TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue IV, November, 2013

GEOMETRIC MODELING OF A 5R-TYPE INDUSTRIAL ROBOT

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Abstract: *To implement a mechanical robot structure in an industrial working process, as welding, spray-painting, inspections and s.o, it has to be modelled mathematically. The first step of mathematical modelling, is the determination of the geometrical control functions, consisting in the expressions of the position and orientation of the end effector, which contains the working tool. According to this, the paper contains the geometrical modelling for an articulated robot, having five rotations.*

Key words: *articulated robot, mathematical modeling, homogeneous transformation, working process.*

1. DESCRIPTION OF 5R-TYPE MODULAR ARTICULATED ROBOT

In order to apply the methods of geometrical modeling by symbolic calculations, it has been considered a mechanical structure of industrial robot of 5R-type. This is a modular robot, articulated, whose kinematic scheme is shown in Figure 1.

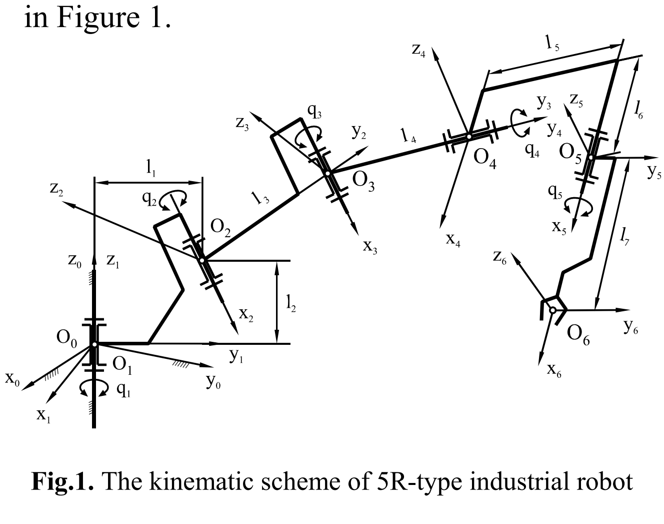

Fig.1. The kinematic scheme of 5R-type industrial robot

The mechanical robot system, presented in Figure 1 is consisting in five rotational joints, as follows below:

- joint (1) performs a rotation around the vertical fixed axis $O_1z_1 \equiv O_0z_0$, with q_1 ;
- joint (2), is rotating with the angle q_2 around the axis O_2x_2;
- joint (3) is rotating with q_3 around the axis O_3x_3;
- joint (4) is rotating with q_4 around the axis O_4x_4;
- joint (5) is rotating with q_5 around the axis O_5x_5;

At the end of the fifth joint, is attached the gripper, oriented along O_6x_6 axis. The robot`s geometrical dimensions, according to the same Figure 1 are denoted as: $l_1 ... l_7$

2. THE DEFINING OF MECHANICAL 5R-TYPE ROBOT STRUCTURE

In order to define the mechanical structure of the robot, there are described the position and orientation of the reference systems $\{i\}$ attached to the every joint (i), and the gripping device, noted (6), expressed by the locating matrices ${}^{i-1}_{i}[T], i = 1 \div n+1$. They are containing the rotation submatrices ${}^{i-1}_{i}[R]$, which are expressing the orientation of system $\{i\}$ with respect to system $\{i-1\}$, and the position vectors ${}^{i-1}\overline{p}_i$, expressing the position of the origin $\{O_i\}$, of the systems $\{i\}$ in relation to the origins $\{O_{i-1}\}$, of the systems $\{i-1\}$. In keeping with the previous considerations, and [1], [2], the expressions of the locating matrices are:

$$ {}^{0}_{1}[T]=\begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1) $$

$$ {}^{1}_{2}[T]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_2 & -sq_2 & l_1 \\ 0 & sq_2 & cq_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2) $$

$$ {}^{2}_{3}[T]=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_3 & -sq_3 & l_3 \\ 0 & sq_3 & cq_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (3) $$

$$ {}^{3}_{4}[T]=\begin{bmatrix} cq_4 & 0 & sq_4 & 0 \\ 0 & 1 & 0 & l_4 \\ -sq_4 & 0 & cq_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4) $$

$$ {}^{4}_{5}[T]=\begin{bmatrix} 1 & 0 & 0 & l_6 \\ 0 & cq_5 & -sq_5 & l_5 \\ 0 & sq_5 & cq_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (5) $$

$$ {}^{5}_{6}[T]=\begin{bmatrix} 1 & 0 & 0 & l_7 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (6) $$

The matrices presented in the expressions (1)-(6), are describing the position and orientation of each kinetic element belonging to 5R–type robot, with respect to the reference system attached to the previous element. [3], [4]

3. THE EQUATIONS OF THE GEOMETRICAL MODEL FOR THE 5R TYPE ROBOT

According to [1],[2], in order to establish the geometry equations for a mechanical robot structure, there are applied some matrix multiplications, as:

$$ {}^{0}_{i}[T]={}^{0}_{i-1}[T]\cdot{}^{i-1}_{i}[T] \quad (7) $$

Being determined the matrices for position and orientation of the reference systems $\{i\}$, attached to each joint (i), with respect to $\{0\}$ fixed frame. According to (7), there are obtained the following expressions:

$$ {}^{0}_{2}[T]=\begin{bmatrix} cq_1 & -cq_2sq_1 & sq_1sq_2 & -l_1sq_1 \\ sq_1 & cq_1cq_2 & -cq_1sq_2 & l_1cq_1 \\ 0 & sq_2 & cq_2 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8) $$

$$ {}^{0}_{3}[T]=[T_1 \mid T_2 \mid T_3 \mid T_4] \quad (9) $$

where:

$$ T_1=\begin{bmatrix} cq_1 \\ sq_1 \\ 0 \\ 0 \end{bmatrix};\quad T_2=\begin{bmatrix} -c(q_2+q_3)\cdot sq_1 \\ c(q_2+q_3)\cdot cq_1 \\ s(q_2+q_3) \\ 0 \end{bmatrix} $$

$$ T_3=\begin{bmatrix} s(q_2+q_3)\cdot sq_1 \\ -s(q_2+q_3)\cdot cq_1 \\ c(q_2+q_3) \\ 0 \end{bmatrix};\quad T_4=\begin{bmatrix} -sq_1\cdot(l_1+l_3cq_2) \\ cq_1\cdot(l_1+l_3cq_2) \\ l_2+l_3sq_2 \\ 1 \end{bmatrix} $$

Due to the complexity of the homogeneous transformation matrices for the joints (4), (5) and for the gripping device, there will be presented separately the rotation matrices and vectors of the position, as follows:

$$ {}^{0}_{4}[T]=\begin{bmatrix} {}^{0}_{4}[R]=[R_1 \;\; R_2 \;\; R_3] & \bar{p}_4 \\ 0\;\;0\;\;0 & 1 \end{bmatrix} \quad (10) $$

where:

$$ R_1=\begin{bmatrix} cq_1cq_4-s(q_2+q_3)\cdot sq_1sq_4 \\ cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4 \\ -c(q_2+q_3)\cdot sq_4 \end{bmatrix}; $$

$$ R_2=\begin{bmatrix} -c(q_2+q_3)\cdot sq_1 \\ c(q_2+q_3)\cdot cq_1 \\ s(q_2+q_3) \end{bmatrix} $$

$$ R_3=\begin{bmatrix} cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1 \\ sq_1sq_4-s(q_2+q_3)\cdot cq_1cq_4 \\ c(q_2+q_3)\cdot cq_4 \end{bmatrix} $$

$$ \bar{p}_4=\begin{bmatrix} -sq_1(l_1+l_3cq_2)-l_4c(q_2+q_3)\cdot sq_1 \\ cq_1(l_1+l_3cq_2)+l_4\cdot c(q_2+q_3)\cdot cq_1 \\ l_2+l_4s(q_2+q_3)+l_3sq_2 \end{bmatrix} \quad (11) $$

and:

$$ {}^{0}_{5}[T]=\left[\begin{array}{c|c} {}^{0}_{5}[R]=[R_4 \quad R_5 \quad R_6] & \overline{p}_5 \\ \hline 0 \;\; 0 \;\; 0 & 1 \end{array}\right] \tag{12} $$

where:

$$ R_4=\begin{bmatrix} cq_1cq_4 - \\ s(q_2+q_3)\cdot sq_1sq_4 \\ cq_4sq_1 + \\ s(q_2+q_3)\cdot cq_1sq_4 \\ -c(q_2+q_3)\cdot sq_4 \end{bmatrix} $$

$$ R_5=\begin{bmatrix} sq_5cq_1sq_4+s(q_2+q_3)\cdot \\ cq_4sq_1-c(q_2+q_3)\cdot cq_5sq_1 \\ sq_5(sq_1sq_4-s(q_2+q_3)\cdot cq_1cq_4)+ \\ c(q_2+q_3)\cdot cq_1cq_5 \\ s(q_2+q_3)\cdot cq_5 + \\ c(q_2+q_3)\cdot cq_4sq_5 \end{bmatrix} $$

$$ R_6=\begin{bmatrix} cq_5(cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1)+ \\ c(q_2+q_3)\cdot sq_1sq_5 \\ cq_5(sq_1sq_4-s(q_2+q_3)\cdot cq_1cq_4)- \\ c(q_2+q_3)\cdot cq_1sq_5) \\ c(q_2+q_3)\cdot cq_4cq_5 - \\ s(q_2+q_3)\cdot sq_5 \end{bmatrix} $$

$$ \overline{p}_5=\left[\begin{array}{c} l_6\cdot(cq_1cq_4-s(q_2+q_3)\cdot sq_1sq_4)- \\ -sq_1(l_1+l_3cq_2)-(l_4+l_5)\cdot c(q_2+q_3)\cdot sq_1 \\ \hdashline l_6\cdot(cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4)+ \\ +cq_1(l_1+l_3cq_2)+(l_4+l_5)\cdot c(q_2+q_3)\cdot cq_1 \\ \hdashline l_2+(l_4+l_5)\cdot s(q_2+q_3)+l_3sq_2- \\ -l_6\cdot c(q_2+q_3)\cdot sq_4 \end{array}\right] \tag{13} $$

After performing the calculus, imposed by (7), there is obtained the matrix:

$$ {}^{0}_{6}[T]=\left[\begin{array}{c|c} {}^{0}_{6}[R]=[R_7 \quad R_8 \quad R_9] & \overline{p}_6 \\ \hline 0 \;\; 0 \;\; 0 & 1 \end{array}\right] \tag{14} $$

where:

$$ R_7=\begin{bmatrix} cq_1cq_4 - \\ s(q_2+q_3)\cdot sq_1sq_4 \\ cq_4sq_1 + \\ s(q_2+q_3)\cdot cq_1sq_4 \\ -c(q_2+q_3)\cdot sq_4 \end{bmatrix} \tag{15} $$

$$ R_8=\begin{bmatrix} sq_5cq_1sq_4+s(q_2+q_3)\cdot \\ cq_4sq_1-c(q_2+q_3)\cdot cq_5sq_1 \\ sq_5(sq_1sq_4-s(q_2+q_3)\cdot cq_1cq_4)+ \\ c(q_2+q_3)\cdot cq_1cq_5 \\ s(q_2+q_3)\cdot cq_5 + \\ c(q_2+q_3)\cdot cq_4sq_5 \end{bmatrix} \tag{16} $$

$$ R_9=\begin{bmatrix} cq_5(cq_1sq_4+s(q_2+q_3)\cdot cq_4sq_1)+ \\ c(q_2+q_3)\cdot sq_1sq_5 \\ cq_5(sq_1sq_4-s(q_2+q_3)\cdot cq_1cq_4)- \\ c(q_2+q_3)\cdot cq_1sq_5) \\ c(q_2+q_3)\cdot cq_4cq_5 - \\ s(q_2+q_3)\cdot sq_5 \end{bmatrix} \tag{17} $$

and:

$$ \overline{p}_6=\left[\begin{array}{c} (l_6+l_7)\cdot(cq_1cq_4-s(q_2+q_3)\cdot sq_1sq_4)- \\ -sq_1(l_1+l_3cq_2)-(l_4+l_5)\cdot c(q_2+q_3)\cdot sq_1 \\ \hdashline (l_6+l_7)\cdot(cq_4sq_1+s(q_2+q_3)\cdot cq_1sq_4)+ \\ +cq_1(l_1+l_3cq_2)+(l_4+l_5)\cdot c(q_2+q_3)\cdot cq_1 \\ \hdashline l_2+(l_4+l_5)\cdot s(q_2+q_3)+l_3sq_2- \\ -(l_6+l_7)\cdot c(q_2+q_3)\cdot sq_4 \end{array}\right] \tag{18} $$

The form (14) contents the expressions for the position (18) and orientation (15)- (17) of the characteristic point of the gripping device, which coincides with the origin O_6 of the system $\{6\}$, with respect to the system $\{0\}$. According to previous statement, the orientation of the axes of the system $\{6\}$ relative to $\{0\}$ frame, is expressed by the elements of the rotation matrix ${}^{0}_{6}[R]$, determined by (14), whose columns are the Cartesian components of the unit vectors of the system $\{6\}$ related to the $\{0\}$ fixed frame. The other component of (14), $\overline{p}_6$, determined by (18) represents the position vector of the same system $\{6\}$ relative to $\{0\}$ frame. [1]-[6]

4. CONCLUSIONS

As can be seen from previous considerations, by locating matrices has been studied the transformations, between a coordinate frame located in the end effector of an articulated industrial robot and a coordinate frame located at the base of the mechanical structure. The geometric model equations, for a 5R-type mechanical system, as presented in the Figure 1, are expressed by (14). They represent the position and orientation of the reference system attached to the gripping device, with respect to the fixed frame $\{0\}$, attached to the robot`s base.

This transformations specifies the location (position and orientation) of the end-effectors in space with respect to the base of the robot, but doesn't express which configuration of the arm is required to achieve this location.

5. REFERENCES

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Modelarea geometrică a unui robot industrial de tipul 5R

Rezumat: Pentru a implementa o structură mecanică de robot într-un proces de lucru industrial, precum sudură, vopsire, inspecţii, etc., acesta trebuie să fie modelat matematic. Primul pas al modelarii matematice, este determinarea funcţiilor de control geometrice, adică stabilirea expresiilor pentru poziţie şi orientare a efectorului final, care conţine de scula de lucru. Astfel, lucrarea prezintă modelarea geometrică a unui robot articulat având cinci rotaţii.

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