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SOLVING DYNAMIC MODELLING OF TTTR GANTRY MODULAR ROBOT BY NEWTON EULER FORMALISM

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Abstract. Dynamic equations of a robot shall be determined by an iterative method. This method highlights the generalized variables: generalized driving forces and liaison forces that appear in the components of the robot. So, knowing the position vectors of the centers of masses, the work aims to determine the accelerations associated with mass centres, the external forces and moments torsor, the liaison forces and moments torsor, respectively generalized drivinf forces of a Gantry modular robot. In this paper the authors present the dynamic modeling of a 4 dof Gantry robotic structure, type TTTR.
Key words: geometric model, kinematics model, dynamic model, modular robot, Gantry structure.

1. INTRODUCTION

The dynamic equations of a robot are determined by an iterative method, which emphasizes the generalized variables, the driving generalized forces and the contact forces that arise between linked components of the robot. The calculation algorithm is based on the Luth-Walker-Paul method [3] and consists of two iteration to the mechanical structure of the parts robot, respectively:

- Iterations to the exterior structure,
- Iterations inside the structure.

2. THEORETICAL JUSTIFICATION

In accordance with the kinematics structure of a robot with n degrees of freedom (fig.1), it can be established the dynamic equations of a robot [1]. The kinematics structure of the robot it's geometric modeling, thus, for each item i is determined the homogeneous transformation matrix:

$$[T]_i^{i-1} = \begin{bmatrix} [R]_i^{i-1} & | & [\bar{r}]_i^{i-1} \\ --- & | & --- \\ 0\ 0\ 0 & | & 1 \end{bmatrix} \quad (1)$$

If the method of compounds operators DH in the second variant is applied, the matrix elements (1) following meanings:

$[R]_i^{i-1}$ is the rotation matrix defining the orientation of each axis of the system (T_i) with the system (T_{i-1}) and has the expression:

$$[R]_i^{i-1} = \begin{bmatrix} c\theta_i & -s\theta_i & 0 \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} \end{bmatrix} \quad (2)$$

$\bar{r}_i^{i-1}$ is the column vector which defines the position of origin O_i of the system (T_i) in relation with the origin O_{i+1} of the system (T_{i+1}), with the matrix expression:

$$[\bar{r}]_i^{i-1} = [a_i \ \ -d_i s\alpha_{i-1} \ \ d_i c\alpha_{i-1}]^T. \quad (3)$$

It's determine the inverse rotation matrix $[R]_i^{i-1}$:

$[R]_{i-1}^{i} = [R_{i-1}^{i}]^{-1} = [R_{i-1}^{i}]^T$, and hence,

$$[R]_{i-1}^{i} = \begin{bmatrix} c\theta_i & s\theta_i c\alpha_{i-1} & s\theta_i s\alpha_{i-1} \\ -s\theta_i & c\theta_i c\alpha_{i-1} & c\theta_i s\alpha_{i-1} \\ 0 & -s\alpha_{i-1} & c\alpha_{i-1} \end{bmatrix}. \quad (4)$$

For each item i are determined the following parameters:

M_i – the mass of the element i of a robot with n degrees of freedom

$M_i = \sum_{j=1}^{k_i} \sigma_j m_j$, in witch

$\sigma_j = +1$ if the item j remains in the item i composition;

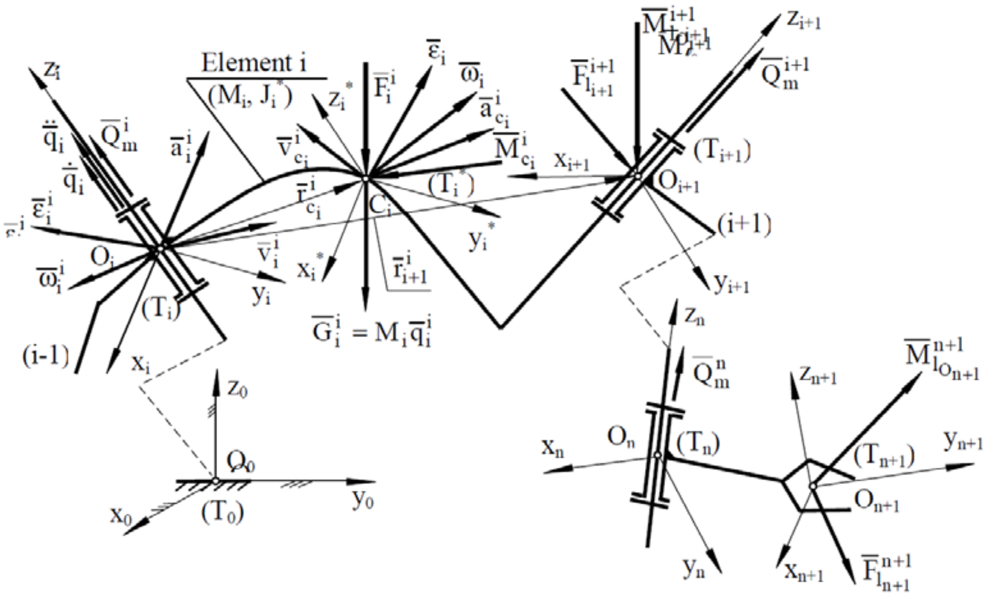

Figure 1. The kinematic structure of a robot with n degrees of freedom

$\sigma_j = -1$ if the item j is eliminated;

$\bar{r}_{c_i}^{i}$ - the position vector of the mass center C_i in relation with the origin O_i of the reference system (T_i), with the relation:

$$[\bar{r}_c]_i^{iD} = [T]_i^{iD} \cdot [\bar{r}_c]_i^{i} = \begin{bmatrix} x_{c_i}^{iD} & y_{c_i}^{iD} & z_{c_i}^{iD} & 1 \end{bmatrix}^T, \quad (5)$$

where $[T]_i^{iD}$ is the matrix that defines the position and the orientation of each axis of the reference system (T_i) in relation to the reference system DH, (T_{iD}) which can be determined, according to [5] the relation:

$$[T]_i^{iD} = \left[\begin{array}{c|c} \begin{bmatrix} \bar{x}_{iD}^T \\ \bar{y}_{iD}^T \\ \bar{z}_{iD}^T \end{bmatrix} \cdot \begin{bmatrix} \bar{x}_i & \bar{y}_i & \bar{z}_i \end{bmatrix} & \begin{matrix} \bar{x}_{iD}^T(\bar{p}_i - \bar{p}_{iD}) \\ \bar{y}_{iD}^T(\bar{p}_i - \bar{p}_{iD}) \\ \bar{z}_{iD}^T(\bar{p}_i - \bar{p}_{iD}) \end{matrix} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]; \quad (6)$$

J_i^{*i} – the inertial tensor of the element i compared with the reference system (T_i^*) with the origin in the mass center C_i. This is determined according to [Isp 04], with the following relation:

$$J_i^{*iD} = \begin{bmatrix} J_x^{*iD} & -J_{xy}^{*iD} & -J_{xz}^{*iD} \\ -J_{yx}^{*iD} & J_y^{*iD} & -J_{yz}^{*iD} \\ -J_{zx}^{*iD} & J_{zy}^{*iD} & J_z^{*iD} \end{bmatrix} \quad (7)$$

where the matrix elements are the axial and centrifugal mechanical moments of inertia of the element i determined in relation with the reference system (T_{iD}), with the origin in the mass center C_i.

On the robot acts the system of the weight forces for every element i (i=1÷n) and a system of external forces situated at the end of the robot. The system of external forces is reduced compared to the origin O_{n+1} of the reference system (T_{n+1}), jointly with the gripper and the manipulated object caught in the grip handle.
Thus, it's obtain the reduction torsos of the external forces, with the elements: the resultant force $\bar{F}_{n+1}^{n+1}$ and the resultant moment $\bar{M}_{O_{n+1}}^{n+1}$, expressed in the reference system (T_{n+1}).

If the robot is in motion, then on each axis of motion the generalized variables $q_i, \dot{q}_i, \ddot{q}_i, i = 1 \div n$ are highlighted. The kinematic parameters that characterize the movement of the element I, at a time, are $\bar{\omega}_i^i, \bar{\varepsilon}_i^i, \bar{v}_{c_i}, \bar{a}_{c_i}, i = 1 \div n$, which are added the kinematics parameters of the fixed element (0):

$$[\bar{\omega}]_0^0 = [\dot{\bar{\omega}}]_0^0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$
$$[\dot{\bar{v}}]_0^0 = \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T, \quad (8)$$

where g is the gravitational acceleration.

By applying the Newton-Euler method the robot's dynamic equations are determined, from which are obtained the generalized driving forces. $\overline{Q}_m^i$. These results are obtained covering the two stages of the calculation algorithm and by introducing the notation:

$$\begin{cases} \Delta_i = 1, \text{ in the case of rotation (i=1÷n)} \\ \Delta_i = 0, \text{ in the case of translation} \end{cases} \quad (9)$$

1) Iterations to the exterior of the robot mechanical structure.

Using the Newton-Euler dynamic equations, is determined for each element i, (i=1÷n), component of the robot, the linear and angular velocities and accelerations, the forces and moments exterior forces.

Applying the calculation algorithm of the iterative method presented in the kinematics modeling, [2] and [8] determine the following kinematics parameters:

$\overline{\omega}_i^i$ - the angular velocity of the element i relative to the fixed system (T_0) from the base of the robot, expressed in the system (T_i) with the relation:

$$\overline{\omega}_i^i = [R]_{i-1}^i \cdot \overline{\omega}_{i-1}^{i-1} + \Delta_i \dot{q}_i \overline{k}_i^i; \quad (10)$$

$\overline{\varepsilon}_i^i$ - the angular acceleration of the element i relative to the system (T_0), expressed in the system (T_i) with the relation:

$$\overline{\varepsilon}_i^i = [R]_{i-1}^i \cdot \overline{\varepsilon}_{i-1}^{i-1} + \Delta_i \left\{ [R]_{i-1}^i \overline{\omega}_{i-1}^{i-1} \times \dot{q}_i \overline{k}_i^i + \ddot{q}_i \overline{k}_i^i \right\}; \quad (11)$$

$\overline{a}_i^i$ - the linear acceleration of the origin of system (T_i) relative to the fixed system (T_0), expressed in the system (T_i) with the relation:

$$\overline{a}_i^i = [R]_{i-1}^i \begin{Bmatrix} \overline{a}_{i-1}^{i-1} + \overline{\varepsilon}_{i-1}^{i-1} \times \overline{r}_i^{i-1} + \overline{\omega}_{i-1}^{i-1} \\ \times \left(\overline{\omega}_{i-1}^{i-1} \times \overline{r}_i^{i-1} \right) \end{Bmatrix} + \quad (12)$$
$$(1-\Delta_i)\left(2\overline{\omega}_i^i \times \dot{q}_i \overline{k}_i^i + \ddot{q}_i \overline{k}_i^i\right);$$

$\overline{a}_{ci}$ - the mass center acceleration C_i of the element i, determined in relation to the fixed system (T_0) and expressed relative to the system (T_i), by the relation:

$$\overline{a}_{c_i}^i = \overline{a}_i^i + \overline{\varepsilon}_i^i \times \overline{r}_{c_i}^i + \overline{\omega}_i^i \times \left(\overline{\omega}_i^i \times \overline{r}_{c_i}^i \right). \quad (13)$$

By applying to each element i, the dynamic equations of the Newton-Euler are obtained the reduction torsos elements for the external forces with the following expressions:

$$\overline{R}_i^i = M_i \overline{a}_{c_i}^i$$
$$\overline{M}_{c_i}^i = J_i^{*i} \overline{\varepsilon}_i^i + \overline{\omega}_i^i \times J_i^{*i} \overline{\omega}_i^i. \quad (14)$$

2) Iterations inside of the robot mechanical structure.

Under this case, is determined for each element i, (i=1÷n), of the robot, the forces torsos between the elements i, i+1, respectively the generalized driving forces of kinematics axes.

For each element i, it can be determined the reduction torsos of the contact forces. The elements of this torsos have the expressions:

$$\overline{F}_{l_i}^i = M_i \overline{a}_{c_i} - \overline{F}_i^i - [R]_{i+1}^i \overline{F}_{l_{i+1}}^{i+1}$$
$$\overline{M}_{l_{o_i}}^i = \overline{r}_{c_i} \times M_i \overline{a}_{c_i} + J_i^* \overline{\varepsilon}_i + \overline{\omega}_i \times J_i^* \overline{\omega}_i - \overline{M}_{c_i}^i - \quad (15)$$
$$\overline{r}_{c_i} \times \overline{F}_i^i - [R]_{i+1}^i \overline{M}_{l_{o_{i+1}}}^{i+1} - \overline{r}_{i+1}^i \times [R]_{i+1}^i \overline{F}_{l_{i+1}}^{i+1}.$$

From above relation, by transforming vectors $\overline{F}_{l_{i+1}}^i$ and $\overline{M}_{l_{o_{i+1}}}^i$ in the vectors, you get:

$$\overline{F}_{l_{i+1}}^i = [R]_{i+1}^i \overline{F}_{l_{i+1}}^{i+1}; \quad \overline{M}_{l_{o_{i+1}}}^i = [R]_{i+1}^i \overline{M}_{l_{o_{i+1}}}^{i+1}, \quad (16)$$

and the generalized driving forces Q_m^i, which actually represents the dynamic model of the robot:

$$Q_m^i = \Delta_i \left[M_{l_{O_i}}^i \right]^T \cdot \overline{k}_i^i + (1-\Delta_i) \left[\overline{F}_{l_i}^i \right]^T \cdot \overline{k}_i^i + Q_f^i, \quad (17)$$

where Q_f^i, according to [8], reprezent the force caused by friction and has the expressions:

$$Q_f^i = b_i \dot{q}_i + Q_{f_c}^i. \quad (18)$$

The parameters b_i şi $Q_{f_c}^i$ from above relation represent:

b_i – the viscous friction coefficient;

$Q_{f_c}^i$ -the generalized force due to dry friction (Coulomb friction) and it has the expression:

$$Q_{f_c}^i = \Delta_i c_i \frac{d_i}{2} \left| \overline{k}_i^i \times \overline{F}_{l_i}^i \right| \operatorname{sgn} \dot{q}_i +$$
$$(1-\Delta_i) c_i \left| \overline{k}_i^i \times \overline{F}_{l_i}^i \right| \operatorname{sgn} \dot{q}_i. \quad (19)$$

In the above expression, c_i is the dry friction coefficient, and d_i is the diameter of spindle torque.

The dynamic equations system (17) can be written as:

$$\overline{Q}_m(t) = \left[Q_m^i(t) = f^{-1}(q_j(t), \ j = 1 \div n), \ i = 1 \div n \right]^T \quad (20)$$

and represents the dynamic model of the robot with n degrees of freedom.

In the direct problem of robot dynamics are known the column vector of the generalized driving forces. Thus, the functions can be deduced:

$$\overline{q}(t) = f\{\overline{Q}_m(t)\} = [q_i(t), i = 1 \div n]^T, \quad (21)$$

which is the robot law of motion in configurative states space.

In the inverse problem of robot dynamics, called inverse dynamic model, functions $\overline{q}(t)$ are known, and with the relation (17) the generalized driving forces $\overline{Q}_m(t)$ are determinated [7]. Using Newton-Euler iterative method, with relation (15) can be determined the elements of contact forces torsor between the components of the robot.

In conclusion, the Newton-Euler iterative method includes the following steps:

- It's shaped geometrical the mechanical structure of the robot with n degrees of freedom and are determined for each i=1÷n, the matrices (3) and their inverse.

Is calculated for each element i, i=1÷n, parameters which characterizing the mass distribution, namely: the mass M_i, the position vector $\overline{r}^i_{c_i}$ of mass center C_i relative with O_i (7), (8) and the tensor J^{*i}_i with relation (9).

It is calculated, by the iterations to the outside, the kinematics parameters (10)-(15) and the external forces torsor (16).

By iterations to the outside, are determined, the contact forces torsors, whose elements are given by (17), and the generalized driving forces using, the dynamic equations (19).

The dynamic model of TTTR serial robot

The mechanical structural diagram of the TTTR industrial robot, shown in fugure 2 consists of: a traslation module 1 on the horizontal axis O_0y_0, the translation module 2 on the horizontal axis O_2x_2, the translation module 3 of the vertical sled on axis O_3z_3 and the orientation module 4 of the gripping device, witch executes a rotation on axis O_4z_4 [1].

The dynamic modeling of the TTTR robot, whose kinematics scheme is shown in figure 2 will be achieved by applying the Newton-Euler method, implemented in the symbolic modeling program *Robot Symbolic, Robot Dynamics module of the Matlab 7.1 program* [2].

For applying the method is required to develop the geometric and kinematics modeling, as well the determining of mass distribution parameters.

On the basis of recommendations from the articles [4] and [6], it is noted:

$l^i_1, l_i, i = 1 \div 4$ – constructive parameters of the robot i=1÷4

Δ_i, i = 1÷4 – motion axis;

Δ_5 – parallel axis with axis Δ_3 of rotation of the arm, passing through the mass center C_5 of the gripping device;

$q_k, \dot{q}_k, \ddot{q}_k$ -, the generalized coordinates positions, velocities and accelerations

O_i, i = 1÷4 – origins of the systems $O_ix_iy_iz_i$, witch coincides with the mass centers of the robot modules;

O_0 – measurement base (zero point);

$O_0x_0y_0z_0$ – Cartesian reference fixed system

$O_ix_iy_iz_i$, i = 1÷4 – Cartesian reference mobile system, solider with the mobile parts of the robot modules

$\overline{G}_i$, i = 1÷5 – the mass forces related to the modules respectively the gripping device

$\overline{F}_1$, $\overline{F}_2$, $\overline{F}_3$ - driving forces in the couplings 1. 2 and 3;

$\overline{M}_4$ - the coupling torque 4;

m_i, i = 1÷5 – all the masses including, modules and gripping device;

$J^{(3)}_{z_3}$ - mechanical inertial moment of the module 3 in relation to the axis O_3z_3;

$J^{(4)}_{\Delta_4}$ - mechanical inertial moment of the mobile equipment of the orientation module 4 in relation to the axis Δ_4.

It is also required some simplifying assumptions:

- It is considered the mass centers C_i located in in the origins O_i of the Cartesian reference system $O_ix_iy_iz_i$, i=1÷4 and so the position vectors of mass center are void;
- Choosing the moving reference system so that their axes coincide with the main directions of

inertial forces, associated with the origins of these systems, result that the mechanical centrifugal moments of inertia are void.

According to the Newton-Euler method, [6] and [7], first, the mechanical structure is walked by iteration to outward of the robot mechanical structure.

The mass distribution parameters are included in the table 1.

Table 1

Element i	Masa M_i	Centrul maselor $\bar{r}^i_{c_i}$	Tensorul inerțial J^{*i}_i
1	M_1	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$\begin{bmatrix}J^{*1}_x & 0 & 0\\0 & J^{*1}_y & 0\\0 & 0 & J^{*1}_z\end{bmatrix}$
2	M_2	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$\begin{bmatrix}J^{*2}_x & 0 & 0\\0 & J^{*2}_y & 0\\0 & 0 & J^{*2}_z\end{bmatrix}$
3	M_3	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$\begin{bmatrix}J^{*3}_x & 0 & 0\\0 & J^{*3}_y & 0\\0 & 0 & J^{*3}_z\end{bmatrix}$
4	M_4	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$\begin{bmatrix}J^{*4}_x & 0 & 0\\0 & J^{*4}_y & 0\\0 & 0 & J^{*4}_z\end{bmatrix}$

In table 1 J^{*i}_x, J^{*i}_y, J^{*i}_z, $i = 1 \div 4$, are the mechanical axial moments of inertia relative to the system i, with the origin in the mass center C_i, and having the same guidance with the system attached to each element of the robot [1].

Next, are determining the accelerations corresponding to mass centers, with the following relations:

$$\bar{a}^1_{c_1} = \bar{a}^1_1 + \bar{\varepsilon}^1_1 \times \bar{r}^1_{c_1} + \bar{\omega}^1_1 \times \left(\bar{\omega}^1_1 \times \bar{r}^1_{c_1}\right), \quad [\bar{a}_c]^1_1 = \begin{bmatrix}0\\ \ddot{q}_1\\ g\end{bmatrix} \qquad (22)$$

$$\bar{a}^2_{c_2} = \bar{a}^2_2 + \bar{\varepsilon}^2_2 \times \bar{r}^2_{c_2} + \bar{\omega}^2_2 \times \left(\bar{\omega}^2_2 \times \bar{r}^2_{c_2}\right), \quad [\bar{a}_c] = \begin{bmatrix}\ddot{q}_2\\ \ddot{q}1\\ g\end{bmatrix} \qquad (23)$$

$$\bar{a}^3_{c_3} = \bar{a}^3_3 + \bar{\varepsilon}^3_3 \times \bar{r}^3_{c_3} + \bar{\omega}^3_3 \times \left(\bar{\omega}^3_3 \times \bar{r}^3_{c_3}\right), \quad [\bar{a}_c]^3_3 = \begin{bmatrix}\ddot{q}_2\\ \ddot{q}_1\\ g + \ddot{q}_3\end{bmatrix} \qquad (24)$$

$$\bar{a}^4_{c_4} = \bar{a}^4_4 + \bar{\varepsilon}^4_4 \times \bar{r}^4_{c_4} + \bar{\omega}^4_4 \times \left(\bar{\omega}^4_4 \times \bar{r}^4_{c_4}\right)$$

$$[\bar{a}_c]^4_4 = \begin{bmatrix}cq_4 \cdot \ddot{q}_2 + sq_4 \cdot \ddot{q}_1\\ -sq_4 \cdot \ddot{q}_2 + cq_4 \cdot \ddot{q}_1\\ g + \ddot{q}_3\end{bmatrix} \qquad (25)$$

According to [6], first, the mechanical structure is walked by iteration to outward of the robot mechanical structure. Thus, the reduction torsos elements for the external forces system are determined, achieving the following relations:

$$[\bar{F}]^1_1 = M_1 [\bar{a}_c]^1_1, \quad F^1_1 = \begin{bmatrix}0\\ M_1 \cdot \ddot{q}_1\\ M_1 \cdot g\end{bmatrix} \qquad (26)$$

$$[\bar{F}]^2_2 = M_2 [\bar{a}_c]^2_2, \quad [\bar{F}]^2_2 = \begin{bmatrix}M_2 \cdot \ddot{q}_2\\ M_2 \cdot \ddot{q}_1\\ M_2 \cdot g\end{bmatrix} \qquad (27)$$

$$[\bar{F}]^3_3 = M_3 [\bar{a}_c]^3_3, \quad [\bar{F}]^3_3 = \begin{bmatrix}M_3 \cdot \ddot{q}_2\\ M_3 \cdot \ddot{q}_1\\ M_3 \cdot (g + \ddot{q}_3)\end{bmatrix} \qquad (28)$$

$$[\bar{F}]^4_4 = M_4 [\bar{a}_c]^4_4, \quad [\bar{F}]^4_4 = \begin{bmatrix}M_4 \cdot (cq_4 \cdot \ddot{q}_2 + sq_4 \cdot \ddot{q}_1)\\ M_4 \cdot (-sq_4 \cdot \ddot{q}_2 + cq_4 \cdot \ddot{q}_1)\\ M_4 \cdot (g + \ddot{q}_3)\end{bmatrix} \qquad (29)$$

The moments of external forces are:

$$\bar{M}^1_{c_1} = J^{*1}_1 \bar{\varepsilon}^1_1 + \bar{\omega}^1_1 \times J^{*1}_1 \bar{\omega}^1_1,$$
$$[\bar{M}_c]^1_1 = [0 \quad 0 \quad 0]^T \qquad (30)$$

$$\bar{M}^2_{c_2} = J^{*2}_2 \bar{\varepsilon}^2_2 + \bar{\omega}^2_2 \times J^{*2}_2 \bar{\omega}^2_2,$$
$$[\bar{M}_c]^2_2 = [0 \quad 0 \quad 0]^T \qquad (31)$$

$$\bar{M}^3_{c_3} = J^{*3}_3 \bar{\varepsilon}^3_3 + \bar{\omega}^3_3 \times J^{*3}_3 \bar{\omega}^3_3,$$
$$[\bar{M}_c]^3_3 = [0 \quad 0 \quad 0]^T \qquad (32)$$

$$\bar{M}^4_{c_4} = J^{*4}_4 \bar{\varepsilon}^4_4 + \bar{\omega}^4_4 \times J^{*4}_4 \bar{\omega}^4_4$$
$$[\bar{M}_c]^4_4 = [0 \quad 0 \quad I^4_z \cdot \ddot{q}_4]^T \qquad (33)$$

In the second part of the Newton-Euler method, the mechanical structure is walked by iteration to inward of the robot mechanical structure. Thus, the contact forces torsos between elements and their moments are determined, respectively the generalized driving forces from the robot's couplers. The reduction torsos elements of the payload handling are expressed by the relations:

$$F^5_{l_5} = \begin{bmatrix}F^5_{lx}\\ F^5_{ly}\\ F^5_{lz}\end{bmatrix}; \qquad M^5_{O_5} = \begin{bmatrix}M^5_x\\ M^5_y\\ M^5_z\end{bmatrix} \qquad (34)$$

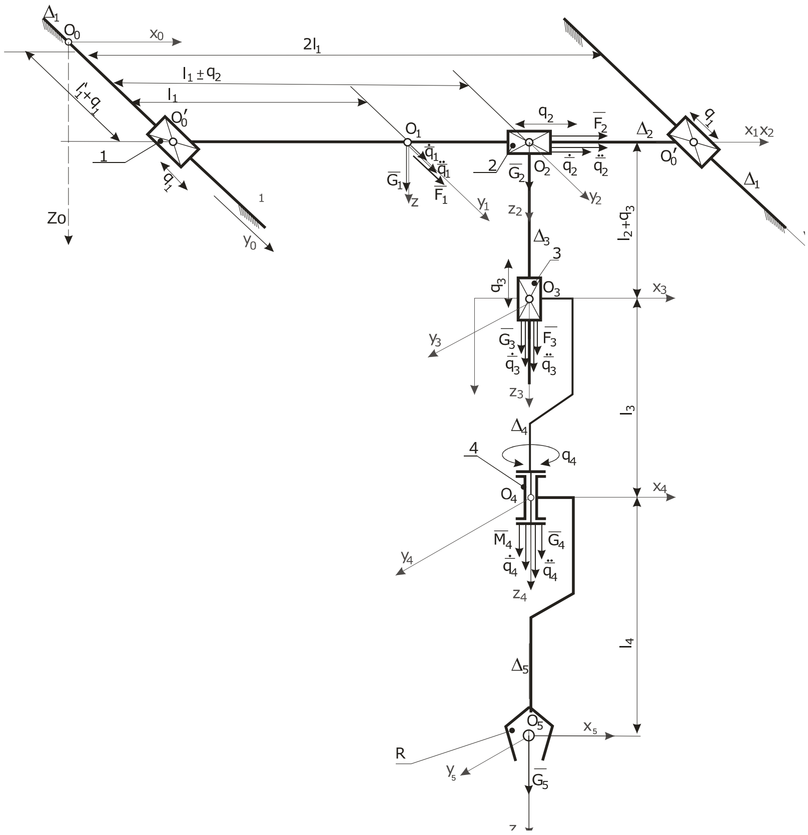

Figure 2. The kinematics structure of the TTTR robot

According to relation (16), the contact forces have the following expressions:

$$\overline{F}_{l_4}^{4}=[R]_5^4\cdot\overline{F}_{l_5}^{5}+\overline{F}_4^{4}$$

$$[\overline{F}_l]_4^4=\begin{bmatrix} F_{l_x}^5+M_4\cdot cq_4\cdot\ddot{q}_2+M_4\cdot sq_4\cdot\ddot{q}_1 \\ F_{l_y}^5-M_4\cdot sq_4\cdot\ddot{q}_2+M_4\cdot cq_4\cdot\ddot{q}_1 \\ F_{l_z}^5+M_4\cdot g+M_4\cdot\ddot{q}_3 \end{bmatrix} \quad (35)$$

$$\overline{F}_{l_3}^{3}=[R]_4^3\cdot\overline{F}_{l_4}^{4}+\overline{F}_3^{3},$$

$$[\overline{F}_l]_3^3=\begin{bmatrix} F_{l_x}^5\cdot cq_4-F_{ly}^5\cdot sq_4+M_4\cdot\ddot{q}_2+M_3\cdot\ddot{q}_2 \\ F_{l_z}^5\cdot sq_4+M_4\cdot\ddot{q}_1+F_{l_y}^5\cdot cq_4+M_3\ddot{q}_1 \\ M_4\cdot g+F_{l_z}^5+M_4\cdot\ddot{q}_3+M_3\cdot g+M_3\cdot\ddot{q}_3 \end{bmatrix} \quad (36)$$

$$\overline{F}_{l_2}^{2}=[R]_3^2\cdot\overline{F}_{l_3}^{3}+\overline{F}_2^{2}$$

$$[\overline{F}_l]_2^2=\begin{bmatrix} F_{l_x}^5\cdot cq_4-F_{l_y}^5\cdot sq_4+M_4\cdot\ddot{q}_2 \\ +M_3\cdot\ddot{q}_2+M_2\cdot\ddot{q}_2 \\ F_{l_x}^5\cdot sq_4+M_4\cdot\ddot{q}_1+F_{l_y}^5\cdot cq_4 \\ +M_3\cdot\ddot{q}_1+M_2\cdot\ddot{q}_1 \\ F_{l_z}^5+M_4\cdot g+M_4\cdot\ddot{q}_3+M_3\cdot g \\ +M_3\cdot\ddot{q}_3+M_2\cdot g \end{bmatrix} \quad (37)$$

$$\overline{F}_{l_1}^{1}=[R]_2^1\cdot\overline{F}_{l_2}^{2}+\overline{F}_1^{1}$$

$$[\overline{F}_l]_1^1=\begin{bmatrix} F_{l_x}^5\cdot cq_4-F_{l_y}^5\cdot sq_4+M_4\cdot\ddot{q}_2+ \\ M_3\cdot\ddot{q}_2+M_2\cdot\ddot{q}_2 \\ F_{l_z}^5\cdot sq_4+M_4\cdot\ddot{q}_1+F_{l_y}^5\cdot cq_4+ \\ M_3\cdot\ddot{q}_1+M_2\cdot\ddot{q}_1+M_1\cdot\ddot{q}_1 \\ F_{l_z}^5+M_4\cdot g+M_4\cdot\ddot{q}_3+M_3\cdot g+ \\ M_3\cdot\ddot{q}_3+M_2\cdot g+M_1\cdot g \end{bmatrix} \quad (38)$$

The contact forces have the expressions:

$$\overline{M}_{l_{O_4}}^{4}=[R]_5^4\cdot\overline{M}_x^{5}+\overline{r}_{c_4}^{4}\times\overline{F}_4^{4}+\overline{r}_5^{4}\times[R]_5^4\cdot\overline{F}_{l_5}^{5}+\overline{M}_{c_4}^{4},$$

$$[\overline{M}_l]_{O_4}^4=\begin{bmatrix} M_{l_x}^5-l_4\cdot F_{l_y}^5 \\ M_{l_y}^5+l4\cdot F_{l_x}^5 \\ M_{l_z}^5+J_z^{*4}\cdot\ddot{q}_4 \end{bmatrix} \quad (39)$$

$$\overline{M}_{l_{O_3}}^{3}=[R]_4^3\cdot\overline{M}_{l_{O_4}}^{4}+\overline{r}_{c_3}^{3}\times\overline{F}_3^{3}+\overline{r}_4^{3}\times[R]_4^3\cdot\overline{F}_{l_4}^{4}+\overline{M}_{c_3}^{3},$$

$$[\overline{M}_l]_{O_3}^3=\begin{bmatrix} cq_4\cdot M_{l_x}^5-cq_4\cdot l_4\cdot F_{l_y}^5-sq_4\cdot M_{l_y}^5-sq_4\cdot l_4\cdot F_{l_x}^5- \\ sq_4\cdot l_3\cdot F_{l_x}^5-l_3\cdot M_4\cdot\ddot{q}_1-cq_4\cdot l_3\cdot F_{l_y}^5 \\ \hdashline sq_4\cdot M_{l_x}^5-sq_4\cdot l_4\cdot F_{l_y}^5+cq_4\cdot M_{l_y}^5+cq_4\cdot l_4\cdot F_{l_x}^5 \\ +cq_4\cdot l_3\cdot F_{l_x}^5-sq_4\cdot l_3\cdot F_{l_y}^5+l3\cdot M_4\cdot\ddot{q}_2 \\ \hdashline M_{l_z}^5+J_z^{*4}\cdot\ddot{q}_4 \end{bmatrix} \quad (40)$$

$$\overline{M}^2_{l_{O_2}} = [R]^2_3 \cdot \overline{M}^3_{l_{O_3}} + \bar{r}^2_{c_2} \times \overline{F}^2_2 +$$
$$\bar{r}^2_3 \times [R]^2_3 \cdot \overline{F}^3_{l_3} + \overline{M}^2_{c_2},$$

$$[\overline{M}_l]^2_{O_2} = \begin{bmatrix} cq_4 \cdot M^5_{l_x} - cq_4 \cdot l_4 \cdot F^5_{l_y} - sq_4 \cdot M^5_{l_y} - \\ sq_4 \cdot l_4 \cdot F^5_{l_x} - sq_4 \cdot l_3 \cdot F^5_{l_x} - l_3 \cdot M_4 \cdot \ddot{q}_1 - \\ cq_4 \cdot l_3 \cdot F^5_{l_y} - -sq_4 \cdot q_3 \cdot F^5_{l_x} - q_3 \cdot M_4 \cdot \ddot{q}_1 - \\ cq_4 \cdot q_3 \cdot F^5_{l_y} - q_3 \cdot M_3 \cdot \ddot{q}_1 - sq_4 \cdot l_2 \cdot F^5_{l_x} - \\ l_2 \cdot M_4 \cdot \ddot{q}_1 - cq_4 \cdot l_2 \cdot F^5_{l_y} - l_2 \cdot M_3 \cdot \ddot{q}_1 \\ \hline sq_4 \cdot M^5_{l_x} - sq_4 \cdot l_4 \cdot F^5_{l_y} + cq_4 \cdot M^5_{l_y} + \\ cq_4 \cdot l_4 \cdot F^5_{l_x} + cq_4 \cdot l_3 \cdot F^5_{l_x} - sq_4 \cdot l_3 \cdot F^5_{l_y} + \\ l_3 \cdot M_4 \cdot \ddot{q}_2 + +cq_4 \cdot q_3 \cdot F^5_{l_x} - sq_4 \cdot q_3 \cdot F^5_{l_y} + \\ q_3 \cdot M_4 \cdot \ddot{q}_2 + q_3 \cdot M_3 \cdot \ddot{q}_2 + cq_4 \cdot l_2 \cdot F^5_{l_x} - \\ sq_4 \cdot l_2 \cdot F^5_{l_y} + +l_2 \cdot M_4 \cdot \ddot{q}_2 + \cdot l_2 \cdot M_3 \cdot \ddot{q}_2 \\ \hline M^5_{l_z} + J^{*4}_z \cdot \ddot{q}_4 \end{bmatrix} \quad (41)$$

$$\overline{M}^1_{l_{O_1}} = [R]^1_2 \cdot \overline{M}^2_{l_{O_2}} + \bar{r}^1_{c_1} \times \overline{F}^1_1 +$$
$$\bar{r}^1_2 \times [R]^1_2 \cdot \overline{F}^2_{l_2} + \overline{M}^1_{c_1},$$

$$[\overline{M}_l]^2_{O_2} = \begin{bmatrix} cq_4 \cdot M^5_{l_x} - cq_4 \cdot l_4 \cdot F^5_{l_y} - sq_4 \cdot M^5_{l_y} - \\ sq_4 \cdot l_4 \cdot F^5_{l_x} - sq_4 \cdot l_3 \cdot F^5_{l_x} - l_3 \cdot M_4 \cdot \ddot{q}_1 - \\ cq_4 \cdot l_3 \cdot F^5_{l_y} - -sq_4 \cdot q_3 \cdot F^5_{l_x} - q_3 \cdot M_4 \cdot \ddot{q}_1 - \\ cq_4 \cdot q_3 \cdot F^5_{l_y} - q_3 \cdot M_3 \cdot \ddot{q}_1 - sq_4 \cdot l_2 \cdot F^5_{l_x} - \\ l_2 \cdot M_4 \cdot \ddot{q}_1 - cq_4 \cdot l_2 \cdot F^5_{l_y} - l_2 \cdot M_3 \cdot \ddot{q}_1 \\ \hline sq_4 \cdot M^5_{l_x} - sq_4 \cdot l_4 \cdot F^5_{l_y} + cq_4 \cdot M^5_{l_y} + \\ cq_4 \cdot l_4 \cdot F^5_{l_x} + cq_4 \cdot l_3 \cdot F^5_{l_x} - sq_4 \cdot l_3 \cdot F^5_{l_y} + \\ l_3 \cdot M_4 \cdot \ddot{q}_2 + +cq_4 \cdot q_3 \cdot F^5_{l_x} - sq_4 \cdot q_3 \cdot F^5_{l_y} + \\ q_3 \cdot M_4 \cdot \ddot{q}_2 + q_3 \cdot M_3 \cdot \ddot{q}_2 + cq_4 \cdot l_2 \cdot F^5_{l_x} - \\ sq_4 \cdot l_2 \cdot F^5_{l_y} + +l_2 \cdot M_4 \cdot \ddot{q}_2 + \cdot l_2 \cdot M_3 \cdot \ddot{q}_2 \\ \hline M^5_{l_z} + J^{*4}_z \cdot \ddot{q}_4 \end{bmatrix} \quad (42)$$

The generalized driving forces have the following expressions:

For axis 1 O_1y_1, translation:

$$Q^1_m = \left[\overline{F}^1_{l_1}\right]^T \cdot \bar{j}^1_1 = \left[F^1_{l_x} \quad F^1_{l_y} \quad F^1_{l_z}\right] \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = F^1_{l_y}$$

$$Q^1_m = sq_4 \cdot F^5_{l_x} + M_4 \cdot \ddot{q}_1 + cq_4 F^5_{l_y} +$$
$$M_3 \cdot \ddot{q}_1 + M_2 \cdot \ddot{q}_1 + M_1 \cdot \ddot{q}_1 \quad (43)$$

For axis 2 O_2x_2, translation:

$$Q^2_m = \left[\overline{F}^2_{l_2}\right]^T \cdot \bar{i}^2_2 = \left[F^2_{l_x} \quad F^2_{l_y} \quad F^2_{l_z}\right] \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = F^2_{l_x}$$

$$Q^2_m = cq_4 \cdot F^5_{l_x} - sq_4 \cdot F^5_{l_y} +$$
$$M_4 \cdot \ddot{q}_2 + M_3 \cdot \ddot{q}_2 + M_2 \cdot \ddot{q}_2 \quad (44)$$

For axis 3 O_3z_3, translation:

$$Q^3_m = \left[\overline{F}^3_{l_3}\right]^T \cdot \bar{k}^3_3 = \left[F^3_{l_x} \quad F^3_{l_y} \quad F^3_{l_z}\right] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = F^3_{l_z}$$

$$Q^3_m = F^5_{l_z} + M_4 \cdot g + M_4 \cdot \ddot{q}_3 +$$
$$M_3 \cdot g_2 + M_3 \cdot \ddot{q}_3 \quad (45)$$

For axis 4 O_4z_4, rotation:

$$Q^4_m = \left[\overline{M}^4_{l_{O_4}}\right]^T \cdot \bar{k}^4_4 = \left[M^4_{l_x} \quad M^4_{l_y} \quad M^4_{l_z}\right] \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = M^4_{l_z}$$

$$Q^4_m = M^5_{l_z} + J^{*4}_z \ddot{q}_4 . \quad (46)$$

These generalized driving forces represent the system of differential dynamic equations characterizing the dynamic model of serial modular TTTR robot.

3. CONCLUSIONS

To determine the robot's dynamic equations using the Newton-Euler method, is primarily needed the geometric and kinematics modeling. Secondly are required the distribution parameters mass and certain simplifying assumptions of choosing the mass centers C, but also the mechanical centrifugal moments of inertia. With these are determined the mass centers accelerations and the reduction torsos elements for the external forces.

The next step is to determine the torsos of the contact forces and the moments of these contact forces. The last step is to determine the driving generalized forces from the couplers robot, their expressions representing the dynamic equations of the TTTR robot.

Dynamics study of serial robot structure, give the possibility to obtain the variants of arranging modules in a optimal structure, as well to the choice of the motion laws on each kinematics axes, so that energy consumption to be reduced.

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Elaborarea modelului dinamic pentru un robot modular Gantry, de tipul TTTR, cu ajutorul formalismului Newton Euler

Rezumat: În prima parte, lucrarea prezintă premisele teoretice necesare abordării problemei modelarii dinamice a unei structuri robotice seriale de tipul TTTR, folosind metoda Newton-Euler. În acest scop este nevoie în primul rând de modelarea geometrică şi cinematică a structurii propuse iar în al doilea rând este necesară cunoaşterea parametrilor de distribuţie a maselor. Se fac anumite ipoteze simplificatoare legate de alegerea centrelor de masă. Deasemenea este necesară cunoaşterea momentelor de inerţie mecanice centrifugale a elementelor în mişcare de rotaţie. Pe baza acestor date s-a determinat în continuare acceleraţiile corespunzătoare centrelor de masă, apoi elementele torsorului de reducere pentru sistemul forţelor exterioare, iar în continuare s-a determinat torsorul forţelor de legătură şi cel al momentelor de legătură. Ultimul pas a fost determinarea forţelor generalizate motoare din cuplele robotului, expresiile acestora reprezentând ecuaţiile dinamice ale structurii robotice seriale, de tipul TTTR. Studiul dinamic al unei structuri robotice seriale dă posibilitatea obţinerii variantelor de combinare a modulelor pentru o structură optimizata, deasemenea dă posibilitatea alegerii legilor de mişcare pe fiecare axă cinematică, astfel încât consumul energetic, pe fiecare motor şi pe întreaga structură, să fie minim.

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