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THE DIRECT KINEMATIC MODEL OF THE SERIAL MODULAR ROBOT TRTTRR1

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Abstract: *This paper present the direct kinematic model of the serial modular robot TRTTRR1 intended to be implemented in a flexible manufacturing cell. By this model the disadvantages of geometric modeling are eliminated due to non-linearity of the geometrical equations and the lack of control over velocity and acceleration on the motion trajectory.*
Key words: *kinematic modeling, geometrical equations, generalized velocities, generalized accelerations, parameter.*

1. INTRODUCTION

The static assumption is removed in the kinematic modeling. Thus the generalized and operational coordinates of the robot are functions of time.

The kinematic modeling involves solving the two fundamental mutual problems of the robots kinematics: direct and inverse.

In the case of kinematic modeling the generalized coordinates, velocities and accelerations are known and are determined the operational velocities and accelerations which defines together with $\overline{X}^{(n)0}$ the gripper movement relative to the fixed system (T_0).

For solving the problem of the direct kinematic modeling, is used the iterative method. This method is one of the frequently used in the kinematic modeling, according to [1] and [2]. It is based on the introduction in calculus of the position vectors, of the rotation matrices and their derivatives with respect to time.

2. THE DIRECT KINEMATIC MODEL OF THE TRTTRR1 ROBOT

The robot TRTTRR1 shown in figure 1 was geometric modelated in [3]. The dates obtained from the geometric modelling can be used to the direct kinematic modelling. The homogeneous transformation matrices can be determined using the rotation matrices and position vectors.

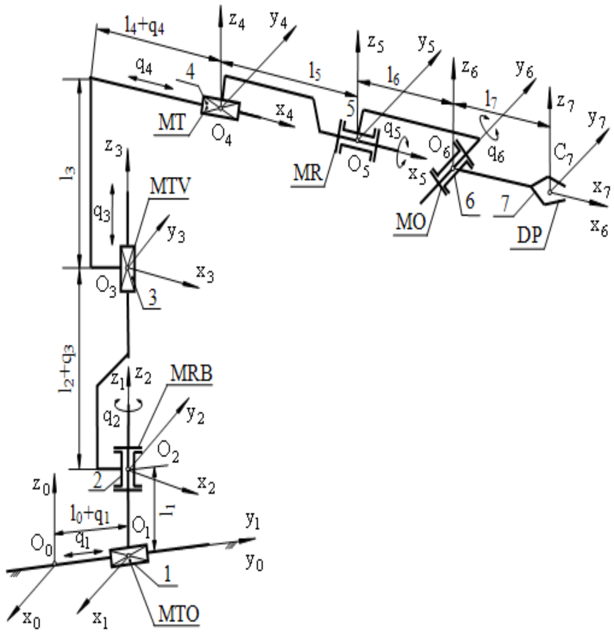

Fig. 1. The kinematic scheme of the industrial serial modular TRTTRR1 robot

Thus, the matrix expression of these matrices is given below:

$$[T]_1^0(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&l_0+q_1\\0&0&1&0\\\hline 0&0&0&1\end{array}\right];\ [T]_2^1(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&0\\sq_2&cq_2&0&0\\0&0&1&l_1\\\hline 0&0&0&1\end{array}\right];\tag{1}$$

$$[T]_3^2(t)=\left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&l_2+q_3\\\hline 0&0&0&1\end{array}\right];\ [T]_4^3(t)=\left[\begin{array}{ccc|c}1&0&0&l_4+q_4\\0&1&0&0\\0&0&1&l_3\\\hline 0&0&0&1\end{array}\right];\tag{2}$$

$$[T]_5^4(t)=\left[\begin{array}{ccc|c}1&0&0&l_5\\0&cq_5&-sq_5&0\\0&sq_5&cq_5&0\\\hline 0&0&0&1\end{array}\right];\tag{3}$$

$$[T]_6^5(t)=\left[\begin{array}{ccc|c}cq_6&0&sq_6&l_6\\0&1&0&0\\-sq_6&0&cq_6&0\\\hline 0&0&0&1\end{array}\right];\ [T]_7^6(t)=\left[\begin{array}{ccc|c}1&0&0&l_7\\0&1&0&0\\0&0&1&0\\\hline 0&0&0&1\end{array}\right];\tag{4}$$

respectively,

$$[T]_2^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&0\\sq_2&cq_2&0&l_0+q_1\\0&0&1&l_1\\\hline 0&0&0&1\end{array}\right];\tag{5}$$

$$[T]_3^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&1\\sq_2&cq_2&0&l_0+q_1\\0&0&1&l_1+l_2+q_3\\\hline 0&0&0&1\end{array}\right];\tag{6}$$

$$[T]_4^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2&0&(l_4+q_4)cq_2\\sq_2&cq_2&0&l_0+q_1+(l_4+q_4)sq_2\\0&0&1&l_1+l_2+l_3+q_3\\\hline 0&0&0&1\end{array}\right];\tag{7}$$

$$[T]_5^0(t)=\left[\begin{array}{ccc|c}cq_2&-sq_2cq_5&sq_2sq_5&(l_4+l_5+q_4)cq_2\\sq_2&cq_2cq_5&-cq_2sq_5&l_0+q_1+(l_4+l_5+q_4)sq_2\\0&sq_5&cq_5&l_1+l_2+l_3+q_3\\\hline 0&0&0&1\end{array}\right];\tag{8}$$

$$[T]_6^0(t)=\left[\begin{array}{ccc|c}cq_2cq_6-sq_2sq_5sq_6&-sq_2cq_5&cq_2sq_6+sq_2sq_5cq_6&(l_4+l_5+l_6+q_4)cq_2\\\hline sq_2cq_6+cq_2sq_5sq_6&cq_2cq_5&sq_2sq_6-cq_2sq_5cq_6&l_0+q_1+(l_4+l_5+l_6+q_4)sq_2\\\hline -cq_5sq_6&sq_5&cq_5cq_6&l_1+l_2+l_3+q_3\\\hline 0&0&0&1\end{array}\right];\tag{9}$$

$$[T]_7^0(t)=\left[\begin{array}{ccc|c}cq_2cq_6-sq_2sq_5sq_6&-sq_2cq_5&cq_2sq_6+sq_2sq_5cq_6&(l_4+l_5+l_6+q_4)cq_2+(cq_2cq_6-sq_2sq_5sq_6)l_7\\\hline sq_2cq_6+cq_2sq_5sq_6&cq_2cq_5&sq_2sq_6-cq_2sq_5cq_6&l_0+q_1+(l_4+l_5+l_6+q_4)sq_2+(sq_2cq_6+cq_2sq_5sq_6)l_7\\\hline -cq_5sq_6&sq_5&cq_5cq_6&l_1+l_2+l_3+q_3-cq_5sq_6l_7\\\hline 0&0&0&1\end{array}\right].\tag{10}$$

The inverse rotation matrices are calculated with the following relation below:

$$[R]_{i-1}^{i}=\left[R_i^{i-1}\right]^{-1}=\left[R_i^{i-1}\right]^{T}.\tag{11}$$

Thus, it resulted:

$$[R]_0^1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\ [R]_1^2=\begin{bmatrix}cq_2&sq_2&0\\-sq_2&cq_2&0\\0&0&1\end{bmatrix};\tag{12}$$

$$[R]_2^3=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\ [R]_3^4=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix};\tag{13}$$

$$[R]_4^5=\begin{bmatrix}1&0&0\\0&cq_5&sq_5\\0&-sq_5&cq_5\end{bmatrix};\ [R]_5^6=\begin{bmatrix}cq_6&0&-sq_6\\0&1&0\\sq_6&0&cq_6\end{bmatrix};\tag{14}$$

$$[R]_6^7=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}.\tag{15}$$

The matrix expression of the kinematic axes vectors, respectively of the O_7x_7, is as follows:

$$[\bar{j}]_1^1=\begin{bmatrix}0\\1\\0\end{bmatrix};\ [\bar{k}]_2^2=\begin{bmatrix}0\\0\\1\end{bmatrix};\ [\bar{k}]_3^3=\begin{bmatrix}0\\0\\1\end{bmatrix};\ [\bar{i}]_4^4=\begin{bmatrix}1\\0\\0\end{bmatrix};$$
$$[\bar{i}]_5^5=\begin{bmatrix}1\\0\\0\end{bmatrix};\ [\bar{j}]_6^6=\begin{bmatrix}0\\1\\0\end{bmatrix};\ [\bar{i}]_7^7=\begin{bmatrix}1\\0\\0\end{bmatrix}.\tag{16}$$

The kinematic parameters expressed matrix, corresponding to the robot base are:

$$[\bar{\omega}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\quad [\bar{v}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\quad [\bar{\varepsilon}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\quad [\bar{a}]_0^0=\begin{bmatrix}0\\0\\g\end{bmatrix}.\tag{17}$$

With the use of the inverse matrices and the kinematic parameters the operational angular velocities matrix transposed, of the elements i, i=1÷7. Thus:

$$[\bar{\omega}]_1^1=[R]_0^1\cdot[\bar{\omega}]_0^0=\begin{bmatrix}0\\0\\0\end{bmatrix};\tag{18}$$

$$[\bar{\omega}]_2^2=[R]_1^2\cdot[\bar{\omega}]_1^1+\dot{q}_2\cdot[\bar{k}]_2^2=\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix};\tag{19}$$

$$[\bar{\omega}]_3^3=[R]_2^3\cdot[\bar{\omega}]_2^2=\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix};\ [\bar{\omega}]_4^4=[R]_3^4\cdot[\bar{\omega}]_3^3=\begin{bmatrix}0\\0\\\dot{q}_2\end{bmatrix};\tag{20}$$

$$[\bar{\omega}]_5^5=[R]_4^5\cdot[\bar{\omega}]_4^4+\dot{q}_5\cdot[\bar{i}]_5^5=\begin{bmatrix}\dot{q}_5\\\dot{q}_2sq_5\\\dot{q}_2cq_5\end{bmatrix};\tag{21}$$

$$[\overline{\omega}]_6^6 = [R]_5^6 \cdot [\overline{\omega}]_5^5 + \dot{q}_6 \cdot [\overline{j}]_6^6 = \begin{bmatrix} \dot{q}_5 cq_6 - \dot{q}_2 cq_5 sq_6 \\ \dot{q}_2 sq_5 + \dot{q}_6 \\ \dot{q}_5 sq_6 + \dot{q}_2 cq_5 cq_6 \end{bmatrix}; \quad (22)$$

$$[\overline{\omega}]_7^7 = [R]_6^7 \cdot [\overline{\omega}]_6^6 = \begin{bmatrix} \dot{q}_5 cq_6 - \dot{q}_2 cq_5 sq_6 \\ \dot{q}_2 sq_5 + \dot{q}_6 \\ \dot{q}_5 sq_6 + \dot{q}_2 cq_5 cq_6 \end{bmatrix}. \quad (23)$$

The antisymetric matrix 3x3, type $\{\overline{\omega}\times\}$, in which $\overline{\omega}$ is the vector angular velocity, can be expressed by relation below:

$$\{\overline{\omega}\times\} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}. \quad (24)$$

In the following matrix relations, the vector products and double vector that appear are expressed according to the relation above.

The expressions of the operational linear velocity transposed matrix, are given by relations:

$$[\overline{v}]_1^1 = [R]_0^1 \cdot \{\overline{v}_0^0 + \overline{\omega}_0^0 \times \overline{r}_1^0\} + \dot{q}_1 \cdot [\overline{j}]_1^1; [\overline{v}]_1^1 = \begin{bmatrix} 0 \\ \dot{q}_1 \\ 0 \end{bmatrix}; \quad (25)$$

$$[\overline{v}]_2^2 = [R]_1^2 \cdot \{\overline{v}_1^1 + \overline{\omega}_1^1 \times \overline{r}_2^1\}; [\overline{v}]_2^2 = \begin{bmatrix} \dot{q}_1 sq_2 \\ \dot{q}_1 cq_2 \\ 0 \end{bmatrix}; \quad (26)$$

$$[\overline{v}]_3^3 = [R]_2^3 \cdot \{\overline{v}_2^2 + \overline{\omega}_2^2 \times \overline{r}_3^2\} + \dot{q}_3 \cdot [\overline{k}]_3^3; [\overline{v}]_3^3 = \begin{bmatrix} \dot{q}_1 sq_2 \\ \dot{q}_1 cq_2 \\ \dot{q}_3 \end{bmatrix}; \quad (27)$$

$$[\overline{v}]_4^4 = [R]_3^4 \cdot \{\overline{v}_3^3 + \overline{\omega}_3^3 \times \overline{r}_4^3\} + \dot{q}_4 \cdot [\overline{i}]_4^4;$$

$$[\overline{v}]_4^4 = \begin{bmatrix} \dot{q}_1 sq_2 + \dot{q}_4 \\ \dot{q}_1 cq_2 + (l_4 + q_4)\dot{q}_2 \\ \dot{q}_3 \end{bmatrix}; \quad (28)$$

$$[\overline{v}]_5^5 = [R]_4^5 \cdot \{\overline{v}_4^4 + \overline{\omega}_4^4 \times \overline{r}_5^4\};$$

$$[\overline{v}]_5^5 = \begin{bmatrix} \dot{q}_1 sq_2 + \dot{q}_4 \\ cq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \dot{q}_3 sq_5 \\ -sq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \dot{q}_3 cq_5 \end{bmatrix}; \quad (29)$$

$$[\overline{v}]_6^6 = [R]_5^6 \cdot \{\overline{v}_5^5 + \overline{\omega}_5^5 \times \overline{r}_6^5\};$$

$$[\overline{v}]_6^6 = \begin{bmatrix} cq_6(\dot{q}_1 sq_2 + \dot{q}_4) - \\ -sq_6\{-sq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \\ +\dot{q}_3 cq_5 - \dot{q}_2 l_6 sq_5\} \\ \hline cq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \dot{q}_3 sq_5 + \dot{q}_2 l_6 cq_5 \\ \hline sq_6(\dot{q}_1 sq_2 + \dot{q}_4) + \\ +cq_6\{-sq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \\ +\dot{q}_3 cq_5 - \dot{q}_2 l_6 sq_5\} \end{bmatrix}; \quad (30)$$

$$[\overline{v}]_7^7 = [R]_6^7 \cdot \{\overline{v}_6^6 + \overline{\omega}_6^6 \times \overline{r}_7^6\}; \quad (31)$$

$$[\overline{v}]_7^7 = \begin{bmatrix} cq_6(\dot{q}_1 sq_2 + \dot{q}_4) - \\ -sq_6\{-sq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \\ +\dot{q}_3 cq_5 - \dot{q}_2 l_6 sq_5\} \\ \hline cq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \dot{q}_3 sq_5 + \\ +\dot{q}_2 l_6 cq_5 + (\dot{q}_5 sq_6 + \dot{q}_2 cq_5 cq_6) l_7 \\ \hline sq_6(\dot{q}_1 sq_2 + \dot{q}_4) + \\ +cq_6\{-sq_5[\dot{q}_1 cq_2 + \dot{q}_2(l_4 + l_5 + q_4)] + \\ +\dot{q}_3 cq_5 - \dot{q}_2 l_6 sq_5\} + (-\dot{q}_2 sq_5 - \dot{q}_6) l_7 \end{bmatrix}; \quad (32)$$

The angular operational accelerations can be matrix expressed by the following relations:

$$[\overline{\varepsilon}]_1^1 = [R]_0^1 \cdot [\overline{\varepsilon}]_0^0; \quad [\overline{\varepsilon}]_1^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad (33)$$

$$[\overline{\varepsilon}]_2^2 = [R]_1^2 \cdot [\overline{\varepsilon}]_1^1 + \{[R]_1^2 \cdot \overline{\omega}_1^1 \times \dot{q}_2 \cdot \overline{k}_2^2 + \ddot{q}_2 \cdot \overline{k}_2^2\};$$

$$[\overline{\varepsilon}]_2^2 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix}; \quad (34)$$

$$[\overline{\varepsilon}]_3^3 = [R]_2^3 \cdot [\overline{\varepsilon}]_2^2; \quad [\overline{\varepsilon}]_3^3 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix}; \quad (35)$$

$$[\overline{\varepsilon}]_4^4 = [R]_3^4 \cdot [\overline{\varepsilon}]_3^3; \quad [\overline{\varepsilon}]_4^4 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_2 \end{bmatrix}; \quad (36)$$

$$[\overline{\varepsilon}]_5^5 = [R]_4^5 \cdot [\overline{\varepsilon}]_4^4 + \{[R]_4^5 \cdot \overline{\omega}_4^4 \times \dot{q}_5 \cdot \overline{i}_5^5 + \ddot{q}_5 \cdot \overline{i}_5^5\};$$

$$[\overline{\varepsilon}]_5^5 = \begin{bmatrix} \ddot{q}_5 \\ \ddot{q}_2 sq_5 + \dot{q}_2 \dot{q}_5 cq_5 \\ \ddot{q}_2 cq_5 - \dot{q}_2 \dot{q}_5 sq_5 \end{bmatrix}; \quad (37)$$

$$[\overline{\varepsilon}]_6^6 = [R]_5^6 \cdot [\overline{\varepsilon}]_5^5 + \{[R]_5^6 \cdot \overline{\omega}_5^5 \times \dot{q}_6 \cdot \overline{j}_6^6 + \ddot{q}_6 \cdot \overline{j}_6^6\};$$

$$[\overline{\varepsilon}]_6^6 = \begin{bmatrix} \ddot{q}_5 cq_6 - sq_6(\ddot{q}_2 cq_5 - \dot{q}_2 \dot{q}_5 sq_5) + \\ +\dot{q}_6(-\dot{q}_2 cq_5 cq_6 - \dot{q}_5 sq_6) \\ \hline \ddot{q}_2 sq_5 + \dot{q}_2 \dot{q}_5 cq_5 + \ddot{q}_6 \\ \hline \ddot{q}_5 sq_6 + cq_6(\ddot{q}_2 cq_5 - \dot{q}_2 \dot{q}_5 sq_5) + \\ +\dot{q}_6(-\dot{q}_2 cq_5 sq_6 + \dot{q}_5 cq_6) \end{bmatrix}; \quad (38)$$

$$[\overline{\varepsilon}]_7^7 = [R]_6^7 \cdot [\overline{\varepsilon}]_6^6;$$

$$[\overline{\varepsilon}]_7^7 = \begin{bmatrix} \ddot{q}_5 cq_6 - sq_6(\ddot{q}_2 cq_5 - \dot{q}_2 \dot{q}_5 sq_5) + \\ +\dot{q}_6(-\dot{q}_2 cq_5 cq_6 - \dot{q}_5 sq_6) \\ \hline \ddot{q}_2 sq_5 + \dot{q}_2 \dot{q}_5 cq_5 + \ddot{q}_6 \\ \hline \ddot{q}_5 sq_6 + cq_6(\ddot{q}_2 cq_5 - \dot{q}_2 \dot{q}_5 sq_5) + \\ +\dot{q}_6(-\dot{q}_2 cq_5 sq_6 + \dot{q}_5 cq_6) \end{bmatrix}. \quad (39)$$

The linear operational accelerations matrix expressed can be determined, as follows:

$$[\overline{a}]_1^1 = [R]_0^1 \cdot \{\overline{a}_0^0 + \overline{\varepsilon}_0^0 \times \overline{r}_1^0 + \overline{\omega}_0^0 \times (\overline{\omega}_0^0 \times \overline{r}_1^0)\} + \{2\overline{\omega}_1^1 \times \dot{q}_1 \cdot \overline{j}_1^1 + \ddot{q}_1 \cdot \overline{j}_1^1\}; \quad (40)$$

$$[\overline{a}]_1^1 = \begin{bmatrix} 0 \\ \ddot{q}_1 \\ g \end{bmatrix}; \quad (41)$$

$$[\bar a]_2^2=[R]_1^2\cdot\left\{\bar a_1^1+\bar\varepsilon_1^1\times\bar r_2^1+\bar\omega_1^1\times\left(\bar\omega_1^1\times\bar r_2^1\right)\right\};\ [\bar a]_2^2=\begin{bmatrix}\ddot q_1 sq_2\\ \ddot q_1 cq_2\\ g\end{bmatrix};\tag{42}$$

$$[\bar a]_3^3=[R]_2^3\cdot\left\{\bar a_2^2+\bar\varepsilon_2^2\times\bar r_3^2+\bar\omega_2^2\times\left(\bar\omega_2^2\times\bar r_3^2\right)\right\}+ \\ +\left\{2\bar\omega_3^3\times\dot q_3\cdot\bar k_3^3+\ddot q_3\cdot\bar k_3^3\right\};\tag{43}$$

$$[\bar a]_3^3=\begin{bmatrix}\ddot q_1 sq_2\\ \ddot q_1 cq_2\\ \ddot q_3+g\end{bmatrix};$$

$$[\bar a]_4^4=[R]_3^4\cdot\left\{\bar a_3^3+\bar\varepsilon_3^3\times\bar r_4^3+\bar\omega_3^3\times\left(\bar\omega_3^3\times\bar r_4^3\right)\right\}+ \\ +\left\{2\bar\omega_4^4\times\dot q_4\cdot\bar i_4^4+\ddot q_4\cdot\bar i_4^4\right\};\tag{44}$$

$$[\bar a]_4^4=\begin{bmatrix}\ddot q_1 sq_2+\ddot q_4-\dot q_2^2\left(l_4+q_4\right)\\ \ddot q_1 cq_2+\ddot q_2\left(l_4+q_4\right)+2\dot q_2\dot q_4\\ \ddot q_3+g\end{bmatrix};$$

$$[\bar a]_5^5=[R]_4^5\cdot\left\{\bar a_4^4+\bar\varepsilon_4^4\times\bar r_5^4+\bar\omega_4^4\times\left(\bar\omega_4^4\times\bar r_5^4\right)\right\};$$

$$[\bar a]_5^5=\begin{bmatrix}\ddot q_1 sq_2-\dot q_2^2\left(l_4+l_5+q_4\right)+\ddot q_4\\ \hline cq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+ \\ +sq_5\left(\ddot q_3+g\right)\\ \hline -sq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\\ +cq_5\left(\ddot q_3+g\right)\end{bmatrix};\tag{45}$$

$$[\bar a]_6^6=[R]_5^6\cdot\left\{\bar a_5^5+\bar\varepsilon_5^5\times\bar r_6^5+\bar\omega_5^5\times\left(\bar\omega_5^5\times\bar r_6^5\right)\right\};$$

$$[\bar a]_6^6=\begin{bmatrix}cq_6\left[\ddot q_1 sq_2-\dot q_2^2\left(l_4+l_5+q_4\right)+\ddot q_4+\right.\\ \left.+\left(-c^2q_5\dot q_2^2-s^2q_5\dot q_2^2\right)l_6\right]-\\ -sq_6\left\{-sq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\right.\\ +cq_5\left(\ddot q_3+g\right)+\left(-\ddot q_2 sq_5-\dot q_2\dot q_5 cq_5\right)l_6+\\ \left.+\dot q_2\dot q_5 l_6 cq_5\right\}\\ \hline cq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\\ +sq_5\left(\ddot q_3+g\right)+\left(\ddot q_2 cq_5-\dot q_2\dot q_5 sq_5\right)l_6+\\ +\dot q_2\dot q_5 l_6 sq_5\\ \hline sq_6\left[\ddot q_1 sq_2-\dot q_2^2\left(l_4+l_5+q_4\right)+\ddot q_4+\right.\\ \left.+\left(c^2q_5\dot q_2^2-s^2q_5\dot q_2^2\right)l_6\right]+\\ +cq_6\left\{-sq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\right.\\ +cq_5\left(\ddot q_3+g\right)+\left(-\ddot q_2 sq_5-\dot q_2\dot q_5 cq_5\right)l_6+\\ \left.+\dot q_2\dot q_5 l_6 cq_5\right\}\end{bmatrix};\tag{46}$$

$$[\bar a]_7^7=[R]_6^7\cdot\left\{\bar a_6^6+\bar\varepsilon_6^6\times\bar r_7^6+\bar\omega_6^6\times\left(\bar\omega_6^6\times\bar r_7^6\right)\right\};\tag{47}$$

$$[\bar a]_7^7=\\ =\begin{bmatrix}cq_6\begin{bmatrix}\ddot q_1 sq_2-\dot q_2^2\left(l_4+l_5+q_4\right)+\ddot q_4+\\ +\left(-c^2q_5\dot q_2^2-s^2q_5\dot q_2^2\right)l_6\end{bmatrix}-\\ -sq_6\left\{-sq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\right.\\ +cq_5\left(\ddot q_3+g\right)+\left(-\ddot q_2 sq_5-\dot q_2\dot q_5 cq_5\right)l_6+\\ \left.+\dot q_2\dot q_5 l_6 cq_5\right\}+\\ +\begin{bmatrix}\left(-\dot q_2 cq_6 cq_5-\dot q_5 sq_6\right)\left(\dot q_5 sq_6-\dot q_2 cq_6 cq_5\right)+\\ +\left(\dot q_2 sq_5+\dot q_6\right)\left(-\dot q_2 sq_5-\dot q_6\right)\end{bmatrix}l_7\\ \hline cq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\\ +sq_5\left(\ddot q_3+g\right)+\left(\ddot q_2 cq_5-\dot q_2\dot q_5 sq_5\right)l_6+\\ +\dot q_2\dot q_5 l_6 sq_5\}+\begin{bmatrix}\ddot q_5 sq_6+cq_6\left(\ddot q_2 cq_5-\dot q_2\dot q_5 sq_5\right)+\\ +\dot q_6\left(-\dot q_2 sq_6 cq_5+\dot q_5 cq_6\right)\end{bmatrix}l_7+\\ +\left[\left(\dot q_2 sq_6 cq_5-\dot q_5 cq_6\right)\cdot\left(-\dot q_2 sq_5-\dot q_6\right)\right]l_7\\ \hline sq_6\begin{bmatrix}\ddot q_1 sq_2-\dot q_2^2\left(l_4+l_5+q_4\right)+\ddot q_4+\\ +\left(c^2q_5\dot q_2^2-s^2q_5\dot q_2^2\right)l_6\end{bmatrix}+\\ +cq_6\left\{-sq_5\left[\ddot q_1 cq_2+\ddot q_2\left(l_4+l_5+q_4\right)+2\dot q_2\dot q_4\right]+\right.\\ +cq_5\left(\ddot q_3+g\right)+\left(-\ddot q_2 sq_5-\dot q_2\dot q_5 cq_5\right)l_6+\\ \left.+\dot q_2\dot q_5 l_6 cq_5\right\}+\left(-\ddot q_2 sq_5-\dot q_2\dot q_5 cq_5-\ddot q_6\right)l_7+\\ +\left(-\dot q_2 sq_6 cq_5+\dot q_5 cq_6\right)\cdot\left(\dot q_5 sq_6+\dot q_2 cq_6 cq_5\right)l_7\end{bmatrix}.\tag{48}$$

The operational kinematic parameters can be expressed as follows:

$$\left[\dot{\bar X}\right]^7=\\ =\begin{bmatrix}cq_6\left(\dot q_1 sq_2+\dot q_4\right)-\\ -sq_6\begin{Bmatrix}-sq_5\left[\dot q_1 cq_2+\dot q_2\left(l_4+l_5+q_4\right)\right]+\\ +\dot q_3 cq_5-\dot q_2 l_6 sq_5\end{Bmatrix}\\ \hline cq_5\left[\dot q_1 cq_2+\dot q_2\left(l_4+l_5+q_4\right)\right]+\\ +\dot q_3 sq_5+\dot q_2 l_6 cq_5+\\ +\left(\dot q_5 sq_6+\dot q_2 cq_5 cq_6\right)l_7\\ \hline sq_6\left(\dot q_1 sq_2+\dot q_4\right)+\\ +cq_6\begin{Bmatrix}-sq_5\left[\dot q_1 cq_2+\dot q_2\left(l_4+l_5+q_4\right)\right]+\\ +\dot q_3 cq_5-\dot q_2 l_6 sq_5\end{Bmatrix}+\\ +\left(-\dot q_2 sq_5-\dot q_6\right)l_7\\ \hline \dot q_5 cq_6-\dot q_2 cq_5 sq_6\\ \dot q_2 sq_5+\dot q_6\\ \dot q_5 sq_6+\dot q_2 cq_5 cq_6\end{bmatrix};\tag{49}$$

$$\left[\ddot{\bar{X}}\right]^{7}=\begin{bmatrix} cq_6\left[\begin{matrix}\ddot{q}_1 sq_2-\dot{q}_2^2\left(l_4+l_5+q_4\right)+\\ +\ddot{q}_4+\left(-c^2q_5\dot{q}_2^2-s^2q_5\dot{q}_2^2\right)l_6\end{matrix}\right]- \\ -sq_6\left\{-sq_5\left[\ddot{q}_1 cq_2+\ddot{q}_2\left(l_4+l_5+q_4\right)+2\dot{q}_2\dot{q}_4\right]+\right. \\ +cq_5\left(\ddot{q}_3+g\right)+\left(-\ddot{q}_2 sq_5-\dot{q}_2\dot{q}_5 cq_5\right)l_6+ \\ \left.+\dot{q}_2\dot{q}_5 l_6 cq_5\right\}+ \\ +\left[\begin{matrix}\left(-\dot{q}_2 cq_6 cq_5-\dot{q}_5 sq_6\right)\left(\dot{q}_5 sq_6-\dot{q}_2 cq_6 cq_5\right)+\\ +\left(\dot{q}_2 sq_5+\dot{q}_6\right)\left(-\dot{q}_2 sq_5-\dot{q}_6\right)\end{matrix}\right]l_7 \\ \hdashline cq_5\left[\ddot{q}_1 cq_2+\ddot{q}_2\left(l_4+l_5+q_4\right)+2\dot{q}_2\dot{q}_4\right]+ \\ +sq_5\left(\ddot{q}_3+g\right)+\left(\ddot{q}_2 cq_5-\dot{q}_2\dot{q}_5 sq_5\right)l_6+ \\ +\dot{q}_2\dot{q}_5 l_6 sq_5\}+\left[\begin{matrix}\ddot{q}_5 sq_6+cq_6\left(\ddot{q}_2 cq_5-\dot{q}_2\dot{q}_5 sq_5\right)+\\ +\dot{q}_6\left(-\dot{q}_2 sq_6 cq_5+\dot{q}_5 cq_6\right)\end{matrix}\right]l_7+ \\ +\left[\left(\dot{q}_2 sq_6 cq_5-\dot{q}_5 cq_6\right)\cdot\left(-\dot{q}_2 sq_5-\dot{q}_6\right)\right]l_7 \\ \hdashline sq_6\left[\begin{matrix}\ddot{q}_1 sq_2-\dot{q}_2^2\left(l_4+l_5+q_4\right)+\\ +\ddot{q}_4+\left(c^2q_5\dot{q}_2^2-s^2q_5\dot{q}_2^2\right)l_6\end{matrix}\right]+ \\ +cq_6\left\{-sq_5\left[\ddot{q}_1 cq_2+\ddot{q}_2\left(l_4+l_5+q_4\right)+2\dot{q}_2\dot{q}_4\right]+\right. \\ +cq_5\left(\ddot{q}_3+g\right)+\left(-\ddot{q}_2 sq_5-\dot{q}_2\dot{q}_5 cq_5\right)l_6+ \\ \left.+\dot{q}_2\dot{q}_5 l_6 cq_5\right\}+\left(-\ddot{q}_2 sq_5-\dot{q}_2\dot{q}_5 cq_5-\ddot{q}_6\right)l_7+ \\ +\left(-\dot{q}_2 sq_6 cq_5+\dot{q}_5 cq_6\right)\cdot\left(\dot{q}_5 sq_6+\dot{q}_2 cq_6 cq_5\right)l_7 \\ \hline \ddot{q}_5 cq_6-sq_6\left(\ddot{q}_2 cq_5-\dot{q}_2\dot{q}_5 sq_5\right)+ \\ +\dot{q}_6\left(-\dot{q}_2 cq_5 cq_6-\dot{q}_5 sq_6\right) \\ \hdashline \ddot{q}_2 sq_5+\dot{q}_2\dot{q}_5 cq_5+\ddot{q}_6 \\ \hdashline \ddot{q}_5 sq_6+cq_6\left(\ddot{q}_2 cq_5-\dot{q}_2\dot{q}_5 sq_5\right)+ \\ +\dot{q}_6\left(-\dot{q}_2 cq_5 sq_6+\dot{q}_5 cq_6\right)\end{bmatrix}. \qquad (50)$$

Using the transformation relations, the kinematic operational parameters from the fixed system from the robot's base, can be determined, according to [4]. The matrix expression of these parameters is as follows:

$$[\bar{v}]_7^0=[R]_7^0\cdot[\bar{v}]_7^7,$$

$$[\bar{v}]_7^0=\begin{bmatrix}-\dot{q}_2 l_5 sq_2-\dot{q}_2 l_7 sq_2 cq_6-\dot{q}_2 q_4 sq_2- \\ -\dot{q}_2 l_4 sq_2-\dot{q}_2 l_6 sq_2-\dot{q}_6 l_7 cq_2 sq_6+ \\ +\dot{q}_4 cq_2-\dot{q}_5 l_7 sq_2 cq_5 sq_6- \\ -\dot{q}_2 l_7 cq_2 sq_5 sq_6-\dot{q}_6 l_7 sq_2 sq_5 cq_6 \\ \dot{q}_1+\dot{q}_5 l_7 cq_2 cq_5 sq_6-\dot{q}_6 l_7 cq_2 sq_5 cq_6- \\ -\dot{q}_2 l_7 sq_2 sq_5 sq_6+\dot{q}_4 sq_2-\dot{q}_6 l_7 sq_2 sq_6+ \\ +\dot{q}_2 q_4 cq_2+\dot{q}_2 l_4 cq_2+\dot{q}_2 l_5 cq_2+ \\ +\dot{q}_2 l_6 cq_2+\dot{q}_2 l_7 cq_2 cq_6 \\ \dot{q}_3+\dot{q}_5 l_7 sq_5 sq_6-\dot{q}_6 l_7 cq_5 cq_6\end{bmatrix}; \qquad (51)$$

$$[\bar{\omega}]_7^0=[R]_7^0\cdot[\bar{\omega}]_7^7,\ [\bar{\omega}]_7^0=\begin{bmatrix}-\dot{q}_6 sq_2 cq_5+\dot{q}_5 cq_2 \\ \dot{q}_6 cq_2 cq_5+\dot{q}_5 sq_2 \\ \dot{q}_2+\dot{q}_6 sq_5\end{bmatrix}; \qquad (52)$$

$$[\bar{a}]_7^0=[R]_7^0\cdot[\bar{a}]_7^7,$$

$$[\bar{a}]_7^0=\begin{bmatrix}-\ddot{q}_2 l_7 cq_2 sq_5 sq_6-2\dot{q}_2\dot{q}_5 l_7 cq_2 cq_5 sq_6+\ddot{q}_2 q_4 sq_2- \\ -2\dot{q}_2\dot{q}_6 l_7 sq_2 sq_6-\ddot{q}_2 l_4 sq_2-2\dot{q}_2\dot{q}_4 sq_2- \\ -\dot{q}_2^2 q_4 cq_2-\dot{q}_2^2 l_4 cq_2-\dot{q}_2^2 l_5 cq_2- \\ -\ddot{q}_6 l_7 sq_2 sq_5 cq_6-\ddot{q}_5 l_7 sq_2 sq_6 cq_5- \\ -2\dot{q}_5\dot{q}_6 l_7 sq_2 cq_5 cq_6-\ddot{q}_6 l_7 cq_2 sq_6- \\ -\dot{q}_2^2 l_7 cq_2 cq_6-\dot{q}_2^2 l_6 cq_2+\dot{q}_5^2 l_7 sq_2 sq_5 sq_6- \\ -\ddot{q}_2 l_5 sq_2-\ddot{q}_2 l_7 sq_2 cq_6-\ddot{q}_2 l_7 sq_2+ \\ +\dot{q}_2^2 l_7 sq_2 sq_5 sq_6+\dot{q}_6^2 l_7 sq_2 sq_5 sq_6+\ddot{q}_4 cq_2- \\ -2\dot{q}_2\dot{q}_5 l_7 cq_2 cq_6 sq_5-\dot{q}_6^2 l_7 cq_2 cq_6 \\ \hdashline -\ddot{q}_6 l_7 sq_2 sq_6-\dot{q}_2^2 l_4 sq_2-\dot{q}_2^2 l_5 sq_2+\ddot{q}_2 q_4 cq_2+ \\ +\ddot{q}_2 l_5 cq_2+2\dot{q}_2\dot{q}_4 cq_2-\dot{q}_2^2 l_6 sq_2+\ddot{q}_2 l_6 cq_2- \\ -\dot{q}_2^2 q_4 sq_2+2\dot{q}_5\dot{q}_6 l_7 cq_2 cq_5 cq_6- \\ -2\dot{q}_2\dot{q}_6 l_7 cq_6 sq_2 sq_5-2\dot{q}_2\dot{q}_5 l_7 sq_2 cq_5 sq_6- \\ -\ddot{q}_1-\dot{q}_6^2 l_7 cq_2 sq_5 sq_6+\ddot{q}_5 l_7 cq_2 sq_6 cq_5+ \\ +\ddot{q}_2 l_7 cq_2 cq_6-\dot{q}_2^2 l_7 sq_2 cq_6+\ddot{q}_4 sq_2- \\ -\dot{q}_5^2 l_7 cq_2 sq_5 sq_6-\dot{q}_2^2 l_7 cq_2 sq_6 sq_5+ \\ +\ddot{q}_6 l_7 cq_2 cq_6 sq_5-2\dot{q}_2\dot{q}_6 l_7 cq_2 sq_6- \\ -\ddot{q}_2 l_7 sq_2 sq_5 sq_6-\dot{q}_6^2 l_7 sq_2 cq_6 \\ \hdashline \ddot{q}_3+g+\dot{q}_6^2 l_7 cq_5 sq_6+\ddot{q}_5 l_7 sq_5 sq_6+ \\ +2\dot{q}_2\dot{q}_6 l_7 sq_5 cq_6-\ddot{q}_6 l_7 cq_5 cq_6+ \\ +\dot{q}_5^2 l_7 cq_5 sq_6\end{bmatrix}; \qquad (53)$$

$$[\bar{\varepsilon}]_7^0=[R]_7^0\cdot[\bar{\varepsilon}]_7^7, \qquad (54)$$

$$[\bar{\varepsilon}]_7^0=\begin{bmatrix}-\dot{q}_2\dot{q}_5 sq_2+\dot{q}_5\dot{q}_6 sq_2 sq_5-\ddot{q}_6 sq_2 cq_5+\ddot{q}_5 cq_2- \\ -\dot{q}_2\dot{q}_6 cq_2 cq_5 \\ \dot{q}_2\dot{q}_5 cq_2-\dot{q}_5\dot{q}_6 cq_2 sq_5+\ddot{q}_6 cq_2 cq_5+\ddot{q}_5 sq_2- \\ -\dot{q}_2\dot{q}_6 sq_2 cq_5 \\ \dot{q}_5\dot{q}_6 cq_5+\ddot{q}_2+\ddot{q}_6 sq_5\end{bmatrix}. \qquad (55)$$

Using the relations from (51)-(55), the velocity and acceleration of the fixed system $O_0x_0y_0z_0$ from the robot base, can be determined. Thus:

$$\left[\dot{\bar{X}}\right]^{0}=\begin{bmatrix}-\dot{q}_2 l_5 sq_2-\dot{q}_2 l_7 sq_2 cq_6-\dot{q}_2 q_4 sq_2- \\ -\dot{q}_2 l_4 sq_2-\dot{q}_2 l_6 sq_2-\dot{q}_6 l_7 cq_2 sq_6+ \\ +\dot{q}_4 cq_2-\dot{q}_5 l_7 sq_2 cq_5 sq_6- \\ -\dot{q}_2 l_7 cq_2 sq_5 sq_6-\dot{q}_6 l_7 sq_2 sq_5 cq_6 \\ \hdashline \dot{q}_1+\dot{q}_5 l_7 cq_2 cq_5 sq_6-\dot{q}_6 l_7 cq_2 sq_5 cq_6- \\ -\dot{q}_2 l_7 sq_2 sq_5 sq_6+\dot{q}_4 sq_2- \\ -\dot{q}_6 l_7 sq_2 sq_6+\dot{q}_2 q_4 cq_2+\dot{q}_2 l_4 cq_2+ \\ +\dot{q}_2 l_5 cq_2+\dot{q}_2 l_6 cq_2+\dot{q}_2 l_7 cq_2 cq_6 \\ \hdashline \dot{q}_3+\dot{q}_5 l_7 sq_5 sq_6-\dot{q}_6 l_7 cq_5 cq_6 \\ \hline -\dot{q}_6 sq_2 cq_5+\dot{q}_5 cq_2 \\ \hdashline \dot{q}_6 cq_2 cq_5+\dot{q}_5 sq_2 \\ \hdashline \dot{q}_2+\dot{q}_6 sq_5\end{bmatrix}; \qquad (56)$$

$$\left[\ddot{X}\right]^0 = \left[\begin{array}{c}
-\ddot{q}_2 l_7 cq_2 sq_5 sq_6 - 2\dot{q}_2\dot{q}_5 l_7 cq_2 cq_5 sq_6 + \ddot{q}_2 q_4 sq_2 - \\
-2\dot{q}_2\dot{q}_6 l_7 sq_2 sq_6 - \ddot{q}_2 l_4 sq_2 - 2\dot{q}_2\dot{q}_4 sq_2 - \dot{q}_2^2 q_4 cq_2 - \\
-\dot{q}_2^2 l_4 cq_2 - \dot{q}_2^2 l_5 cq_2 - \ddot{q}_6 l_7 sq_2 sq_5 cq_6 - \ddot{q}_5 l_7 sq_2 sq_6 cq_5 - \\
-2\dot{q}_5\dot{q}_6 l_7 sq_2 cq_5 cq_6 - \ddot{q}_6 l_7 cq_2 sq_6 - \dot{q}_2^2 l_7 cq_2 cq_6 - \\
-\dot{q}_2^2 l_6 cq_2 + \dot{q}_5^2 l_7 sq_2 sq_5 sq_6 - \ddot{q}_2 l_5 sq_2 - \ddot{q}_2 l_7 sq_2 cq_6 - \\
-\ddot{q}_2 l_7 sq_2 + \dot{q}_2^2 l_7 sq_2 sq_5 sq_6 + \dot{q}_6^2 l_7 sq_2 sq_5 sq_6 + \ddot{q}_4 cq_2 - \\
-2\dot{q}_2\dot{q}_5 l_7 cq_2 cq_6 sq_5 - \dot{q}_6^2 l_7 cq_2 cq_6 \\
\hdashline
-\ddot{q}_6 l_7 sq_2 sq_6 - \dot{q}_2^2 l_4 sq_2 - \dot{q}_2^2 l_5 sq_2 + \ddot{q}_2 q_4 cq_2 + \ddot{q}_2 l_5 cq_2 + \\
+2\dot{q}_2\dot{q}_4 cq_2 - \dot{q}_2^2 l_6 sq_2 + \ddot{q}_2 l_6 cq_2 - \dot{q}_2^2 q_4 sq_2 + \\
+2\dot{q}_5\dot{q}_6 l_7 cq_2 cq_5 cq_6 - 2\dot{q}_2\dot{q}_6 l_7 cq_6 sq_2 sq_5 - \\
-2\dot{q}_2\dot{q}_5 l_7 sq_2 cq_5 sq_6 - \ddot{q}_1 - \dot{q}_6^2 l_7 cq_2 sq_5 sq_6 + \\
+\ddot{q}_5 l_7 cq_2 sq_6 cq_5 + \ddot{q}_2 l_7 cq_2 cq_6 - \dot{q}_2^2 l_7 sq_2 cq_6 + \\
+\ddot{q}_4 sq_2 - \dot{q}_5^2 l_7 cq_2 sq_5 sq_6 - \dot{q}_2^2 l_7 cq_2 sq_6 sq_5 + \\
+\ddot{q}_6 l_7 cq_2 cq_6 sq_5 - 2\dot{q}_2\dot{q}_6 l_7 cq_2 sq_6 - \ddot{q}_2 l_7 sq_2 sq_5 sq_6 - \\
-\dot{q}_6^2 l_7 sq_2 cq_6 \\
\hdashline
\ddot{q}_3 + g + \dot{q}_6^2 l_7 cq_5 sq_6 + \ddot{q}_5 l_7 sq_5 sq_6 + 2\dot{q}_2\dot{q}_6 l_7 sq_5 cq_6 - \\
-\ddot{q}_6 l_7 cq_5 cq_6 + \dot{q}_5^2 l_7 cq_5 sq_6 \\
\hline
-\dot{q}_2\dot{q}_5 sq_2 + \dot{q}_5\dot{q}_6 sq_2 sq_5 - \ddot{q}_6 sq_2 cq_5 + \ddot{q}_5 cq_2 - \dot{q}_2\dot{q}_6 cq_2 cq_5 \\
\dot{q}_2\dot{q}_5 cq_2 - \dot{q}_5\dot{q}_6 cq_2 sq_5 + \ddot{q}_6 cq_2 cq_5 + \ddot{q}_5 sq_2 - \dot{q}_2\dot{q}_6 sq_2 cq_5 \\
\dot{q}_5\dot{q}_6 cq_5 + \ddot{q}_2 + \ddot{q}_6 sq_5
\end{array}\right]. \quad (57)$$

The equations (49), (50), (56) and (57) are the direct kinematic equations, that determine the operational kinematic parameters of the gripper in relation with the reference systems (T_7) and (T_0).

3. CONCLUSION

The iterative method used to determine de direct kinematic equations, is consists in doing the robot kinematic chain from a fixed base to the gripper and determination by successive iterations of the following kinematic parameters: $\left\{\overline{k}_i^i, \overline{\omega}_i^i, \overline{\varepsilon}_i^i, \overline{v}_i^i, \overline{a}_i^i, i = 1 \div n\right\}$.

The mentioned kinematic parameters characterized the movement of each element i, i=1÷n, in relation to the fixed system (T_0) from robot's base.

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Modelul cinematic direct al robotului serial modular TRTTRR1

Rezumat: Lucrare de faţă prezintă modelul cinematic direct al robotului TRTTRR1. Prin acest model sunt eliminate dezavantajele modelării geometrice determinate de neliniaritatea ecuațiilor geometrice şi de lipsa controlului asupra vitezei şi accelerației pe traiectoria de mişcare.

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