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SOLVING DIRECT KINEMATICS MODELLING OF TTTR GANTRY MODULAR ROBOT BY ITERATIVE METHOD

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Abstract Solving the direct kinematic modelling of robots can be done through various methods. The iterative method is one of the methods commonly used in the study of kinematic modeling. At the base of the method is the introduction in calculus of vectors position, the rotation matrix, and their derivates with respect to time. Using the iterative method, this paper develops methodologies to determine the kinematics parameters: operational angular speeds and accelerations, respectively operational linear velocities and accelerations, witch characterize the movement of each element of a TTTR modular Gantry robot.

Key words: kinematic modeling, geometric modeling, serial robots, Gantry robots.

1. INTRODUCTION

Applying the kinematic modeling in the robots control eliminates the disadvanteges of the geometric modeling, due to the nonlinearity of the geometric equations $\overline{x} = f(\overline{q})$ and the lack of control over the velocity and acceleration over the trajectory of movement.

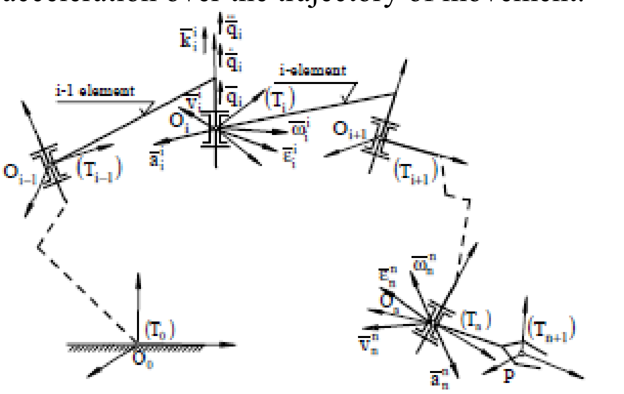

Figure 1. Kinematic structure of a (n) degrees of freedom robot

According with [3] and [5], figure 1 shows the kinematic structure of a (n) degrees of freedom robot. This structure is compose of (n) rigide elements, which perform moves determined by the cinematic V class cuples, considered perfect from mechanical point of view.

$$\dot{\overline{X}}^{(n)0} = f(\overline{q}, \dot{\overline{q}}), \quad \ddot{\overline{X}}^{(n)0} = f(\overline{q}, \dot{\overline{q}}, \ddot{\overline{q}}) \qquad (1)$$

$$\dot{\overline{q}} = f^{-1}\left(\overline{q}, \dot{\overline{X}}^{(n)0}\right), \quad \ddot{\overline{q}} = f^{-1}\left(\overline{q}, \dot{\overline{X}}^{(n)0}, \ddot{\overline{X}}^{(n)0}\right), \qquad (2)$$

2. THEORETICAL JUSTIFICATION

According to [5] the iterative method consists of browsing the robot kinematic chain from the fixed base to the gripping device (n) and determination in succesive iterations of the following the kinematic parameters:

$$\left\{\overline{k}_i^i, \overline{\omega}_i^i, \overline{\varepsilon}_i^i, \overline{v}_i^i, \overline{a}_i^i, i = 1 \div n\right\}. \qquad (3)$$

These parameters characterize the movement of each element i, $i = 1 \div n$, in reation to the fixed system (T_0) from the robot base, as expressed in the reference system (T_i), having the following meaning:

$\overline{k}_i^i$ - unit vector of the motion axis of order „i"

$\overline{\omega}_i^i$, $\overline{\varepsilon}_i^i$ - angular velocity and acceleration with witch the „i" element is it rotates around it's point O_i (origin of (T_i) system , expressed in relation to (T_0));

$\overline{v}_i^i$, $\overline{a}_i^i$ - speed and acceleration of O_i point, expressed in relation to (T_0)

The iterative method it is used in browsing of Newton-Euler method of dynamic modeling of industrial robots. In this method must specify the kinematic parameters coresponding to the robot base. These parameters have the following expressions:

$$[\overline{\omega}]_0^0 = [0\ \ 0\ \ 0]^T,\ [\overline{v}]_0^0 = [0\ \ 0\ \ 0]^T$$
$$[\overline{\varepsilon}]_0^0 = [0\ \ 0\ \ 0]^T,\ [\overline{a}]_0^0 = [0\ \ 0\ \ g]^T, \quad (4)$$

where g is the gravitational aceleration.

The determination of the kinematic parameters in the relation (3) is the result of the robot geometric modeling that has to be solved in advance. Thus, it is possible to determine the homogeneous transformation matrices:

$$[T]_i^{i-1}(q_i(t)) = \begin{bmatrix} [R]_i^{i-1} & | & \overline{r}_i^{i-1} \\ --- & | & --- \\ 0\,0\,0 & | & 1 \end{bmatrix} \cdot \left[[T]_i^{i-1}\right]^{-1} =$$
$$\begin{bmatrix} [R]_i^{i-1} & | & -\left[[R]_i^{i-1}\right]^T \cdot \overline{r}_i^{i-1} \\ --- & | & ------- \\ 0\,0\,0 & | & 1 \end{bmatrix}$$

respectively:

$$[T]_i^0(q_j(t),\ j = 1 \div i) = \begin{bmatrix} [R]_i^0 & | & \overline{p}_i \\ --- & | & --- \\ 0\ 0\ 0 & | & 1 \end{bmatrix}. \quad (5)$$

In accordance with the paper [3], following relations must be applied:

$$\overline{\omega}_i^0 \times [R]_i^0 = \left\{\overline{\omega}_{i-1}^0 + [R]_{i-1}^0 \cdot \overline{\omega}_i^{i-1}\right\} \times [R]_i^0$$
$$\overline{\omega}_i^0 = \overline{\omega}_{i-1}^0 + [R]_{i-1}^0 \cdot \overline{\omega}_i^{i-1} \quad (6)$$
$$\overline{\varepsilon}_i^0 = \overline{\varepsilon}_{i-1}^0 + [R]_{i-1}^0 \cdot \overline{\varepsilon}_{i-1}^{i-1} + \overline{\omega}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{\omega}_i^{i-1}$$
$$\overline{\omega}_i^0 = \overline{\omega}_{i-1}^0\ ,\ \overline{\varepsilon}_i^0 = \overline{\varepsilon}_{i-1}^0\ ,$$

and

$$\overline{v}_i^0 = \overline{v}_{i-1}^0 + [R]_{i-1}^0 \cdot \overline{v}_i^{i-1} + \overline{\omega}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{r}_i^{i-1}$$
$$\overline{a}_i^0 = \overline{a}_{i-1}^0 + [R]_{i-1}^0 \cdot \overline{a}_i^{i-1} + \overline{\varepsilon}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{r}_i^{i-1} +$$
$$\overline{\omega}_{i-1}^0 \times \left(\overline{\omega}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{r}_i^{i-1}\right) + 2\,\overline{\omega}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{v}_i^{i-1} \quad (7)$$

If the reference of (T_i) system position, in relation with the (T_{i-1}) system, remains constant in connection with time, then:

$$\overline{v}_i^0 = \overline{v}_{i-1}^0 + \overline{\omega}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{r}_i^{i-1}$$
$$\overline{a}_i^0 = \overline{a}_{i-1}^0 + \overline{\varepsilon}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{r}_i^{i-1} + \overline{\omega}_{i-1}^0 \times \left(\overline{\omega}_{i-1}^0 \times [R]_{i-1}^0 \cdot \overline{r}_i^{i-1}\right). \quad (8)$$

Since the kinematic parameters of equation (3) are expressed in the (T_i) reference system, it is necessary for ecuatiile (6) to be multiplied to the left with the rotational matrix.

$$\left[[R]_i^0\right]^T = [R]_0^i = [R]_{i-1}^i \cdot [R]_0^{i-1} \quad (9)$$

Thus, the relative angular velocity of the motion axis (i) can be expressed as follows:

$$[R]_{i-1}^i \cdot \overline{\omega}_i^{i-1} = \begin{cases} [R]_{i-1}^i \cdot \overline{k}_i^{i-1} \cdot \dot{q}_i = \dot{q}_i \cdot \overline{k}_i^i, \\ \quad \text{in rotational couple} \\ 0\ ,\ \text{in translational couple}\ . \end{cases} \quad (10)$$

Thus, you get the relative angular velocity of the motion axis (i) that has the expression:

$$[R]_{i-1}^i \cdot \overline{\varepsilon}_i^{i-1} = \begin{cases} [R]_{i-1}^i \cdot \overline{k}_i^{i-1} \cdot \ddot{q}_i = \ddot{q}_i \cdot \overline{k}_i^i, \\ \quad \text{in rotation couple} \\ 0\ ,\ \text{in translation couple}. \end{cases} \quad (11)$$

The expression below represents the linear speed $\overline{v}_i^i$:

$$\overline{v}_i^i = [R]_0^i \cdot \overline{v}_i^0 =$$
$$[R]_{i-1}^i \left\{\overline{v}_{i-1}^{i-1} + \overline{\omega}_{i-1}^{i-1} \times \overline{r}_i^{i-1}\right\} + [R]_{i-1}^i \cdot \overline{v}_i^{i-1}, \quad (12)$$

Thus, the relative linear velocity of the (i) motion axis it is represented by expression :

$$[R]_{i-1}^i \cdot \overline{v}_i^{i-1} = \begin{cases} 0\ ,\ \text{in rotation couple} \\ [R]_{i-1}^i \cdot \overline{k}_i^{i-1} \cdot \dot{q}_i = \dot{q}_i \cdot \overline{k}_i^i, \\ \text{in translation couple.} \end{cases} \quad (13)$$

The relative linear acceleration of the axis of motion (i) has the expression:

$$[R]_{i-1}^i \cdot \overline{a}_i^{i-1} = \begin{cases} 0\ ,\ \text{in rotation couple} \\ [R]_{i-1}^i \cdot \overline{k}_i^{i-1} \cdot \ddot{q}_i = \ddot{q}_i \cdot \overline{k}_i^i\ , \\ \text{in translation couple.} \end{cases} \quad (14)$$

According to relations (10)-(14), the kinematics parameters of relation (3) will have the following expressions:

$$\overline{\omega}_i^i = [R]_{i-1}^i \cdot \overline{\omega}_{i-1}^{i-1} + \begin{cases} \dot{q}_i \cdot \overline{k}_i^i,\ \text{in rotation couple} \\ 0\ ,\ \text{in translation couple.} \end{cases} \quad (15)$$

$$\overline{v}_i^i = [R]_{i-1}^i \left\{\overline{v}_{i-1}^{i-1} + \overline{\omega}_{i-1}^{i-1} \times \overline{r}_i^{i-1}\right\} +$$
$$+ \begin{cases} 0\ ,\ \text{in rotation couple} \\ \dot{q}_i \cdot \overline{k}_i^i\ ,\ \text{in translation couple.} \end{cases} \quad (16)$$

$$\overline{\varepsilon}_i^i = [R]_{i-1}^i \cdot \overline{\varepsilon}_{i-1}^{i-1} +$$
$$+ \begin{cases} [R]_{i-1}^i \cdot \overline{\omega}_{i-1}^{i-1} \times \dot{q}_i \cdot \overline{k}_i^i + \ddot{q}_i \cdot \overline{k}_i^i\ , \\ \text{in rotation couple} \\ 0\ ,\ \text{in translation couple.} \end{cases} \quad (17)$$

$$\overline{a}_i^i = [R]_{i-1}^i \left\{\overline{a}_{i-1}^{i-1} + \overline{\varepsilon}_{i-1}^{i-1} \times \overline{r}_i^{i-1} + \overline{\omega}_{i-1}^{i-1} \times \left(\overline{\omega}_{i-1}^{i-1} \times \overline{r}_i^{i-1}\right)\right\} +$$
$$+ \begin{cases} 0\ ,\ \text{in rotation couple} \\ 2\,\overline{\omega}_i^i \times \dot{q}_i \cdot \overline{k}_i^i + \ddot{q}_i \cdot \overline{k}_i^i, \\ \text{in translation couple.} \end{cases} \quad (18)$$

If i=1, then the values of the specific robot base parameters represented by the expression (4), must be replaced in the (15)-(18) expressions. When i=n the operational velocities and accelerations are expressed as an matrix in the reference system (Tn).

$$\left[\dot{\bar{X}}^n\right]=\left[\left[\bar{v}_n^n\right]^T \ \left[\bar{\omega}_n^n\right]^T\right]^T,$$

$$\left[\ddot{\bar{X}}^n\right]=\left[\left[\bar{a}_n^n\right]^T \ \left[\bar{\omega}_n^n\right]^T \ \left[\bar{\varepsilon}_n^n\right]^T\right]^T. \qquad (19)$$

Usins the following transformation equations, the kinematic parameters of (19) can be expressed in relation to the fixed referece system (T_0),

$$\bar{v}_n^0=[R]_n^0\cdot\bar{v}_n^n,\ \bar{a}_n^0=[R]_n^0\cdot\bar{a}_n^n$$

$$\bar{\omega}_n^0=[R]_n^0\cdot\bar{\omega}_n^n,\ \bar{\varepsilon}_n^0=[R]_n^0\cdot\bar{\varepsilon}_n^n. \qquad (20)$$

After calculation, the relation (20) becomes:

$$\left[\dot{\bar{X}}\right]^0=\begin{bmatrix}\bar{v}_n^0\\ --\\ \bar{\omega}_n^0\end{bmatrix}=[R]^0\cdot\left[\dot{\bar{X}}\right]^n \quad (21) \qquad [\ddot{\bar{X}}]^0=[R]^0\cdot[\ddot{\bar{X}}]^n \quad (22)$$

$$\begin{bmatrix}\dot{X}^0\\ --\\ \ddot{X}^0\end{bmatrix}=\begin{bmatrix}[R]^0 & | & [0]\\ --- & | & ---\\ [0] & | & [R]^0\end{bmatrix}\cdot\begin{bmatrix}\dot{X}^n\\ --\\ \ddot{X}^n\end{bmatrix}, \qquad (23)$$

In expressions (21), (22) and (23), $[R]^0$ is a 6x6 square matrix, used to transform the column vectors of velocity and accelerations of the (T_n) operating system and (T_0) system.

Finally, to determine the operational kinematic parameters of the gripping device, relative to the (T_0) fixed system, must be used the relations (19) and (23) witch reprisents the direct kinematic model equations.

Development of direct kinematics model for a TTTR Gantry modular robot.

The direct kinematic modeling of industrial robots involves determining kinematic operational parameters of the gripping device relative to the reference mobile system (T_n) jointly with the gripping device in relation to fixed reference system (T_0) of the robot base.

The kinematic modeling, shown in the figure 2, assumed knowledge of the constructive mechanical parameters and the instantaneous values of the coordinates, generalized velocities and accelerations of the robot couplings [1].

To determine the kinematics parameters $\bar{\omega}_i^i, \bar{v}_i^i, \bar{\varepsilon}_i^i, \bar{a}_i^i$ are used homogeneous matrix transformation obtained from the geometric modeling. Using the relations below, it can determine the inverse rotation matrix:

$$[R]_{i-1}^i=\left[R_i^{i-1}\right]^{-1}=\left[R_i^{i-1}\right]^T. \qquad (23)$$

Thus, you get:

$$[R]_0^1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}; \qquad [R]_1^2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};$$

$$[R]_2^3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}; \qquad (25)$$

$$[R]_3^4=\begin{bmatrix}cq_4&sq_4&0\\-sq_4&cq_4&0\\0&0&1\end{bmatrix}; \ [R]_4^5=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}. \quad (26)$$

Matrix shape of the kinematics axis vector unit of O_5z_5 (figure 2), are:

$$[j]_1^1=\begin{bmatrix}0\\1\\0\end{bmatrix}; \qquad [i]_2^2=\begin{bmatrix}1\\0\\0\end{bmatrix}; \qquad k_3^3=\begin{bmatrix}0\\0\\1\end{bmatrix};$$

$$[k]_4^4=\begin{bmatrix}0\\0\\1\end{bmatrix}; \qquad [k]_5^5=\begin{bmatrix}0\\0\\1\end{bmatrix}. \qquad (27)$$

The kinematic parameters transposed in the matrix form, coresponding to the robot base, will have the expressions:

$$[\bar{\omega}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix}; [\bar{v}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix}; [\bar{\varepsilon}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix}; [\bar{a}]_0^0=\begin{bmatrix}0\\0\\g\end{bmatrix}. (28)$$

Further will determine the operationale angular velocities, which after calculation becomes:

For axis $1\,(O_1y_1)$, translation:

$$[\bar{\omega}]_1^1=[R]_0^1\cdot[\bar{\omega}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix}; \qquad (29)$$

For axis 2 (O_2x_2), translation:

$$[\bar{\omega}]_2^2=[R]_1^2\cdot[\bar{\omega}]_1^1=\begin{bmatrix}0\\0\\0\end{bmatrix}; \qquad (30)$$

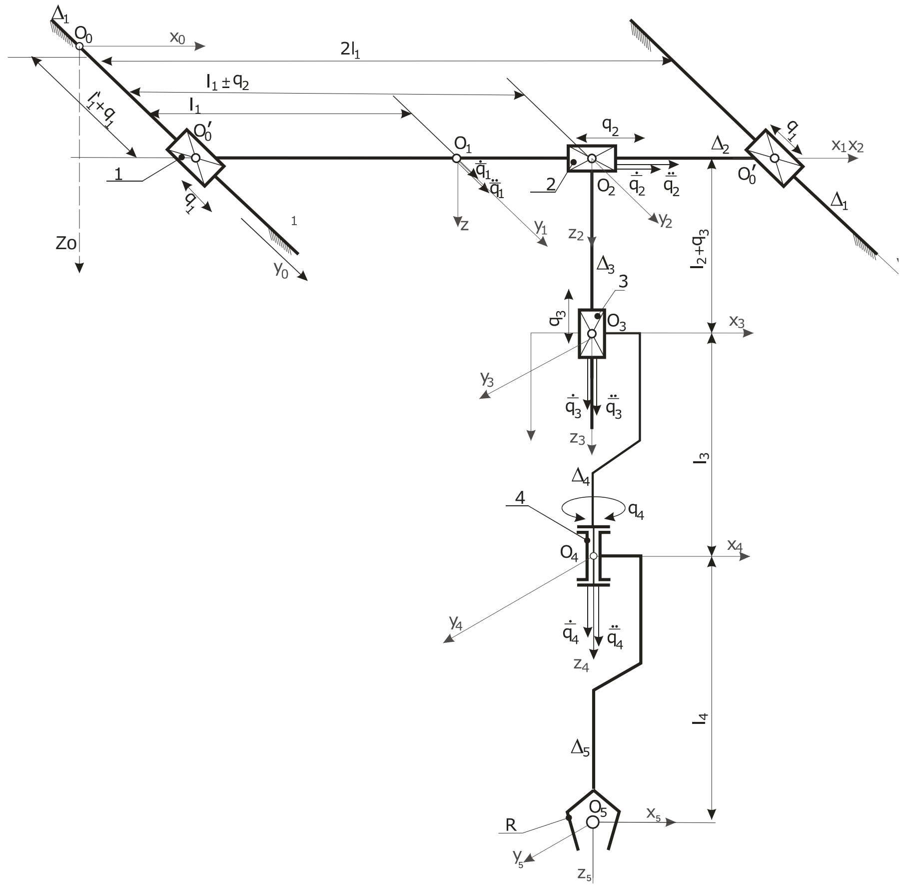

Figure 2. Kinematics scheme of a TTTR Gantry robot.

For axis 3 (O_3z_3), translation:

$$[\overline{\omega}]_3^3 = [R]_2^3 \cdot [\overline{\omega}]_2^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \qquad (31)$$

For axis 4 (O_4z_4), rotation:

$$[\overline{\omega}]_4^4 = [R]_3^4 \cdot [\overline{\omega}]_3^3 + \dot{q}_4 \cdot [\overline{k}]_4^4 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_4 \end{bmatrix}; \qquad (32)$$

For the gripping mechanism 5, axis O_5z_5:

$$[\overline{\omega}]_5^5 = [R]_4^5 \cdot [\overline{\omega}]_4^4 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_4 \end{bmatrix} \qquad (33)$$

$$\{\overline{\omega}\times\} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \qquad (34)$$

In accordance with the [6], the 3x3 antisymetric matrix $\{\overline{\omega}\times\}$,, witch $\overline{\omega}$ reprisents the angular velocity vector, can be written as above. Acoding to works [2] and [4] and based on the relations (12) and (16), will determine transposed matrix of the operational linear velocity expressions, which after calculation becomes:

For axis 1 (O_1z_1), translation:

$$[\overline{v}]_1^1 = [R]_0^1 \cdot \{\overline{v}_0^0 + \overline{\omega}_0^0 \times \overline{r}_1^0\} + \dot{q}_1 \cdot [\overline{j}]_1^1 \; ; \; [\overline{v}]_1^1 = \begin{bmatrix} 0 \\ \dot{q}_1 \\ 0 \end{bmatrix} \qquad (35)$$

For axis 2 (O_2x_2), translation:

$$[\overline{v}]_2^2 = [R]_1^2 \cdot \{\overline{v}_1^1 + \overline{\omega}_1^1 \times \overline{r}_2^1\} + \dot{q}_2 \cdot [\overline{i}]_2^2; \quad [\overline{v}]_2^2 = \begin{bmatrix} \dot{q}_2 \\ \dot{q}_1 \\ 0 \end{bmatrix} \qquad (36)$$

For axis 3 (O_3z_3), translation:

$$[\overline{v}]_3^3 = [R]_2^3 \cdot \{\overline{v}_2^2 + \overline{\omega}_2^2 \times \overline{r}_3^2\} + \dot{q}_3 \cdot [\overline{k}]_3^3; \quad [\overline{v}]_3^3 = \begin{bmatrix} \dot{q}_2 \\ \dot{q}_1 \\ \dot{q}_3 \end{bmatrix} \qquad (37)$$

For four axis (O_4z_4) , rotation:

$$[\bar{v}]_4^4=\begin{bmatrix} cq_4\cdot\dot{q}_2+sq_4\cdot\dot{q}_1 \\ -sq_4\cdot\dot{q}_2+cq_4\cdot\dot{q}_1 \\ \dot{q}_3 \end{bmatrix} \quad (38)$$

For the gripping mechanism 5:

$$[\bar{v}]_5^5=\begin{bmatrix} cq_4\cdot\dot{q}_2+sq_4\cdot\dot{q}_1 \\ -sq_4\cdot\dot{q}_2+cq_4\cdot\dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \quad (39)$$

Transposed matrix of the operational angular accelerations, becomes:

For axis 1 (O_1y_1), translation:

$$[\bar{\varepsilon}]_2^2=[R]_2^1\cdot[\bar{\varepsilon}]_1^1; \qquad [\bar{\varepsilon}]_2^2=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad (41)$$

For axis 3 (O_3z_3), translation:

$$[\bar{\varepsilon}]_3^3=[R]_3^2\cdot[\bar{\varepsilon}]_2^2; \qquad [\bar{\varepsilon}]_3^3=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad (42)$$

For axis 4 (O_4z_4), translation:

$$[\bar{\varepsilon}]_4^4=[R]_3^4\cdot[\bar{\varepsilon}]_3^3+\left\{[R]_3^4\cdot\bar{\omega}_3^3\times\dot{q}_4\cdot\bar{k}_4^4+\ddot{q}_4\cdot\bar{k}_4^4\right\};$$

$$[\bar{\varepsilon}]_4^4=\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix}; \quad (43)$$

For the gripping mechanism 5 (O_5z_5) :

$$[\bar{\varepsilon}]_5^5=[R]_4^5\cdot[\bar{\varepsilon}]_4^4; \qquad [\bar{\varepsilon}]_5^5=\begin{bmatrix} 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix} \quad (44)$$

The expressions of operational linear acceleration transposed matrix, after calculation becomes:

For axis 1 (O_1y_1), translation:

$$[\bar{a}]_1^1=[R]_0^1\cdot\left\{\bar{a}_0^0+\bar{\varepsilon}_0^0\times\bar{r}_1^0+\bar{\omega}_0^0\times\left(\bar{\omega}_0^0\times\bar{r}_1^0\right)\right\}+\left\{2\bar{\omega}_1^1\times\dot{q}_1\cdot\bar{j}_1^1+\ddot{q}_1\cdot\bar{j}_1^1\right\};$$

$$[\bar{a}]_1^1=\begin{bmatrix} 0 \\ \ddot{q}_1 \\ g \end{bmatrix}; \quad (45)$$

For axis 2 (O_2x_2), translation:

$$[\bar{a}]_2^2=[R]_1^2\cdot\left\{\bar{a}_1^1+\bar{\varepsilon}_1^1\times\bar{r}_2^1+\bar{\omega}_1^1\times\left(\bar{\omega}_1^1\times\bar{r}_2^1\right)\right\}+\left\{2\bar{\omega}_2^2\times\dot{q}_2\cdot\bar{i}_2^2+\ddot{q}_2\cdot\bar{i}_2^2\right\};$$

$$[\bar{a}]_2^2=\begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_1 \\ g \end{bmatrix}; \quad (46)$$

For axis 3 (O_3z_3), translation:

$$[\bar{a}]_3^3=[R]_2^3\cdot\left\{\bar{a}_2^2+\bar{\varepsilon}_2^2\times\bar{r}_3^2+\bar{\omega}_2^2\times\left(\bar{\omega}_2^2\times\bar{r}_3^2\right)\right\}+\left\{2\bar{\omega}_3^3\times\dot{q}_3\cdot\bar{k}_3^3+\ddot{q}_3\cdot\bar{k}_3^3\right\};$$

$$[\bar{a}]_3^3=\begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_1 \\ g+\ddot{q}_3 \end{bmatrix}; \quad (47)$$

For axis 4 (O_4z_4), rotation:

$$[\bar{a}]_4^4=[R]_3^4\cdot\left\{\bar{a}_3^3+\bar{\varepsilon}_3^3\times\bar{r}_4^3+\bar{\omega}_3^3\times\left(\bar{\omega}_3^3\times\bar{r}_4^3\right)\right\};$$

$$[\bar{a}]_4^4=\begin{bmatrix} cq_4\cdot\ddot{q}_2+sq_4\cdot\ddot{q}_1 \\ -sq_4\cdot\ddot{q}_2+cq_4\cdot\ddot{q}_1 \\ g+\ddot{q}_3 \end{bmatrix} \quad (48)$$

For the gripping mechanism 5 (O_5z_5), rotation:

$$[\bar{a}]_5^5=[R]_4^5\cdot\left\{\bar{a}_4^4+\bar{\varepsilon}_4^4\times\bar{r}_5^4+\bar{\omega}_4^4\times\left(\bar{\omega}_4^4\times\bar{r}_5^4\right)\right\};$$

$$[\bar{a}]_5^5=\begin{bmatrix} cq_4\cdot\ddot{q}_2+sq_4\cdot\ddot{q}_1 \\ -sq_4\cdot\ddot{q}_2+cq_4\cdot\ddot{q}_1 \\ g+\ddot{q}_3 \end{bmatrix}. \quad (49)$$

The kinematic parameters in the operational system $O_5x_5y_5z_5$, has the following form

$$[\dot{\bar{X}}]^5=\begin{bmatrix} cq_4\cdot\dot{q}_2+sq_4\cdot\dot{q}_1 \\ -sq_4\cdot\dot{q}_2+cq_4\cdot\dot{q}_2 \\ \dot{q}_3 \\ 0 \\ 0 \\ \dot{q}_4 \end{bmatrix} \quad (50) \qquad [\ddot{\bar{X}}]^5=\begin{bmatrix} cq_4\cdot\ddot{q}_2+sq_4\cdot\ddot{q}_1 \\ -sq_4\cdot\ddot{q}_2+cq_4\cdot\ddot{q}_1 \\ g+\ddot{q}_3 \\ 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix} \quad (51)$$

Using the transformation relations, the kinematics parameters can be determined in the operational fixed system $O_0x_0y_0z_0$, at the base of the robot.

These equations are:

$$[\bar{v}]_5^0=[R]_5^0\cdot[\bar{v}]_5^5, \qquad [\bar{v}]_5^0=\begin{bmatrix} \dot{q}_2 \\ \dot{q}_1 \\ \dot{q}_3 \end{bmatrix}; \quad (52)$$

$$[\bar{\omega}]_5^0=[R]_5^0\cdot[\bar{\omega}]_5^5, \qquad [\bar{\omega}]_5^0=\begin{bmatrix} 0 \\ 0 \\ \dot{q}_4 \end{bmatrix}; \quad (53)$$

$$[\bar{a}]_5^0 = [R]_5^0 \cdot [\bar{a}]_5^5, \quad [\bar{a}]_5^0 = \begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_1 \\ g + \ddot{q}_3 \end{bmatrix}, \quad (54)$$

$$[\bar{\varepsilon}]_5^0 = [R]_5^0 \cdot [\bar{\varepsilon}]_5^5, \quad [\bar{\varepsilon}]_5^0 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix}. \quad (55)$$

According to (21 and (22), given (52) (53) and (54) (55), velocity and acceleration can be expressed in operational fixed system $O_0x_0y_0z_0$:

$$\left[\dot{X}\right]^0 = \begin{bmatrix} \dot{q}_2 \\ \dot{q}_1 \\ \dot{q}_3 \\ 0 \\ 0 \\ \dot{q}_4 \end{bmatrix} \quad (56) \qquad \left[\ddot{X}\right]^0 = \begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_1 \\ g + \ddot{q}_3 \\ 0 \\ 0 \\ \ddot{q}_4 \end{bmatrix} \quad (57)$$

Equations (50) (51) (56) and (57) represent the direct kinematics model with witch can be determine the kinematics operational parameters of the gripping device, compared with reference systems (Tn) and (T0). Through the kinematics modeling method eliminates the disadvantages caused by nonlinearity of the geometric equations and the lack of control over speed and acceleration on the trajectory of movement.

3. CONCLUSIONS

Using the iterative method was possible to determine the kinematics parameters: operational angular speeds and accelerations, respectively operational linear velocities and accelerations of a TTTR Gantry modular robot.

These parameters define the movement of each "i" mobile element, in accordance with the $O_0x_0y_0z_0$ fixed system from the robot base.

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Elaborarea modelului cinematic direct pentru un robot modular Gantry de tip TTTR, prin metoda iterativă

Rezumat: Lucrarea prezintă premisele teoretice necesare abordării problemei modelarii cinematice directe, prin metoda iterativa, a structurilor specifice roboţilor seriali. Utilizând această metodă, lucrarea dezvolta metodologia de determinare a parametrilor cinematici operaţionali: viteza şi acceleraţie unghiulară, respectiv viteza şi acceleraţie liniara, ce caracterizează mişcarea fiecărui element mobil a unui robot modular suspendat de tip Gantry, de tip TTTR.

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