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THE GEOMETRIC MODELING OF THE ARTICULATE 6R ROBOT

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Abstract: *The authors present in this paper the direct geometrical model for the robot 6R. Firstly we need to establish the geometrical model equations is necessary to determine the rotation matrix. Secondly is necessary to determine the independent parameters of the orientation, the vectors of position and, finally, following the results obtained result the direct geometrical model equations.* ***Key words:*** *kinematic structure, robot, matrix, independent parameters, characteristic point, column vector.*

1. INTRODUCTION

In the figure 1 is shown the mechanical structure of a serial robot with (n) degrees of freedom, having an open kinematic chain. Robot's mechanical structure is made of n+1 rigid elements linked together by (n) kinematic coupling of rotation (R) or translation (T). In the origin of each item k (k=1÷n) is attached a mobile reference system (T_k) and at the base of the robot is inserted the fixed reference system (T_0) in the point O_0.

The direct geometric modeling (DGM), according to [2] implies that the mechanical structure of the robot is in a known configuration, represented by the vector $\overline{q}$ of generalized coordinates.

The position of the reference system (T_n) which is jointly with the gripper of the robot, in relation with the fixed reference system (T_0), it can be determined as:

- determining the position of the origin O_n of the system (T_n) by the vector $\overline{p}_n = [p]^0$;
- using the rotation matrix, the orientation of each axis of the system (T_n) in relation with the system (T_0) it's determined.

Using the successive iterations the problem of direct geometric modelling it can be solved. Thus considering the sequence of elements (q-1, k), k=1÷n, from the kinematic structure of

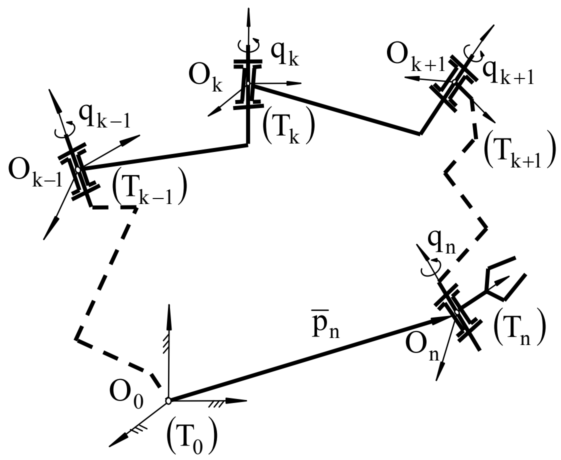

Fig. 1. The kinematic structure of a robot with (n) degrees of freedom

the robot and corresponding to this sequence, are considered known the following:

$[R]_k^{k-1}$ - is the rotation matrix expressing the orientation axes of the system (T_k) to the system (T_{k-1}); it can be expressed mathematically as:

$$[R]_k^{k-1} = \left[\overline{x}_k^{k-1}\ \overline{y}_k^{k-1}\ \overline{z}_k^{k-1}\right] = \begin{cases} R(\{\overline{x}\ \overline{y}\ \overline{z}\}, q_k), if\ rotation\ torque \\ J_3, if\ translation\ torque \end{cases} \quad (1)$$

$\overline{r}_k^{k-1}$ - is the column position vector of the origin O_k of the system (T_k) in relation with the origin O_{k-1} of the system (T_{k-1}), which can be written as:

$$\bar{r}_k^{k-1} = \bar{p}_{k,k-1}^{k-1} = \begin{cases} \left[x_k^{k-1}\ y_k^{k-1}\ z_k^{k-1}\right], \text{if rotation torque} \\ r_k^{k-1}(q_k), \text{if translation torque} \end{cases} \quad (2)$$

$\bar{p}^k$ - is the position of a point P in relation with the system (T_k), which can be written as:

$$\bar{p}^k = [p]^k = \left[p_x^k\ p_y^k\ p_z^k\right]^T \quad (3)$$

Next, using the matrix equation below, we calculate the position of the ponit P relative to the reference system (T_{k-1}):

$$\bar{p}^{k-1} = [R]_k^{k-1} \cdot \bar{p}^k = \left[p_x^{k-1}\ p_y^{k-1}\ p_z^{k-1}\right]^T. \quad (4)$$

The relation (4) is an iterative relation to obtain the following expressions, if the index k is from 1÷n:

$$\begin{bmatrix} \bar{p}^0 = \bar{p} \\ \bar{p}^1 \\ \cdots \\ \bar{p}^{k-1} \\ \cdots \\ p^{n-1} \end{bmatrix} = \begin{bmatrix} [R]_0^1 \cdot \bar{p}^1 \\ [R]_2^1 \cdot \bar{p}^2 \\ \cdots \\ [R]_k^{k-1} \cdot \bar{p}^k \\ \cdots \\ [R]_n^{n-1} \cdot \bar{p}^n \end{bmatrix} = \begin{bmatrix} \left[p_x\ p_y\ p_z\right]^T \\ [R]_2^1 \cdot \left[p_x^2\ p_y^2\ p_z^2\right]^T \\ \cdots\cdots \\ [R]_k^{k-1} \cdot \left[p_x^k\ p_y^k\ p_z^k\right]^T \\ \cdots\cdots \\ [R]_n^{n-1} \cdot \left[p_x^n\ p_y^n\ p_z^n\right]^T \end{bmatrix}. \quad (5)$$

For each k=1÷(n-1) and considering that $[\bar{p}]^k = [R]_{k+1}^k \cdot [\bar{p}]^{k+1}$, then the first relation from (5) becomes:

$$\bar{p}^0 = [R]_1^0 \cdot [R]_2^1 \ldots [R]_k^{k-1} \ldots [R]_n^{n-1} \cdot [p]^n \quad (6)$$

and this is the transformation matrix equation of the column vector $\bar{p}^n$, assumed known, the vector column $\bar{p}^0$. The equation above is equivalent to:

$$\bar{p}^0 = \bar{p} = [R]_n^0 \cdot \bar{p}^n \quad (7)$$

or

$$\left[p_x\ p_y\ p_z\right]^T = \left[\bar{x}_n\ \bar{y}_n\ \bar{z}_n\right] \cdot \left[p_x^n\ p_y^n\ p_z^n\right]^T.$$

To determine de rotation matrix, from the relations (6)-(7) it can be obtained a matrix relation that has the following form:

$$[R]_0^n = \prod_{k=1}^{n} [R]_k^{k-1} = \prod_{k=1}^{n} R(\{x\ y\ z\}, q_k) \quad (8)$$

or

$$\left[\bar{x}_n\ \bar{y}_n\ \bar{z}_n\right] = \prod_{k=1}^{n} \left[\bar{x}_k^{k-1}\ \bar{y}_k^{k-1}\ \bar{z}_k^{k-1}\right] = \begin{bmatrix} \alpha_{nx} & \alpha_{ny} & \alpha_{nz} \\ \beta_{nx} & \beta_{ny} & \beta_{nz} \\ \gamma_{nx} & \gamma_{ny} & \gamma_{nz} \end{bmatrix}. \quad (9)$$

The next step is to determine the column vector $\bar{r}_k^{k-1} = \bar{p}_{k,k-1}^{k-1}$ towards the fixed system (T_0), with the relation:

$$\bar{p}_{k,k-1} = [R]_{k-1}^0 \cdot \bar{r}_k^{k-1} = \begin{cases} \bar{r}_1^0, \text{for } k=1 \\ \left\{\prod_{j=1}^{k-1} [R]_j^{j-1}\right\} \cdot \bar{r}_k^{k-1}, \text{for } k = 2 \div (n+1). \end{cases} \quad (10)$$

Also, it's calculated the position of each origins O_k of the system (T_k) in relation to (T_0), with the relations:

$$\bar{p}_k = \bar{p}_{k-1} + \bar{p}_{k,k-1} = \sum_{j=1}^{k} \bar{p}_{j,j-1}, \text{for } k = 1 \div (n+1), \quad (11)$$

with which the column vectors are obtained:

$$\bar{p}_n = \bar{p}_{n-1} + \bar{p}_{n,n-1} = \sum_{k=1}^{n} \bar{p}_{k,k-1} = \left[p_{xn}\ p_{yn}\ p_{zn}\right]^T \quad (12)$$

$$\bar{p} = \bar{p}_{n+1} = \bar{p}_n + \bar{p}_{n+1,n} = \sum_{k=1}^{n+1} \bar{p}_{k,k-1} = \left[p_x\ p_y\ p_z\right]^T. \quad (13)$$

The points O_n and P belong to the mobile system (T_n), thus implicitly to the gripper, so that the relations (12)-(13) determines the coordinates in relation to the fixed system (T_0).

In view of (8), (12) and (13), can write the equations:

$$\left.\begin{matrix} \left[\bar{p}_n^T\ \bar{x}_n^T\ \bar{y}_n^T\ \bar{z}_n^T\right]^T \\ \left[\bar{p}^T\ \bar{x}_n^T\ \bar{y}_n^T\ \bar{z}_n^T\right]^T \end{matrix}\right\} = \left[f_j(q_k, k=1 \div n), j = 1 \div 12\right]^T. \quad (14)$$

The equations above are the direct geometric model equations (DGM), according to [2]. From these equations, only six are independent because three parameters are needed for guidance, so that we can write the identity matrix, [4]:

$$[R]_n^0(q_k, k = 1 \div n) = R(\alpha, \beta, \gamma), \quad (15)$$

where R (α, β, γ) is the orientation matrix corresponding to a set of Euler angles. For example the matrix $R(\alpha_z - \beta_x - \gamma_y)$, and from the identity matrix (15) we obtain the independent parameters of the orientation and for $\gamma_{nz} \neq \pm 1$,

$$\begin{aligned} \alpha_z &= A\tan 2(\alpha_{nz}, -\beta_{nz}) \\ \beta_x &= A\tan 2(\alpha_{nz} s\alpha_z - \beta_{nz} c\alpha_z, \gamma_{nz}) \\ \gamma_z &= A\tan 2(-\alpha_{ny} c\alpha_z - \beta_{ny} s\alpha_z, \alpha_{nx} c\alpha_z + \beta_{nx} s\alpha_z). \end{aligned} \quad (16)$$

According to the equations (14), we can write the column vector of the operational coordinates:

$$\begin{aligned} \bar{X}^0 &= \left[p_x\ p_y\ p_z\ \alpha_z\ \beta_y\ \gamma_z\right]^T = \\ &= \left[f_j\ (q_k, k = 1 \div n), j = 1 \div 6\right]^T. \end{aligned} \quad (17)$$

The relation (17) defines the position of the robot's gripper with the fixed system (T_0) by: coordinates p_x, p_y, p_z of a point and the α_z, β_x, γ_z elements of the rotation matrix R, defining its orientation.

2. THE DIRECT GEOMETRIC MODELLING

In this paragraph is presented the geometric study of the robot 6R (figure 2), composed of a rotating vertical module 1, the module 2, the rotation around the horizontal axis x_2, the rotation module 3, around the horizontal axis x_3 and spherical joint guidance of the gripper comprising: module 4 the vertical rotation around axis z_4, module 5 of rotation around the horizontal axis x_5 and module 6 rotation around the vertical axis z_6, assembled with the gripper.

In the figure 2 are defined as follows:
l_i – the constructive parameters of the robot, $i=1\div4$; q_k – the generalized coordinates, $k=1\div6$. In the origin of each item i ($i = 0 \div 7$), is attached to the Cartesian reference system (O_i).

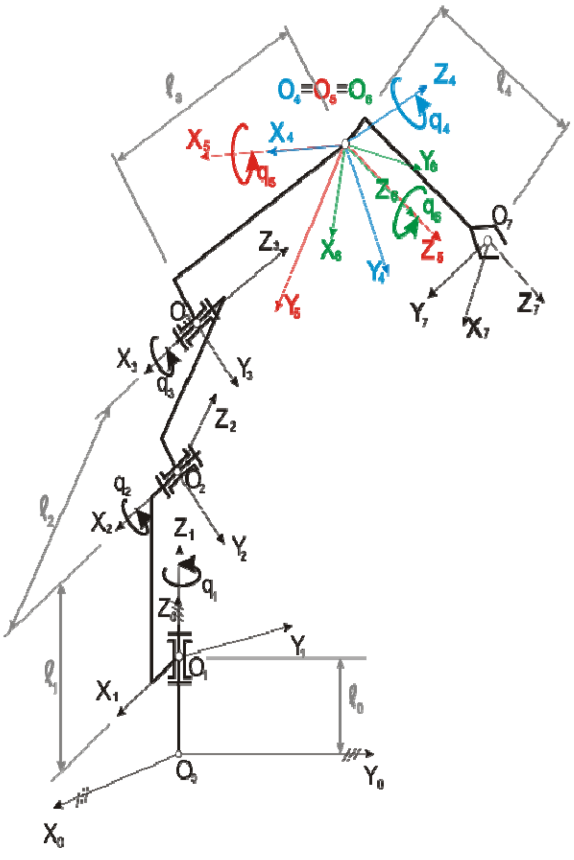

Fig. 2. The kinematic scheme of the 6R robot

The mechanical structure of the robot is in a known configuration, represented by column vector of generalized coordinates:

$$\bar{q} = [q_k;\, k = 1 \div 6]^T . \tag{18}$$

Then is determine the rotation matrix $[R]_7^0$, defined by the relation:

$$[R]_7^0 = [\bar{x}_7 \;\; \bar{y}_7 \;\; \bar{z}_7] = \begin{bmatrix} \alpha_{7x} & \alpha_{7y} & \alpha_{7z} \\ \beta_{7x} & \beta_{7y} & \beta_{7z} \\ \gamma_{7x} & \gamma_{7y} & \gamma_{7z} \end{bmatrix} . \tag{19}$$

By using the recurrence relation (20), is determine the position of the characteristic

point of the gripper relative to fixed reference system, attached to the base of the robot.

$$\bar{p} = [P]^0 = \bar{p}_6 + \bar{p}_{7,6} \, . \tag{20}$$

The orientation system (O_i) in relation to the system (O_{i-1}) is presented as the following matrices:

$$[R]_1^0 = R(\bar{z};q_1) = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{21}$$

$$[R]_2^1 = R(\bar{x};q_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_2 & -sq_2 \\ 0 & sq_2 & cq_2 \end{bmatrix}, \tag{22}$$

$$[R]_3^2 = R(\bar{x};q_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_3 & -sq_3 \\ 0 & sq_3 & cq_3 \end{bmatrix}, \tag{23}$$

$$[R]_4^3 = R(\bar{z};q_4) = \begin{bmatrix} cq_4 & -sq_4 & 0 \\ sq_4 & cq_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{24}$$

$$[R]_5^4 = R(\bar{x};q_5) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cq_5 & -sq_5 \\ 0 & sq_5 & cq_5 \end{bmatrix}, \tag{25}$$

$$[R]_6^5 = R(\bar{z};q_6) = \begin{bmatrix} cq_6 & -sq_6 & 0 \\ sq_6 & cq_6 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{26}$$

Considering the above orientation matrices for the express of each axis orientation system (O_i) in relation to the system (O_0) are written the following relations:

$$[R]_2^0 = \begin{bmatrix} cq_1 & -sq_1 \cdot cq_2 & sq_1 \cdot sq_2 \\ sq_1 & cq_1 \cdot cq_2 & -cq_1 \cdot sq_2 \\ 0 & sq_2 & cq_2 \end{bmatrix}, \tag{27}$$

$$[R]_3^0 = \begin{bmatrix} cq_1 & \begin{matrix} -sq_1.cq_2.cq_3 + \\ +sq_1.sq_2.sq_3 \end{matrix} & \begin{matrix} sq_1.cq_2.sq_3 + \\ +sq_1.sq_2.cq_3 \end{matrix} \\ sq_1 & \begin{matrix} cq_1.cq_2cq_3 - \\ -cq_1.sq_2.sq_3 \end{matrix} & \begin{matrix} -cq_1.cq_2.sq_3 - \\ -cq_1.sq_2.cq_3 \end{matrix} \\ 0 & sq_2.cq_3 + cq_2.sq_3 & sq_2.sq_3 + cq_2.cq_3 \end{bmatrix} \tag{28}$$

$$[R]_4^0 = \begin{bmatrix} \begin{matrix} cq_1.cq_4 + \\ +\begin{pmatrix} sq_1.cq_2.cq_3 + \\ +sq_1.sq_2.sq_3 \end{pmatrix}.sq_4 \end{matrix} & \begin{matrix} -cq_1.sq_4 + \\ +\begin{pmatrix} -sq_1.cq_2.cq_3 + \\ +sq_1.sq_2.sq_3 \end{pmatrix}.cq_4 \end{matrix} & \begin{matrix} sq_1.cq_2.sq_3 + \\ +sq_1.sq_2.cq_3 \end{matrix} \\ \begin{matrix} sq_1.sq_4 + \\ +\begin{pmatrix} sq_1.cq_2.cq_3 - \\ -cq_1.sq_2.sq_3 \end{pmatrix}.sq_4 \end{matrix} & \begin{matrix} sq_1.sq_4 + \\ +\begin{pmatrix} cq_1.cq_2.cq_3 - \\ -cq_1.sq_2.sq_3 \end{pmatrix}.sq_4 \end{matrix} & \begin{matrix} cq_1.cq_2.sq_3 - \\ -cq_1.sq_2.cq_3 \end{matrix} \\ \begin{pmatrix} sq_2.cq_3 + \\ +cq_2.sq_3 \end{pmatrix}.sq_4 & \begin{pmatrix} sq_2.cq_3 + \\ +cq_2.sq_3 \end{pmatrix}.cq_4 & \begin{matrix} -sq_2.sq_3 + \\ +cq_2.cq_3 \end{matrix} \end{bmatrix} \tag{29}$$

$$[R]_5^0 = \begin{bmatrix} \begin{matrix} cq_1.cq_4 + \\ +\begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.sq_4 \end{matrix} & \begin{matrix} \left(-cq_1.sq_4 + \begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.cq_4\right). \\ .cq_5 + (sq_1.cq_2.sq_3 + sq_1.sq_2.cq_3).sq_5 \end{matrix} & \begin{matrix} -\left(-cq_1.sq_4 + \begin{pmatrix} -sq_1.cq_2.cq_3 + \\ +sq_1.sq_2.sq_3 \end{pmatrix} \cdot cq_4\right) \\ .sq_5 (sq_1.cq_2.sq_3 + sq_1.sq_2.cq_3).cq_5 \end{matrix} \\ sq_1.cq_4 + \begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.sq_4 & \begin{matrix} \left(-sq_1.cq_4 + \begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.cq_4\right) \\ .cq_5 + \begin{pmatrix} -cq_1.cq_2.sq_3 \\ -cq_1.sq_2.cq_3 \end{pmatrix}.sq_5 \end{matrix} & \begin{matrix} -\left(-sq_1.sq_4 + \begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.cq_4\right) \\ .sq_5 + \begin{pmatrix} -cq_1.cq_2.sq_3 - \\ -cq_1.sq_2.cq_3 \end{pmatrix}.cq_5 \end{matrix} \\ (sq_2.cq_3 + cq_2.sq_3).sq_4 & \begin{matrix} (sq_2.cq_3 + cq_2.sq_3).cq_4.cq_5 + \\ +(-sq_2.sq_3 + cq_2.cq_3).sq_5 \end{matrix} & \begin{matrix} -(sq_2.cq_3 + cq_2.sq_3)cq_4.sq_5 + \\ +(-sq_2.sq_3 + cq_2.cq_3).cq_5 \end{matrix} \end{bmatrix} \tag{30}$$

$$[R]_6^0 = \begin{bmatrix} \begin{matrix} \left(cq_1.cq_4 + \begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.sq_4\right).cq_6 \\ +\begin{pmatrix} \begin{pmatrix} -cq_1.sq_4 \\ +\begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.cq_4 \end{pmatrix}.cq_5 \\ +\begin{pmatrix} sq_1.cq_2.sq_3 \\ +sq_1.sq_2.cq_3 \end{pmatrix}.sq_5 \end{pmatrix}.sq_6 \end{matrix} & \begin{matrix} -\begin{pmatrix} cq_1.cq_4 \\ +\begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.sq_4 \end{pmatrix}.cq_6 \\ +\begin{pmatrix} \begin{pmatrix} -cq_1.sq_4 \\ +\begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.cq_4 \end{pmatrix}.cq_5 \\ +\begin{pmatrix} sq_1.cq_2.sq_3 \\ +sq_1.sq_2.cq_3 \end{pmatrix}.sq_5 \end{pmatrix}.cq_6 \end{matrix} & \begin{matrix} -\begin{pmatrix} -cq_1.sq_4 \\ +\begin{pmatrix} -sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3 \end{pmatrix}.cq_4 \end{pmatrix}.sq_5 \\ +\begin{pmatrix} sq_1.cq_2.sq_3 \\ +sq_1.sq_2.cq_3 \end{pmatrix}.cq_5 \end{matrix} \\ \begin{matrix} \begin{pmatrix} sq_1.cq_4 \\ +(cq_1.cq_2.cq_3 - cq_1.sq_2.sq_3).sq_4 \end{pmatrix}.cq_6 \\ +\begin{pmatrix} -sq_1.sq_4 \\ +\left(\begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.cq_4\right).cq_5 \\ +(-cq_1.sq_2.cq_3).sq_5 \end{pmatrix}.sq_6 \end{matrix} & \begin{matrix} \begin{pmatrix} sq_1.cq_4 \\ +\begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.sq_4 \end{pmatrix}.sq_6 \\ +\begin{pmatrix} \left(-sq_1.sq_4 + \begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.cq_4\right).cq_5 \\ +(-cq_1.cq_2.sq_3 - cq_1.sq_2.cq_3).sq_5 \end{pmatrix}.cq_6 \end{matrix} & \begin{matrix} -\begin{pmatrix} -sq_1.sq_4 \\ +\begin{pmatrix} cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3 \end{pmatrix}.cq_4 \end{pmatrix}.sq_5 \\ +\begin{pmatrix} -cq_1.cq_2.sq_3 \\ -cq_1.sq_2.cq_3 \end{pmatrix}.cq_5 \end{matrix} \\ \begin{matrix} (sq_2.cq_3 + cq_2.sq_3).sq_4.cq_6 \\ +\begin{pmatrix} (sq_2.cq_3 + cq_2.sq_3).cq_4.cq_5 \\ +(-sq_2.sq_3 + cq_2.cq_3).sq_5 \end{pmatrix}.sq_6 \end{matrix} & \begin{matrix} (sq_2.cq_3 + cq_2.sq_3).sq_4.sq_6 \\ +\begin{pmatrix} (sq_2.cq_3 + cq_2.sq_3).cq_4.cq_5 \\ +(-sq_2.sq_3 + cq_2.cq_3).sq_5 \end{pmatrix}.cq_6 \end{matrix} & \begin{matrix} -(sq_2.cq_3 + cq_2.sq_3) \\ .cq_4.sq_5 + (-sq_2.sq_3.cq_2.cq_3).cq_5 \end{matrix} \end{bmatrix} \tag{31}$$

$$[R]_7^0 =$$

$$= \left[\begin{array}{c:c:c}
\begin{gathered}\left(cq_1 \cdot cq_4 + \begin{pmatrix}-sq_1 \cdot cq_2 \cdot cq_3 \\ +sq_1 \cdot sq_2 \cdot sq_3\end{pmatrix} \cdot sq_4\right) \\ \cdot cq_6 + \left(\begin{pmatrix}-cq_1 \cdot sq_4 + \\ \begin{pmatrix}-sq_1 \cdot cq_2 \cdot cq_3 \\ +sq_1 \cdot sq_2 \cdot sq_3\end{pmatrix}.cq_4\end{pmatrix}\right. \\ \left..cq_5 + (sq_1.cq_2.sq_3.sq_1.sq_2.sq_3).sq_5\right).sq_6\end{gathered} &
\begin{gathered}-\begin{pmatrix}cq_1.cq_4 \\ +(-sq_1.cq_2.cq_3.sq_1.sq_2.sq_3)sq_4\end{pmatrix} \\ .sq_6 + \begin{pmatrix}\begin{pmatrix}-cq_1.sq_4 \\ +\begin{pmatrix}-sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3\end{pmatrix}.cq_4\end{pmatrix} \\ .cq_5 + \begin{pmatrix}sq_1.cq_2.sq_3 \\ +sq_1.sq_2.cq_3\end{pmatrix}.sq_5\end{pmatrix} \\ .cq_6\end{gathered} &
\begin{gathered}-\begin{pmatrix}-cq_1.sq_4 + \\ \begin{pmatrix}-sq_1.cq_2.cq_3 \\ +sq_1.sq_2.sq_3\end{pmatrix}.cq_4\end{pmatrix}.sq_5 \\ +\begin{pmatrix}sq_1.cq_2.sq_3 \\ +sq_1.sq_2.cq_3\end{pmatrix}.cq_5\end{gathered} \\ \hdashline
\begin{gathered}\left(sq_1.cq_4 + \begin{pmatrix}cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3\end{pmatrix}.sq_4\right).cq_6 \\ +\begin{pmatrix}\begin{pmatrix}-sq_1.sq_4 + \begin{pmatrix}cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3\end{pmatrix} \\ .cq_4\end{pmatrix}.cq_5 \\ +(-cq_1.cq_2.sq_3 - cq_1.sq_2.cq_3)sq_5\end{pmatrix} \\ .sq_6\end{gathered} &
\begin{gathered}\left(sq_1.cq_4 + \begin{pmatrix}cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3\end{pmatrix}.sq_4\right).sq_6 \\ +\begin{pmatrix}\begin{pmatrix}-sq_1.sq_4 \\ +\begin{pmatrix}cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3\end{pmatrix}.cq_4\end{pmatrix} \\ +\begin{pmatrix}-cq_1.cq_2.sq_3 \\ -cq_1.sq_2.cq_3\end{pmatrix}.sq_5\end{pmatrix}.cq_6\end{gathered} &
\begin{gathered}-\begin{pmatrix}-sq_1.sq_4 \\ +\begin{pmatrix}cq_1.cq_2.cq_3 \\ -cq_1.sq_2.sq_3\end{pmatrix}.cq_4\end{pmatrix}.sq_5 \\ +\begin{pmatrix}-cq_1.cq_2.sq_3 \\ -cq_1.sq_2.cq_3\end{pmatrix}.cq_5\end{gathered} \\ \hdashline
\begin{gathered}(sq_2.cq_3.cq_2.sq_3)sq_4.cq_6 + \\ \begin{pmatrix}(sq_2.cq_3.sq_2.sq_3)cq_4.cq_5 + \\ (-sq_2.sq_3 + cq_2.cq_3)sq_5\end{pmatrix}.sq_6\end{gathered} &
\begin{gathered}(sq_2.cq_3 + cq_2.sq_3)sq_4.sq_6 \\ +\begin{pmatrix}\begin{pmatrix}sq_2.cq_3 \\ +cq_2.sq_3\end{pmatrix}.cq_4.cq_5 \\ +\begin{pmatrix}-sq_2.sq_3 \\ +cq_2.sq_3\end{pmatrix}.sq_5\end{pmatrix}.cq_6\end{gathered} &
\begin{gathered}-\begin{pmatrix}sq_2.cq_3 \\ +cq_2.sq_3\end{pmatrix}.cq_4.sq_5 \\ +\begin{pmatrix}-sq_2.sq_3 \\ +cq_2.cq_3\end{pmatrix}.cq_5\end{gathered}
\end{array}\right] \quad (32)$$

For establised the independent orintation parameters, for example, $(\alpha_z - \beta_x - \gamma_z)$, according to (22) and [4] relation:

$$[R]_7^0 = (\alpha_z - \beta_x - \gamma_z) \quad (33)$$

$$R(\alpha_z - \beta_x - \gamma_z) =$$

$$= \left[\begin{array}{c:c:c}
\begin{gathered}c\alpha_z.c\gamma_z - \\ -s\alpha_z.s\gamma_z.c\beta_x\end{gathered} & \begin{gathered}-c\alpha_z.s\gamma_z - \\ -s\alpha_z.c\gamma_z.c\beta_x\end{gathered} & s\alpha_z.s\beta_x \\ \hdashline
\begin{gathered}s\alpha_z.c\gamma_z + \\ +c\alpha_z.s\gamma_z.c\beta_x\end{gathered} & \begin{gathered}-s\alpha_z.s\gamma_z + \\ +c\alpha_z.c\gamma_z.c\beta_x\end{gathered} & -c\alpha_z.s\beta_x \\ \hdashline
s\gamma_z.s\beta_x & c\gamma_z.s\beta_x & c\beta_x
\end{array}\right]. \quad (34)$$

In the case of the 6R robot, the relation (9) become:

$$[R]_7^0 =$$

$$= \left[\begin{array}{c:c:c}
\begin{gathered}c\alpha_z.c\gamma_z - \\ -s\alpha_z.s\gamma_z.c\beta_x\end{gathered} & \begin{gathered}-c\alpha_z.s\gamma_z - \\ -s\alpha_z.c\gamma_z.c\beta_x\end{gathered} & s\alpha_z.s\beta_x \\ \hdashline
\begin{gathered}s\alpha_z.c\gamma_z + \\ +c\alpha_z.s\gamma_z.c\beta_x\end{gathered} & \begin{gathered}-s\alpha_z.s\gamma_z + \\ +c\alpha_z.c\gamma_z.c\beta_x\end{gathered} & -c\alpha_z.s\beta_x \\ \hdashline
s\gamma_z.s\beta_x & c\gamma_z.s\beta_x & c\beta_x
\end{array}\right]. \quad (35)$$

Identifying the elements 13, 32 and 33 in equation (32), is determine the orientation independent parameters, specified in the relation, according to [3] and [4]:

$$\alpha_z =$$

$$= arctg\begin{pmatrix}-\left(-cq_1.sq_4 + \left(-sq_1.cq_2.cq_3 + sq_1.sq_2.sq_3\right).cq_4\right).sq_5 \\ +\left(sq_1.cq_2.sq_3 + sq_1.sq_2.cq_3\right).cq_5, \\ \left(-sq_1.sq_4 + \left(cq_1.cq_2\right)\right)\end{pmatrix} \quad (36)$$

$$\beta_x =$$

$$= \begin{pmatrix}\sqrt{\begin{pmatrix}\begin{pmatrix}sq_4^2.sq_5^2 + cq_5^2.cq_2^2.sq_3^2 + sq_5^2.cq_4^2.cq_2^2.cq_3^2 \\ +2.sq_5.cq_4.cq_2^2.cq_3.cq_5.sq_3 \\ -2.sq_5^2.cq_4^2.cq_2.cq_3.sq_2.sq_3 + \\ 2.sq_5.cq_4.cq_2.cq_3^2.cq_5.sq_2 \\ +sq_5^2.cq_4^2.sq_2^2.sq_3^2 + cq_5^2.sq_2^2.cq_3^2 \\ -2.sq_5.cq_4.sq_2.sq_3^2.cq_5.cq_2 \\ +2.cq_5^2.cq_2.sq_3.sq_2.cq_3\end{pmatrix}.cq_1^2 + sq_1^2\end{pmatrix}}, \\ -cq_4.sq_5.cq_2.sq_3 - cq_4.sq_5.sq_2.cq_3 + cq_5.cq_2.cq_3 - cq_5.sq_2.sq_3\end{pmatrix}$$

(37)

$$\gamma_z = arctg\left[\frac{\begin{pmatrix} c(-q_6-q_4-q_5+q_3+q_2) \\ -2.c(q_6+q_3+q_2+q_4) \\ -2.c(-q_6+q_3+q_2+q_4) \\ +c(-q_6-q_4+q_5+q_3+q_2) \\ +2.c(q_6+q_3+q_2-q_4) \\ -c(q_6+q_4+q_5+q_3+q_2) \\ -c(q_6+q_4-q_5+q_3+q_2) \\ +2.c(-q_6+q_3+q_2+q_5) \\ +2.c(-q_6-q_4+q_3+q_2) \\ +c(-q_6+q_4+q_5+q_3+q_2) \\ +c(-q_6+q_4-q_5+q_3+q_2) \\ -2.c(q_6+q_2+q_3+q_5) \\ +2.c(q_6+q_3+q_2-q_5) \\ -2.c(-q_6+q_3+q_2-q_5) \\ -c(q_6-q_4+q_5+q_3+q_2) \\ -c(q_6-q_4-q_5+q_3+q_2) \end{pmatrix}}{\begin{pmatrix} 4.c(2.q_5+2.q_4) \\ -2.c(2.q_5-2.q_4+2.q_3+2.q_2) \\ -2.c(-2.q_5-2.q_4+2.q_3+2.q_2) \\ +4.c(2.q_4+2.q_3+2.q_2) \\ -2.c(2.q_5+2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_4)+40 \\ -2.c(-2.q_5-2.q_4+2.q_3+2.q_2) \\ +4.c(-2.q_5+2.q_4) \\ +4.c(-2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_5+q_4+2.q_3+2.q_2) \\ +8.c(-2.q_5+q_4+2.q_3+2.q_2) \\ -8.c(2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_3+2.q_2) \\ -12.c(2.q_5+2.q_3+2.q_2) \\ +8.c(-2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_5) \\ -12.c(-2.q_5+2.q_3+2.q_2) \end{pmatrix}^{\frac{1}{2}}}, \frac{\begin{pmatrix} -2.s(q_6+q_3+q_2-q_5) \\ +s(q_6+q_4+q_5+q_3+{}_2) \\ +2.s(q_6+q_3+q_2+q_5) \\ -2.s(-q_6+q_3+q_2+q_4) \\ +s(q_6-q_4+q_5+q_3+q_2) \\ +s(q_6-q_4-q_5+q_3+q_2) \\ -2.s(-q_6+q_3+q_2-q_5) \\ +s(-q_6+q_4+q_5+q_3+q_2) \\ +s(-q_6+q_4-q_5+q_3+q_2) \\ +2.s(q_6+q_3+q_2+q_4) \\ +2.s(-q_6-q_4+q_3+q_2) \\ +2.s(-q_6+q_3+q_2+q_5) \\ -2.s(q_6+q_3+q_2-q_4) \\ +s(q_6+q_4-q_5+q_3+q_2) \\ +s(-q_6-q_4+q_5+q_3+q_2) \\ +s(-q_6-q_4-q_5+q_3+q_2) \end{pmatrix}}{\begin{pmatrix} 4.c(2.q_5+2.q_4) \\ -2.c(2.q_5-2.q_4+2.q_3+2.q_2) \\ -2.c(-2.q_5+2.q_4+2.q_3+2.q_2) \\ +4.c(2.q_4+2.q_3+2.q_2) \\ -2.c(2.q_5+2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_4)+40 \\ -2.c(-2.q_5-2.q_4+2.q_3+2.q_2) \\ +4.c(-2.q_5+2.q_4) \\ +4.c(-2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_5+q_4+2.q_3+2.q_2) \\ +8.c(-2.q_5+q_4+2.q_3+2.q_2) \\ -8.c(2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_3+2.q_2) \\ -12.c(2.q_5+2.q_3+2.q_2) \\ +8.c(-2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_5)-12.c(-2.q_5+2.q_3+2.q_2) \end{pmatrix}^{\frac{1}{2}}}\right]. \tag{38}$$

Based on the scheme from figure 2, it can be written relative position vectors of the origins O_i of mobile systems $O_i x_i y_i z_i$ in relation to O_{i-1}, in the following form:

$$[\bar{r}]_1^0 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix};\ [\bar{r}]_2^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix};[\bar{r}]_3^2 = \begin{bmatrix} 0 \\ 0 \\ l_2 \end{bmatrix};\ [\bar{r}]_4^3 = \begin{bmatrix} 0 \\ 0 \\ l_3 \end{bmatrix};$$

$$[\bar{r}]_5^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};[\bar{r}]_6^5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};[\bar{r}]_7^6 = \begin{bmatrix} 0 \\ 0 \\ l_4 \end{bmatrix}; \tag{39}$$

In what follows, by multiplying the position vector of the rotation matrices, is determine the position of origin O_i of each reference system relative to the O_{i-1} system:

$$[\bar{p}]_{10} = [\bar{r}]_1^0 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}; [\bar{p}]_{21} = [R]_1^0 . [\bar{r}]_2^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}; \quad (40)$$

$$[\bar{p}]_{32} = [R]_2^0 . [\bar{r}]_3^2 = \begin{bmatrix} sq_1.sq_2.l_2 \\ -cq_1.sq_2.l_2 \\ cq_2.l_2 \end{bmatrix}; \quad (41)$$

$$[\bar{p}]_{43} = [R]_3^0 . [\bar{r}]_4^3 = \\ = \begin{bmatrix} \left(-\frac{1}{2}.l_3.c(q_3+q_1+q_2)+\frac{1}{2}.l_3.c(q_3-q_1+q_2)\right) \\ \left(-\frac{1}{2}.l_3.s(q_3-q_1+q_2)-\frac{1}{2}.l_3.s(q_3+q_1+q_2)\right) \\ c(q_2+q_3).l_3 \end{bmatrix}; \quad (42)$$

$$[\bar{p}]_{54} = [R]_3^0 . [\bar{r}]_5^4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; [\bar{p}]_{65} = [R]_5^0 . [\bar{r}]_6^5 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad (43)$$

$$[\bar{p}]_{76} = [R]_6^0 . [\bar{r}]_7^6 = \\ = \begin{bmatrix} \left(-\begin{pmatrix} -cq_1.sq_4 \\ +(-sq_1.cq_2.cq_3+sq_1.sq_2.sq_3).cq_4 \end{pmatrix}.sq_5 \\ +(sq_1.cq_2.sq_3+sq_1.sq_2.cq_3).cq_5 \right).l_4 \\ \left(-\begin{pmatrix} -sq_1.sq_4 \\ +(cq_1.cq_2.cq_3-cq_1.sq_2.sq_3).cq_4 \end{pmatrix}.sq_5 \\ +(-cq_1.cq_2.sq_3-cq_1.sq_2.cq_3).cq_5 \right).l_4 \\ \begin{pmatrix} -(sq_2.cq_3+cq_2.sq_3).cq_4.sq_5 \\ +(-sq_2.sq_3+cq_2.cq_3).cq_5 \end{pmatrix}.l_4 \end{bmatrix} \quad (44)$$

Based on relation (14), it expresse the position of the origin O_i, $i = 1 \div 7$, of the reference system of order i, in relation to fixed system from the robot's base. Thus:

$$[\bar{p}]_1 = [\bar{p}]_{10} = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}; [\bar{p}]_2 = [\bar{p}]_1 + [\bar{p}]_{21} = \begin{bmatrix} 0 \\ 0 \\ l_0 + l_1 \end{bmatrix}; \quad (45)$$

$$[\bar{p}]_3 = [\bar{p}]_2 + [\bar{p}]_{32} = \begin{bmatrix} sq_1.sq_2.l_2 \\ -cq_1.sq_2.l_2 \\ l_0 + l_1 + cq_2.l_2 \end{bmatrix}; \quad (46)$$

$$[\bar{p}]_4 = [\bar{p}]_3 + [\bar{p}]_{43} = \\ = \begin{bmatrix} sq_1.sq_2.l_2 - \frac{1}{2}.l_3.c(q_3+q_1+q_2)+\frac{1}{2}.l_3.c(q_3-q_1+q_2) \\ -cq_1.sq_2.l_2 - \frac{1}{2}.l_3.s(q_3-q_1+q_2)-\frac{1}{2}.l_3.s(q_3+q_1+q_2) \\ l_0 + l_1 + cq_2.l_2 + c(q_2+q_3).l_3 \end{bmatrix}; \quad (47)$$

$$[\bar{p}]_5 = [\bar{p}]_4 + [\bar{p}]_{54} = \\ = \begin{bmatrix} sq_1.sq_2.l_2 - \frac{1}{2}.l_3.c(q_3+q_1+q_2)+\frac{1}{2}.l_3.c(q_3-q_1+q_2) \\ -cq_1.sq_2.l_2 - \frac{1}{2}.l_3.s(q_3-q_1+q_2)-\frac{1}{2}.l_3.s(q_3+q_1+q_2) \\ l_0 + l_1 + cq_2.l_2 + c(q_2+q_3).l_3 \end{bmatrix}; \quad (48)$$

$$[\bar{p}]_6 = [\bar{p}]_5 + [\bar{p}]_{65} = \\ = \begin{bmatrix} sq_1.sq_2.l_2 - \frac{1}{2}.l_3.c(q_3+q_1+q_2)+\frac{1}{2}.l_3.c(q_3-q_1+q_2) \\ -cq_1.sq_2.l_2 - \frac{1}{2}.l_3.s(q_3-q_1+q_2)-\frac{1}{2}.l_3.s(q_3+q_1+q_2) \\ l_0 + l_1 + cq_2.l_2 + c(q_2+q_3).l_3 \end{bmatrix}; \quad (49)$$

$$[\bar{p}]_7 = [\bar{p}]_6 + [\bar{p}]_{76} =$$

$$= \begin{bmatrix} sq_1.sq_2.l_2 - \frac{1}{2}.l_3.c(q_3+q_1+q_2) + \frac{1}{2}.l_3.c(q_3-q_1+q_2) \\ + \left(- \begin{pmatrix} cq_1.sq_4 + \\ -(-sq_1.cq_2.cq_3 + sq_1.sq_2.sq_3).cq_4.sq_5 \\ +(sq_1.cq_2.sq_3 + sq_1.sq_2.cq_3).cq_5 \end{pmatrix} .l_4 \right) \\ ----------------- \\ -cq_1.sq_2.l_2 - \frac{1}{2}.l_3.s(q_3-q_1+q_2) - \frac{1}{2}.l_3.s(q_3+q_1+q_2) \\ + \begin{pmatrix} - \begin{pmatrix} -sq_1.sq_4 + \\ +(cq_1.cq_2.cq_3 - cq_1.sq_2.sq_3).cq_4 \end{pmatrix} .sq_5 \\ +(-cq_1.cq_2.sq_3 - cq_1.sq_2.cq_3).cq_5 \end{pmatrix} .l_4 \\ ----------------- \\ l_0 + l_1 + cq_2.l_2 + c(q_2+q_3).l_3 + \\ + \begin{pmatrix} -(sq_2.cq_3 + cq_2.sq_3).cq_4.sq_5 \\ +(-sq_2.sq_3 + cq_2.cq_3).cq_5 \end{pmatrix} .l_4 \end{bmatrix}; \qquad (50)$$

The column vector of the operational coordinates is obtained by meeting the operational results of the relations (50) and the Euler anglers (36), (37) and (38). The column vector, matrix written will show like in the following form:

$$[\bar{x}]_0 = \begin{bmatrix} x_7 \\ y_7 \\ z_7 \\ .... \\ \alpha_z \\ \beta_x \\ \gamma_z \end{bmatrix} =$$

$$
\begin{bmatrix}
sq_1.sq_2.l_2-\frac{1}{2}.l_3.c(q_3+q_1+q_2)+\frac{1}{2}.l_3.c(q_3-q_1+q_2)+\left(-\begin{pmatrix} cq_1.sq_4+(-sq_1.cq_2.cq_3+sq_1.sq_2.sq_3).cq_4.sq_5 \\ +(sq_1.cq_2.sq_3+sq_1.sq_2.cq_3).cq_5 \end{pmatrix}.l_4\right) \\
-cq_1.sq_2.l_2-\frac{1}{2}.l_3.s(q_3-q_1+q_2)-\frac{1}{2}.l_3.s(q_3+q_1+q_2)+\begin{pmatrix} -(-sq_1.sq_4+(cq_1.cq_2.cq_3-cq_1.sq_2.sq_3).cq_4).sq_5 \\ +(-cq_1.cq_2.sq_3-cq_1.sq_2.cq_3).cq_5 \end{pmatrix} \\
l_0+l_1+cq_2.l_2+c(q_2+q_3).l_3+\begin{pmatrix} -(sq_2.cq_3+cq_2.sq_3).cq_4.sq_5 \\ +(-sq_2.sq_3+cq_2.cq_3).cq_5 \end{pmatrix} \\
\ldots \\
arctg\begin{pmatrix} -(-cq_1.sq_4+(-sq_1.cq_2.cq_3+sq_1.sq_2.sq_3).cq_4).sq_5+(sq_1.cq_2.sq_3+sq_1.sq_2.cq_3).cq_5, \\ (-sq_1.sq_4+(cq_1.cq_2)) \end{pmatrix} \\
\begin{pmatrix} \left(\sqrt{\left(\begin{pmatrix} sq_4{}^2.sq_5{}^2+cq_5{}^2.cq_2{}^2.sq_3{}^2+sq_5{}^2.cq_4{}^2.cq_2{}^2.cq_3{}^2+2.sq_5.cq_4.cq_2{}^2.cq_3.cq_5.sq_3 \\ -2.sq_5{}^2.cq_4{}^2.cq_2.cq_3.sq_2.sq_3+2.sq_5.cq_4.cq_2.cq_3{}^2.cq_5.sq_2 \\ +sq_5{}^2.cq_4{}^2.sq_2{}^2.sq_3{}^2+cq_5{}^2.sq_2{}^2.cq_3{}^2-2.sq_5.cq_4.sq_2.sq_3{}^2.cq_5.cq_2+2.cq_5{}^2.cq_2.sq_3.sq_2.cq_3 \end{pmatrix}.cq_1{}^2+sq_1{}^2\right)}\right), \\ -cq_4.sq_5.cq_2.sq_3-cq_4.sq_5.sq_2.cq_3+cq_5.cq_2.cq_3-cq_5.sq_2.sq_3 \end{pmatrix}
\end{bmatrix}
$$

$$
=\begin{bmatrix}
arctg\left[\frac{\begin{pmatrix} c(-q_6-q_4-q_5+q_3+q_2)-2.c(q_6+q_3+q_2+q_4) \\ -2.c(-q_6+q_3+q_2+q_4)+c(-q_6-q_4+q_5+q_3+q_2) \\ +2.c(q_6+q_3+q_2-q_4) \\ -c(q_6+q_4+q_5+q_3+q_2) \\ -c(q_6+q_4-q_5+q_3+q_2) \\ +2.c(-q_6+q_3+q_2+q_5) \\ +2.c(-q_6-q_4+q_3+q_2) \\ +c(-q_6+q_4+q_5+q_3+q_2) \\ +c(-q_6+q_4-q_5+q_3+q_2) \\ -2.c(q_6+q_2+q_3+q_5) \\ +2.c(q_6+q_3+q_2-q_5) \\ -2.c(-q_6+q_3+q_2-q_5) \\ -c(q_6-q_4+q_5+q_3+q_2) \\ -c(q_6-q_4-q_5+q_3+q_2) \end{pmatrix}}{\begin{pmatrix} 4.c(2.q_5+2.q_4)-2.c(2.q_5-2.q_4+2.q_3+2.q_2) \\ -2.c(-2.q_5-2.q_4+2.q_3+2.q_2) \\ +4.c(2.q_4+2.q_3+2.q_2) \\ -2.c(2.q_5+2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_4)+40 \\ -2.c(-2.q_5-2.q_4+2.q_3+2.q_2) \\ +4.c(-2.q_5+2.q_4) \\ +4.c(-2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_5+q_4+2.q_3+2.q_2) \\ +8.c(-2.q_5+q_4+2.q_3+2.q_2) \\ -8.c(2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_3+2.q_2) \\ -12.c(2.q_5+2.q_3+2.q_2) \\ +8.c(-2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_5) \\ -12.c(-2.q_5+2.q_3+2.q_2) \end{pmatrix}^{\frac{1}{2}}}, \frac{\begin{pmatrix} -2.s(q_6+q_3+q_2-q_5) \\ +s(q_6+q_4+q_5+q_3+{}_2) \\ +2.s(q_6+q_3+q_2+q_5) \\ -2.s(-q_6+q_3+q_2+q_4) \\ +s(q_6-q_4+q_5+q_3+q_2) \\ +s(q_6-q_4-q_5+q_3+q_2) \\ -2.s(-q_6+q_3+q_2-q_5) \\ +s(-q_6+q_4+q_5+q_3+q_2) \\ +s(-q_6+q_4-q_5+q_3+q_2) \\ +2.s(q_6+q_3+q_2+q_4) \\ +2.s(-q_6-q_4+q_3+q_2) \\ +2.s(-q_6+q_3+q_2+q_5) \\ -2.s(q_6+q_3+q_2-q_4) \\ +s(q_6+q_4-q_5+q_3+q_2) \\ +s(-q_6-q_4+q_5+q_3+q_2) \\ +s(-q_6-q_4-q_5+q_3+q_2) \end{pmatrix}}{\begin{pmatrix} 4.c(2.q_5+2.q_4)-2.c(2.q_5-2.q_4+2.q_3+2.q_2) \\ -2.c(-2.q_5+2.q_4+2.q_3+2.q_2) \\ +4.c(2.q_4+2.q_3+2.q_2) \\ -2.c(2.q_5+2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_4)+40 \\ -2.c(-2.q_5-2.q_4+2.q_3+2.q_2) \\ +4.c(-2.q_5+2.q_4) \\ +4.c(-2.q_4+2.q_3+2.q_2) \\ -8.c(2.q_5+q_4+2.q_3+2.q_2) \\ +8.c(-2.q_5+q_4+2.q_3+2.q_2) \\ -8.c(2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_3+2.q_2) \\ -12.c(2.q_5+2.q_3+2.q_2) \\ +8.c(-2.q_5-q_4+2.q_3+2.q_2) \\ -8.c(2.q_5)-12.c(-2.q_5+2.q_3+2.q_2) \end{pmatrix}^{\frac{1}{2}}}\right]
\end{bmatrix}
\tag{51}
$$

3. CONCLUSION

The direct geometric modeling of the articulated 6R robot are required the constructive parameters and the generalized coordinates.

Given the rotation matrix that expressed the relative orientation of each system in relation to the previous system and relative position vectors, the position and orientation of the robot's gripper relative to the fixed system it can be determined after following some steps.

The direct geometrical modeling defines the position and orientation of the gripper, expressed by the coordinates of the characteristic point and relative to this point the orientation of the gripper.

4. REFERENCES

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Modelarea geometrică a robotului articulat 6R

Rezumat: Autorii prezintă în această lucrare modelul geometric direct pentru robotul TRTTRR1.
În primul rând este necesar stabilirea ecuațiilor modelului geometric este necesar determinarea matricei de rotație. În al doilea rând este necesar determinarea parametrilor independenți ai orientării, vectorii de poziție şi, în final, urmărind rezultatele obținute rezultă ecuațiile modelului geometric direct.

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