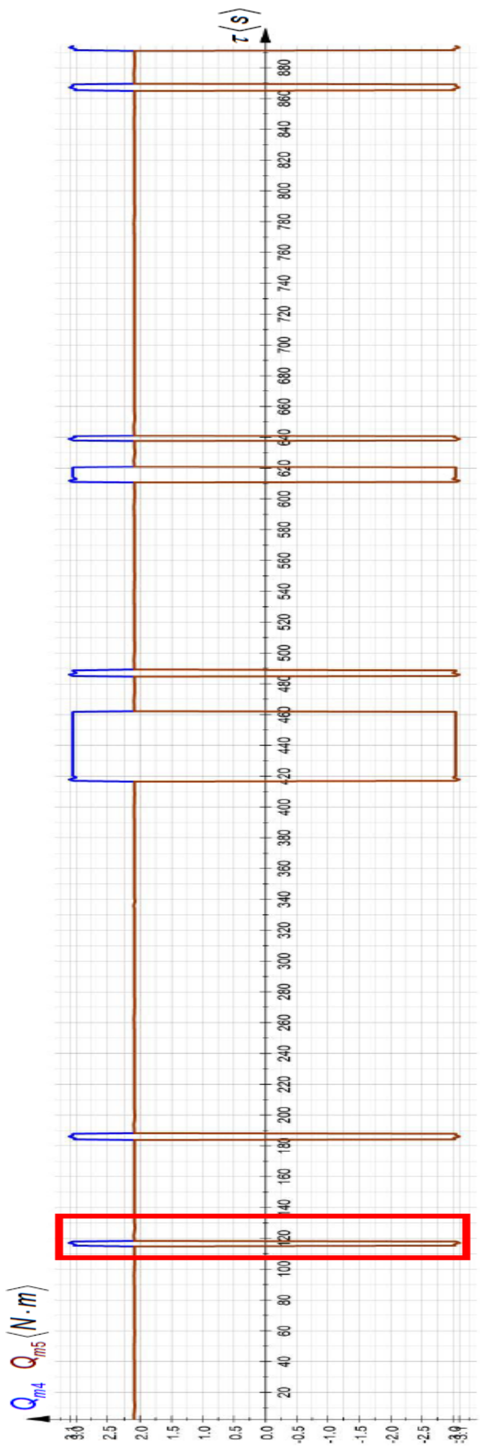

Figure 5 Variation for the driving moments on the entire working process of RmITA

4. CONCLUSIONS

In the above presented paper there have been developed the dynamic control functions for the mobile robot RmITA. Using as starting point the dynamics equations, in keeping with the conditions which must be taken into account for straight line displacement or orientation, there have been deduced the driving moments of the wheels.

The dynamic model of the structure is based on acceleration energy, which substituted in the specific expression of mechanical systems with nonholonomous links are leading to differential equations of motion for the mobile robot RmITA. Having the driving moments of the wheels, the structure was integrated in a technological process.

The process has been modeled using $(3n)$ type with restrictions polynomial functions, in keeping with the conditions necessary to achieve a straight line trajectory, orientation and kinematic constraints of the robot.

For two sequences of the working process, consisting in a linear translation and a rotation for orientation, there were represented graphically the variation in time of the driving moments.

The graphical representation of the variation of accelerations and driving moments on considered sequences of the process is in accordance with the real working process of the mobile robot.

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Utilizarea funcţiilor polinomiale pentru modelarea procesului de lucru a robotului RmITA

Rezumat: In lucrare, pe baza ecuaţiilor dinamicii, care au fost dezvoltate folosind noi concepte în mecanica avansată, precum energia acceleraţiilor, au fost determinate ecuaţiile diferenţiale de mişcare, pentru structura mobilă RmITA. Aceste ecuaţii, au fost utilizate pentru a ilustra modelarea matematica a unui lucru proces de lucru, care constă în verificarea periodică a autovehiculelor. Modelul matematic a fost dezvoltat pe de o parte în conformitate cu restricţiile impuse de particularităţile structurii mecanice a robotului, iar pe de alta parte de cele impuse de procesul de inspecţie auto, în care este implementată structura mobilă RmITA.

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