TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics, Mechanics, and Engineering
Vol. 57, Issue II, June, 2014

MECHANICAL STRESS AND STRAIN PROPERTIES, REGARDING THE ELBOW JOINT

Aurora Felicia CRISTEA

Abstract: This paper is important, because the literature does not treat this topic deeply, and the mechanical properties in elbow joint, considering the thickness of hyaline cartilage of the elbow are neglected, due to its low thickness. The paper proposes a study based on a theoretical platform. Firstly, it approached the study of elbow joint films (cartilage), because a nonlinear study would involve a multitude of internal and external factors that are supposed to keep out, and theoretical research would be particularly cumbersome, if not impossible. In addition, firstly (case1) this study, be it linear, has not been approached by the literature and it is particularly important to know the mechanical behaviour of the joint of the cartilage. It played with the ultimate goal of determining the linear mechanical properties perspective, elongation and resistance to hyaline cartilage from breaking it, regarding the elbow joint. In the case 2 of study, it be analysed the nonlinear evolution, regarding stress and strain properties base on the anterior tissue experiments (not hyaline cartilages), given by specialty literature. Theoretical approach to this problem, from the standpoint of mathematical models developed, they try to observe as accurately model real hand-arm system, it was shown that the mechanical properties (stress and strain) of the articulation the elbow are not neglected.

Key words: cartilage, elbow joint, mechanical characteristics, stress and strain.

1. INTRODUCTION

This paper presents a *theoretical* introduction and a comprehensive study on the determination of mechanical properties (stress and strain) in the human joints (articulation), respective in the joint elbow. The paper proposes a study based on a theoretical platform, but for this purpose it was absolute necessary to use a few experimental notions of the other literature cartilage researches.

1.1. Generality

The literature on the field, it has so far numerical data or graphs of the determination of mechanical properties (stress and strain) on human joints, especially in the elbow joint. *Typically, existing research joint elbow, from the point of view of hyaline cartilage is ignored, because it is considered that it would be too thin, on the order of 2-3 mm.* But, the paper shows that irrespective of the elbow joint and the mechanical properties in it (these being influenced and hyaline cartilage) cannot continue a real study of mechanical modeling of the whole hand-arm, study that then lead to finalize it, solving that match the pattern you created. This is reason the studies that analyzed *two cases: one is the linear behavior* and the other (*case 2*) in the stress and strain *nonlinear behavior* and the point connection between them.

1.2 Elongation and tensile tense

For this reason, for a start, it will define briefly the basics of paper, i.e. elongation tensile-strength (stress and strain) (www.wikipedia.com). The effort (force per unit area), stretching from which the material begins to flow, is called breaking strength (stress).

Stress normal to the plane is usually denoted "norma stress" and can be expressed as:

$$\sigma = F_n / A \qquad (1)$$

where: σ = normal stress [Pa]; F_n = normal component force [N]; A = area [m^2].

Strain is defined as "deformation of a solid due to stress" and can be expressed as:

$$\varepsilon = dl / l_o = \sigma / E \qquad (2)$$

where:

dl = change of length [m]; l_o = initial length [m]; ε = unitless measure of engineering strain; *E* = Young's modulus (Modulus of Elasticity) ([Pa]

Most metals have deformations that are proportional with the imposed loads over a range of loads. *Stress* is proportional to *load* and *strain* is proportional to *deformation,* expressed by the Hooke's law like:

$$E = \text{stress} / \text{strain} = (F_n / A) / (dl / l_o) \qquad \textbf{(3)}$$

where: *E* = Young's modulus [N/m^2]

The relationship between the stress and strain that a material displays is known as *a Stress-Strain curve*. It is unique for each material and, it is found by recording the amount of deformation (strain) at distinct intervals of tensile or compressive loading. These curves reveal many of the properties of a material (including data to establish the Modulus of Elasticity (*E*)) (www.wikipedia.org). Stress-strain diagram of various materials vary widely, and different tensile tests conducted on the same material yield different results, depending upon the temperature of the specimen and the speed of the loading.

1.3 Joint elbow anatomy

In the following, it will define the elbow joint, which is a complex joint that functions as a fulcrum for the forearm lever system that is responsible for positioning the hand in space. It must very important to know very well the inside anatomy of elbow articulation, that's why the following paragraph describes this.

1.3.1 Articular cartilage

Like bone, cartilage come in different form, each suited to a particular application. For the sake of simplicity, it defines three main types of cartilages [1, 2]:

- *"Elastic cartilage; Fibrocartilage; Hyaline cartilage* is the most prevalent type of cartilage in the adult. It is found in the ventral ends of ribs, in the tracheal rings, and covering the joint surfaces of bones (like in elbow joints), where it is known as *articular cartilage*. In the following paragraph, it will describe the cartilage.

Articular cartilage [2] is most often hyaline, although some joints contain fibrocartilaginous disks (ex. menisci of knee joints). The extracellular matrix of articular cartilage is composed mostly of *type II collagen* and *proteoglycans* manufactured by cartilage cell (called *chondrocytes*), and then assembled outside the cell into a mesh of collagen interwoven with aggregated proteoglycan molecules called *aggrecan*. The mechanism for intercellular signal transduction, and how chondrocytes coordinate their activities, if they do, is unknown [2]. Cartilage matrix is ordinal not mineralized and is about 70% (by mass) water. Approximately 40-60% of the dry weight of the matrix is collagen and 15%-40% is proteoglycans. *Proteoglycans* in aggregate form are the other primary polymer in the matrix of cartilage. In fact, articular cartilage has unique chemical properties allowing it to serve as a bearing surface. It is able to transfer loads from one bone to another while simultaneously allowing the load – bearing surfaces to articulate with very low friction [2].

In fact, the large size and negative charge of the aggrecan molecule give cartilage its hydrophilic properties. Thus, compressive loads are borne by a volume of water that cannot escape [2]. This all arrangement works well mechanically. Large tensile stresses within the articular surfaces and at the edges of joint contact areas are resisted primarily by tangentially oriented collagen fibers. The compressive and hydrostatic stresses found in the deeper layers of cartilage are resisted by the incompressibility of water, which is held in place by the hydrophilic aggrecan molecules [2]. In conclusion, it could say that is evidence that chondrocytes also respond to mechanical signals, and that this capacity is instrumental in the development of cartilaginous structures. Finally, the specialty literature say that: "the cartilage does not serve to "cushion" or reduce impulsive forces in joints. It cannot do this because it is so thin (~50 Å) that its capacity to absorb energy is insignificant compared to eccentric contractions of muscles and energy absorption in the bones on either side of joint. Instead, the role of cartilage is to provide a self-renewing, well-lubricated, load-bearing surface". But, these aspects are completed of the next paragraph."

2. BIOMECHANICAL PROPERTIES

1. "*Structural properties* refer to the mechanical properties of an object as a whole and so will depends not only on the material of which the object is composed, but also its shape and size [3].

2. *Material properties*, in contrast, describe the intrinsic mechanical behavior of tissue constituents only, irrespective of the tissue sample's overall geometry. Material properties are described by parameters such as the stress-strain relationship, the modulus and the ultimate stress and strain. Since stress and strain are normalized by the cross-sectional area and length of sample, respectively, material properties are independent of the geometry of the specimen [3]. In fact, the material properties of biological tissues are based on stress-strain relationships of the tissue substance itself. These properties are more difficult to measure than structural properties for several reasons: it is difficult to grip the tissue without damaging it; accurate measurement of tissue cross-sectional area is challenging; strain is best measured without contacting the tissue; and material properties are sensitive to external factors such as the source of the tissue and how the tissue is handled, stored, or prepared prior to testing.

a. The *tensile material properties* of cartilage can be described by the *tensile modulus, E*, which is equal to the slope of the linear portion of the stress-strain curve. When a soft connective tissue is subjected to an applied force, its length will change with time until it reaches an equilibrium length. This process is called "creep" and is the result of the visco-elastic properties of the tissue. Ideally, the tensile modulus should be measured once the tissue stops deforming and has reached equilibrium.

b. Because articular cartilage is loaded in *compression*, it makes sense to study its compressive properties. The compressive properties of cartilage can be measured using several test configurations [3].

c. An important characteristic of the material properties is *visco-elasticity*.

Some of the visco-elastic characteristics displayed by most biological tissue are: *time-dependent responses, hysteresis,* and *history-dependent responses*.

d. *Biphasic mixture theory of cartilage* [3] – while visco-elastic models are useful for describing and predicting the mechanical behaviour of biological tissues, they are phenomenological and reveal little about the actual physical mechanisms responsible for the observed mechanical behaviour".

3. MATHEMATICAL MODEL REGARDING THE ELBOW JOINT CARTILAGE

The paper, wish to study the determination of mechanical properties (stress and strain) in the elbow joint, using the visco-elastic parameters. They were determined by creating a mathematical model for hand-arm system, writing equations that correspond to them, and solving them. For these the resolution shall not be able to perform than knowing these mechanical properties, i.e. *the coefficient of elasticity* and *damping* or in terms of their equivalent *material resistance, stress* and *strain*. For beginning it proposes some simplifying conditions, so that they shall study firstly, a linear cartilage elbow behavior [4]. All these, correspond the studied shape using visco-resilient parameters with linear behaviour (Fig. 1).

Symbols using in the mathematical model of the elbow joint are:

It knows that [2-5]:

$$x(t) = x_{dashpot}(t) + x_{spring}(t) \qquad (4)$$

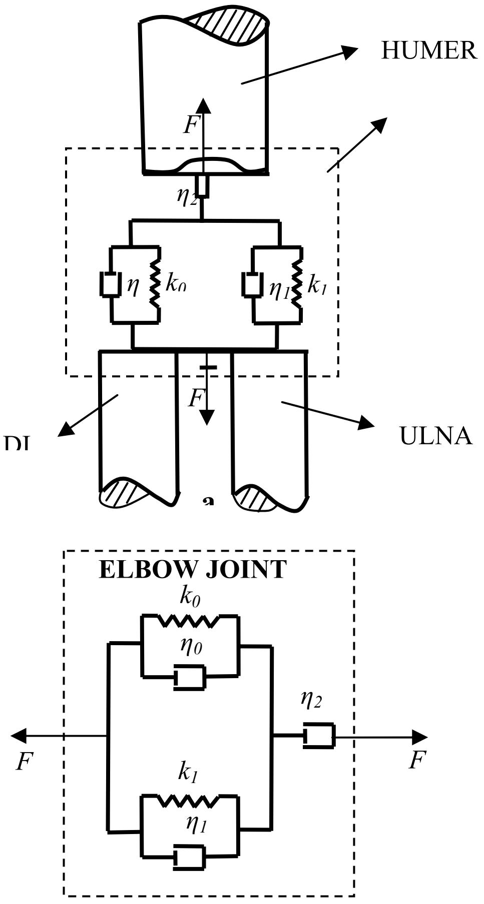
But,

$$F_{spring}(t) = kx_{spring}(t)$$

$$x_{spring}(t) = \frac{F_{spring}(t)}{k} \qquad (5)$$

$$F_{dashpot}(t) = \eta\, \dot{x}_{dashpot}(t)$$

$$\dot{x}_{dashpot}(t) = \frac{F_{dashpot}(t)}{\eta} \qquad (6)$$



b.

Fig.1

a. Linear, simplify model, regarding elbow joints

b. Detail of figure 1a, regarding the elbow joints.

- k_i - spring rate, $i = 1,2$ [N/m];
- η_i - dashpot rate $i = 0,1,2$ [Ns/m];
- μ_i - viscous rate $i = 0,1,2$ [Kg/ms];
- F - tensile force [N];
- $\dot{F}$ - derive force [N/s];
- x - displacement produce of force F [m];
- $\dot{x}$ - velocity produce by displacement $x(t)$ [m/s]; - E_i - Young's modulus $i = 0,1$ [N/m];
- σ - stress [N/m^2];
- $\dot{\sigma}$ - derive stress [N/m^2s];
- ε - strain [-];
- $\dot{\varepsilon}$ - derive strain [-];

If replace relation (5) in (4) will get:

$$x(t) = \frac{F(t)}{k} + x_{dashpot}(t) \qquad (7)$$

$$\dot{x}(t) = \frac{\dot{F}(t)}{k} + \dot{x}_{dashpot}(t) \qquad (8)$$

Using relation (6) in relation (8):

$$\dot{x}(t) = \frac{\dot{F}(t)}{k} + \frac{F(t)}{\eta} \qquad (9)$$

If used relation (9), it will get:

$$\dot{x}(t) = \frac{\dot{F}_{Maxwell}(t)}{k} + \frac{F_{Maxwell}(t)}{\eta} \qquad (10)$$

In relation (9), it using symbol k and η represented the series (11) and parallel (12) connections between visco-elastic parameters ($k_0, k_1, \eta_0, \eta_1, \eta_2$).

$$k = k_0 + k_1 \qquad (11)$$

$$\frac{1}{\eta'} = \frac{1}{\eta_0} + \frac{1}{\eta_1};\quad \eta = \eta' + \eta_2;\quad \eta' = \frac{\eta_0\eta_1}{\eta_0 + \eta_1}$$

$$\eta = \frac{\eta_0\eta_1}{\eta_0 + \eta_1} + \eta_2; \qquad (12)$$

$$\eta = \frac{\eta_0\eta_1 + \eta_0\eta_2 + \eta_1\eta_2}{\eta_0 + \eta_1}.$$

It is known that the graph $\sigma = f(\varepsilon)$ for materials, has an area of elasticity (in the paper is treated as case 1), and an area where entry is elongation up to breaking/flow (denoted case 2). So, in the continuation of this paper will analyze mathematically (theoretic), the two distinct cases of hyaline cartilage behaviour of elbow joint, that case 1 to elastic behaviour (linear) and case 2 the elongation-breaking (non-linear).

3.1. The behaviour of elbow joint cartilage in the area of elasticity (Case 1)

It knows that, the Maxwell force [5] is given by:

$$F_1(t) = F_{Maxwell}(t) + F_{spring}(t) \qquad (13)$$

For continuity of notations in the demonstration, it will note: $F = F_1$

$$F_{Maxwell}(t) = F_1(t) - F_{spring}(t) \tag{14}$$

where:

$$F_{springer}(t) = (k_0 + k_1)x(t) \tag{15}$$

If use the (8) and (9) relations, it results:

$$F_{Maxwell}(t) = F_1(t) - (k_0 + k_1)x(t) \tag{16}$$

If make a derivation to relation (16):

$$\dot{F}_{Maxwell}(t) = \dot{F}_1(t) - (k_0 + k_1)\dot{x}(t) \tag{17}$$

It used relations (15) and (10), it obtained:

$$\dot{x}(t) = \frac{1}{k}\left[\dot{F}_1 - (k_0 + k_1)\dot{x}(t)\right] + \frac{1}{\eta}\left[F_1 - (k_0 + k_1)x(t)\right]$$

(18)

$$\frac{\dot{F}_1}{k} + \frac{F_1}{\eta} = \dot{x}(t)\left[1 + \frac{k_0 + k_1}{k}\right] + \frac{k_0 + k_1}{\eta}x(t)$$

$$\frac{\dot{F}_1}{k} + \frac{F_1}{\eta} = 2\dot{x}(t) + \frac{k}{\eta}x(t) \;|\eta \tag{19}$$

$$\eta\frac{\dot{F}(t)}{k} + F_1(t) = 2\eta\,\dot{x}(t) + k\,x(t)$$

For imposed initial conditions by $t = 0$, it will get:

$$\begin{aligned} &F_1(t) = F_1(0) \\ &\dot{F}(t) = 0 \end{aligned} \tag{20}$$

$$F_1(0) = 2\eta\,\dot{x}(0) + k\,x(0) \tag{21}$$

It notated:

$$\begin{aligned} &F_1(t) = \sigma(t); \quad \dot{F}_1(t) = \dot{\sigma}(t) \\ &x(t) = \varepsilon(t); \quad \dot{x}(t) = \dot{\varepsilon}(t) \end{aligned} \tag{22}$$

It used relations (16) and (18) result:

$$\frac{\eta}{k}\dot{\sigma}(t) + \sigma(t) = 2\eta\dot{\varepsilon}(t) + k\varepsilon(t) \tag{23}$$

For elastically area (linear curve) the differential equations system is follow:

$$\begin{cases} \dfrac{\eta}{k}\dot{\sigma}(t) + \sigma(t) = 2\eta\dot{\varepsilon}(t) + k\varepsilon(t) \\ \sigma(t) = E\,\varepsilon(t) \end{cases} \tag{24}$$

where: E – Young modulus (longitudinal elastically modulus) [N/m^2].

It solved the system (24) will obtained:

$$\begin{aligned} &\dot{\sigma} = E\dot{\varepsilon} \\ &\frac{\eta}{k}E\dot{\varepsilon} + E\varepsilon = 2\eta\dot{\varepsilon} + k\varepsilon \\ &\eta\left(\frac{E}{k} - 2\right)\dot{\varepsilon} + (E - k)\varepsilon = 0 \\ &\frac{\dot{\varepsilon}}{\varepsilon} = \frac{-(E-k)}{\eta\left(\dfrac{E}{k} - 2\right)} \end{aligned} \tag{25}$$

It notated:

$$\frac{\dot{\varepsilon}(t)}{\varepsilon(t)} = A \tag{26}$$

It integrated the relation (23), it will obtain the following solutions of system (21) for the case 1, respectively linear case:

$$\begin{cases} \varepsilon(t) = e^C e^{At} \\ \sigma(t) = Ee^C e^{At} \end{cases} \tag{27}$$

This treated paragraph presents the theoretical solutions regarding the elasticity (linear-area case 1) cartilage of elbow joint hyaline. The following paragraph (3.2) will present the behaviour of elbow joint hyaline cartilage (nonlinear area – case 2) elongation zone and breaking.

3.2 The behaviour of elbow joint cartilage in the elongation zone-breaking (Case 2)

Under normal circumstances, the link between elongation and tensile strength/flow, for a material, you get about experimental, but because for this paper there were no possibilities for experimental determination of mechanical properties of elbow cartilage (stress and strain) and the resulting data from this study were needed for other research in the field, the present paper tried through a theoretical method, but which takes account of certain experimental data, played by literature, to determine the mechanical properties of elbow joint, however taking into account the thickness of the cartilage of the joint influence of thickness, respectively. As a result of this study came the mechanical properties and sizes stress (σ) and strain (ε) [5-8].

So, this feature mechanical stress (σ), noted in the case study 2 (elongation-bursting with σ_s, relationship (28), was taken from the literature [5], where it was determined on experimental path, regarding human tissue, but calculation tubes made of industrial and from several points of view identically regarding the mechanical properties with the human, thus:

For the theoretical continuum study, it was considered an in the research an experimental case give by specialty literature like [5]: a cylindrical tube of radius r_0 and height l_0 made from a material whose Young modulus has the value E = 2.6 *MPa.* It was subjected to tension/elongation length denoted $c = l_0/2$. For the determination of mechanical stress and strain, the experiment will take into account and the potential energy or static γ_s (potential energy = energy that one can develop a body through his movement from the position, in which it is up to a reference index) and as a result of these experiments it was established theoretically, for zone 2, nonlinear, values of breaking strength [5] σ_s.

$$\sigma_s = \sqrt{\frac{E\rho\gamma_s}{4cr_0}} = \sqrt{\frac{3E\gamma_s}{4c}} \tag{28}$$

In the relation (28) it known that:

$\rho = 3r_0$; E – Young modulus (longitudinal elastically modulus) [N/m^2], E = 2.6MPa;

γ_s - Statically energy [J/m], γ_s = 1J/m;

l_0 – testing monster length [mm];

c – l/2 of studied length of testing; r_0 – studied radius of testing experiment.

In the relationship (28) and the first equation of the system (24) yields the solutions for stress and strain on the nonlinear study respectively case 2, analyzed for the elbow joint hyaline cartilage:

$$\sigma_s(t) = \begin{cases} E\varepsilon(t), t \in \left[0, t_{rupere}\right] \\ \sqrt{E\varepsilon(t)},\ t \in \left[t_{rupere}, t\right] \end{cases} \tag{29}$$

Following, it will note: $\sigma_s = \sigma$. Of the (29) relation results:

$$\sigma^2(t) = E\varepsilon(t) \tag{30}$$

Of (29) relation, the second formulas result:

(31)

$$\varepsilon = \frac{\sigma^2}{E}$$

It put relation (32) in (24) and it obtained:

$$\frac{\eta}{k}\dot{\sigma} + \sigma = 2\eta\frac{2}{E}\sigma\dot{\sigma} + \frac{k}{E}\sigma^2$$

$$\left(\frac{\eta}{k} - \frac{4\eta}{E}\sigma\right)\dot{\sigma} = \frac{k}{E}\sigma^2 - \sigma \tag{32}$$

$$\frac{\eta}{kE}\left(E - 4k\sigma\right)\frac{d\sigma}{dt} = \frac{k}{E}\sigma\left(\sigma - \frac{E}{k}\right)$$

$$\frac{\frac{\eta}{kE}(E - 4k\sigma)}{\frac{k}{E}\sigma\left(\sigma - \frac{E}{k}\right)}d\sigma = dt$$

$$\frac{E - 4k\sigma}{\sigma\left(\sigma - \frac{E}{k}\right)}d\sigma = \frac{k^2}{\eta}dt \qquad \left\|\left(-\frac{1}{k}\right)\right.$$

$$\frac{4\sigma - \frac{E}{k}}{\sigma\left(\sigma - \frac{E}{k}\right)} = -\frac{k}{\eta}dt \qquad \begin{matrix}(33)\\(34)\end{matrix}$$

It will notate:

$$\frac{E}{\sigma} = a$$

$$f(\sigma) = \frac{4\sigma - a}{\sigma(\sigma - a)} \tag{35}$$

$$\frac{A}{\sigma} + \frac{B}{\sigma - a} = \frac{(A+B)\sigma - aA}{\sigma(\sigma - a)} \tag{36}$$

To solve the equation (36) we need, in terms of identifying mathematical terms:

$$\begin{cases} A+B=4 \\ -aA=-a \end{cases} \Rightarrow \begin{cases} A+B=4 \\ A=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=3 \end{cases} \tag{37}$$

$$\left(\frac{1}{\sigma} + \frac{3}{\sigma - a}\right)d\sigma = -\frac{k}{\eta}dt \tag{38}$$

The relation (38) becomes:

$$\ln|\sigma| + 3\ln|\sigma - a| = -\frac{k}{\eta}t + C \tag{39}$$

The solution of the equation (29) is given by relation (40):

$$\sigma(\sigma - a)^3 = e^{-\frac{k}{\eta}t} \cdot e^C \tag{40}$$

Thus the solutions of systems of equations (stress and strain unknowns), for the two cases taken into study, case 1, when studying the elbow joint hyaline cartilage in the elasticity and case 2, when studying the properties of cartilage in the area of nonlinear creep-rupture, will result in solving data (41) and (42):

$$\begin{cases} \varepsilon(t) = e^{C} e^{At} \\ \sigma(t) = E e^{C} e^{At} \end{cases} \tag{41}$$

For the second equation, respectively the corresponding case study 2 (elongation-breaking), for using in expression and explanation of constants C, respectively with the constant
C resulting from case 1 study will note C = C_2.

$$\sigma\left(\sigma-\frac{E}{k}\right)^{3} = e^{-\frac{k}{\eta}t}\cdot e^{C} \tag{42}$$

The resulting solutions through solving (41) and (42) are given Matlab programming environment and graphed below.

3.3 Graphical solutions and their interpretation

Table 1 Mechanical properties regarding the soft tissues (Black and Hastings, 1998) [4].

Soft tissue	Longitudinal elasticity modulus, notated E [GPa]	Tensile breaking strength [MPa]
Articular cartilage	10.5	27.5
Fibrocartilaje	159.1	10.4
Ligament	303	29.5
Tendon	401.5	46.5
Skin	0.1-0.2	7.6
Arterial tissue (longitudinal area)	-	0.1
Arterial tissue (transversal area)	-	1.1

Data relations equations solutions (41) and (42) are obtained in Matlab. To resolve them, it was necessary to take into account the numeric data rendered in table 1 for the cartilage articulation, it neglects the value of stretching/compression force, which was taken over by the experimental study of the literature [4, 5] and require some initial conditions, like: It is logical considered: thickness elbow cartilage is 3 mm.

$t_{initial\text{-}breaking}$= 0.015s; $t_{final\text{-}breaking}$ = 0.04s; $\sigma_{0\ breaking}$ = 27.5MPa; E = 10.5 GPa

$\varepsilon_{0\ breaking}$ = σ/E = 0.261; $k_{t1} = k$ = 2Nm/rad [1]; $\eta_{t2} = \eta$ = 6.14Nm/rad [1]

This thinking logic, especially regarding the dates initial and final ($t_{\text{initial-breaking}}$, $t_{\text{final-breaking}}$) it was used, it relies on the analogy with the other anterior experiments, these were effected on the other tissues human (the artery) exposed at elongation, and determination the stress, respective till tissue was broke, of course taking into account the thickness of tissue of artery. Initial data from the given problem data and using the initial conditions problem above, for relations (41) yields the constant $C = C_1$ and for relationship (42) the constant C_2 such that:

At $t = 0$,

$C_1 = 0$ (42); $C_2 = 4752.08$ (43)

and in this case, the solutions of equations and unknowns of stress and strain for the two cases will have values below, case 1, the equation of exponential order 1 (41), it follows by solving a single σ and solution case 2 an equation of order 4 (42), whose resolution it play four solutions, as follows:

$$\begin{gathered} \sigma_{11} = 1.02 N/mm^2 \\ \sigma_{s\_21} = 0 N/mm^2,\ \sigma_{s\_22} = 0.76 N/mm^2,\ \sigma_{s\_23} = 0.95 N/mm^2,\quad \sigma_{s\_24} = 1.019 N/mm^2 \\ \varepsilon_{11} = 0.066 \\ \varepsilon_{s\_21} = 0,\ \varepsilon_{s\_22} = 0.868,\ \varepsilon_{s\_23} = 0.868,\quad \varepsilon_{s\_24} = 1.50 \end{gathered} \tag{44}$$

Graphical representation (Matlab) of equations solutions (40) and (41) (Runge-Kutta method) , that stress and strain is depicted by Figure 2.

The results obtained and the allure of the curve (Fig. 2) shows that have obtained the correct results, and for surface in mm^2 the obtained values are big (~1 N/mm^2), that demonstrates the validity of the study and even if at the point of interconnection of the two areas studied, i.e. linear and nonlinear graph, there is a small breaking, this could arise from choosing the simplifying conditions or perfect undoubting of the properties used in the paper of the experimental formula (29). But, it is mentioned that the allure of the graph in figure 2, to the final correct representation, from the point of view of the relationship between elongation-breaking materials (cartilage).

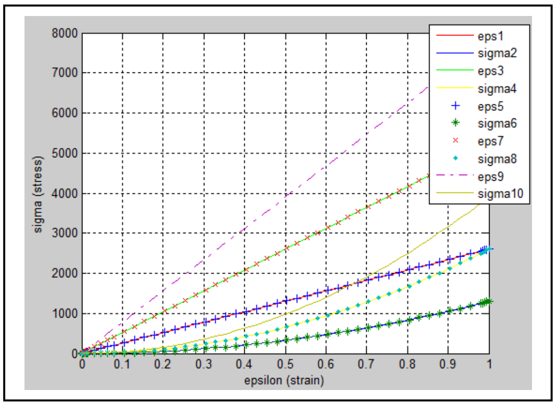


Fig. 2 Linear and nelinear behavior of the elbow cartilage.

where: eps = ε - strain;
sigma = σ - stress;

In the figure 2, we kept constant the initial constants, respectively time $t = 0$, and the strain ε = 0, and varying the stress – sigma from the 0.5 till 3, and your behavior is given by the figure 2. Respectively, we could see the following behavior: at the small values for the sigma (strain), the cartilage has a maximal stress value. This affirmation is in opposite with the linear relation between stress and strain, relation 2. So, if we want to increase the strain, we must to decreasing the stress.

4. CONCLUSION

The theoretical results obtained and are represented in the form of graphics, are consistent (~1N/mm^2) at the elbow hyaline cartilage thickness of 3 mm, with the theoretical curve allure that represents the behaviour of the material depending on stress and strain, allure known from literature.

All of these results, it was obtained after creating a mechanical model presented in mathematical modelling of this paper, model corresponding to the elbow joint. All these, it will continue with writing and solving differential equations that define this model. In the end, one is a graphical analysis of the obtained solutions, in accordance with the allure of the graph, and the link between stress and strain.

New, in the paper is the fact that research for the study and determining the properties of stress and strain in the joints (i.e. determining the characteristics of stress and strain in the elbow cartilage, it initiated firstly, a linear study and then a nonlinear study and connection between them (broken limit). It was conceived at the level of mathematical model, equivalence of mechanical properties, respective coefficients of elasticity and damping with units corresponding to in terms of strength of materials (stress and strain), and all these have not been studied so far in the literature.

For all these reasons, the study of this paper is very important in terms of determining the mechanical properties, the elbow joint and constitutes an important step for determining and analyzing these mechanical properties in other human joints (e.g. hip, knees, etc.).

In conclusion of the paper, if we want to increase the strain, we must to decreasing the stress. This conclusion must be research in the next future and make an experimental model after this. And, if the research studies are confirming, in the next future we can produce the artificial cartilage for the elbow and not only.

5. DISCUSSION

The paper it was proposed, demonstrate the theoretical point of view, regarding the link between stress and strain of the elbow joint (cartilage) leaving of the mechanical visco-elastic properties. Theoretical relationships were obtained between behaviours the mechanical, stress and strain, and they were represented on the same graph for the areas of linearly (case study 1) and geometry (case study 2), mentioning that the interconnection of their being exactly at the boundary of elongation, when cartilage begins to break.

The results of the paper can be taken and used of the other theoretical research, for example to determine the equations that define the movements of the hand-arm system under the action of vibrations, where the knowledge of these mechanical properties of the model is essential. It wants in the future, creating and achieving a more complex mathematical model of the elbow joint, which treats only in nonlinear characteristics of stress and strain, as well as the corresponding experimental. Thus, they will be able to play accurately the experimental characteristics of resistance, especially from breaking.

As to the paper of this research report, the studies mentioned in the literature, it could say the following:

1. Theoretical and experimental studies have not been conducted, the cartilage, the reason being that the elbow of thickness, respectively only 3mm.

2. Of course, there are theoretical and experimental studies, animals or people, it been referring to the mechanical statically behaviour studies. But these are not on human articular elbow [9, 10, 11], but experiments for the other animal or human articular, indeed were effectuated.

For me to argue this, the following paragraphs show in the literature, some of the conclusions relating to studies on human and animal joints:

- "Some evidence and considerable logic suggests static contact stresses per se do not predict cartilage responses, but rather temporal aspects of the contact stress history. Static contact stresses may therefore not be a reasonable surrogate for biomechanical studies. Rather, temporal and spatial aspects of the loading history undoubtedly induce beneficial and deleterious biological responses. **Finally, since all articular cartilage experiences similar stresses, the concept of a "weight-bearing" versus a "non-weight-bearing" joint seems flawed, and should be abandoned"** [11].
- "The interosseous membrane's load transferring ability reduces the forces placed on the radiocapitellar articulation, thereby protecting this joint. However, large chronic loading results in attenuation of the membrane fibers, thereby reducing longitudinal stability. Large sustained loads occur after radial head resection with concurrent interosseous membrane tears, resulting in the proximal migration of the radius and disruption of the distal radioulnar joint. Ultimately, the treatment option for severe membrane disruption combined with proximal migration of the radius is the creation of a single bone forearm" [10].
- "In this chapter a description is given of experiments, in which the previously described tools are used to study the load transmission through collagenous connective tissues. For this study, collagenous connective tissue from the cubital region was used. The primary objective of the experiment was to gain insight into the mechanical behavior of the structure. A secondary goal was to demonstrate of the abilities of the developed tools when used on soft tissues" [9].

In the meantime, knowing in detail the behaviour of mechanical joints, will allow the creation of mathematical models, models that help minimize the adverse effects that might damage the joints. Or, you can also create new mathematical models to innovation, creation of mechanical devices, which are designed to take some of the pressure on the joints, or last but not least to improve biomedical devices (e.g., hearing) and the detection of articular affections.

6. REFERENCES

[1] Nordin Margareta, Frankel H. Victor – *Basic Biomechanics of the Musculoskeletal System*, third edition, Lippincott Williams & Wilkins, Philadelphia, USA, ISBN 13-978-0-683-30247-9, ISBN 0-683-30247-7, 2001, 465 pages.

[2] Bruce R. Martin, Burr B. David, Sharkey A. Neil – *Skeletal Tissue Mechanics*, Springer-Verlag, New York, USA, 1998, 392 pages.

[3] Ross C. Ethier and Simmons A. Craig – *Introductory Biomechanics From Cells to Organisms*, 511 pages, Cambridge University Press, ISBN 978-0-521-84112-2, 2007.

[4] Humphrey J. D. - *Need for a continuum biochemomechanical theory of soft tissue and cellular growth and remodeling*, in: G.A. Holzapfel and R.W. Ogden, eds., Biomechanical Modelling at the Molecular, Cellular and Tissue Levels, Springer-Verlag, Wien, New York, CISM Courses and Lectures no. 508, 2009 pp., 1-82.

[5] *Broken Mechanical Theory* – http://www.ipm.utcluj.ro/Files/Curs_2_MDPR.pdf.

[6] Simona Potorac – *Network interpenetrante collagen-based synthetic polymers with funcționalizat and biomedical applications*, Doctarat thessis, Gh. Asachi Tehnical University Iassy and Universitatea of Rouen, 2011.

[7] Ioan Ciocoară, Elena Onofrei – *General Engineering in textile and leather*, Ed. Performantica 2007, National Institute of Inovation, Iassy, 190 pg.

[8] Raluca Darie – *Mechanical characterization of polymer materials*, Romanian Academy, Chemistry Institution, 2007.

[9] Gerrit Peters – *Tools for the mesurement of stress and strain fields in soft tissue* – application to elbow joint" 161 pag. (http://www.mate.tue.nl/mate/pdfs/210.pdf) access on-line: 04.03.2013;

[10] McGinley, Joseph C. MS; Kozin, Scott H. MD - *Interosseous Membrane Anatomy and Functional Mechanics, Temple University*, Department of Orthopaedic Surgery, Philadelphia, PA, USA (http://www.ncbi.nlm.nih.gov/pubmed/11210945);

[11] Iowa Orthop J. - Joint Contact Stress "*A Reasonable Surrogate for Biological Processes?*", 25: 82–94, PMCID: PMC1888787, , Richard A Brand, Editor-in-Chief (http://www.ncbi.nlm.nih.gov/pmc/articles/PMC1888787/), 2005.

Proprietățile mecanice de alungire şi rezistență la rupere ale articulației cotului

Rezumat: Această lucrare prezintă importanță, deoarece literatura de specialitate nu tratează acest subiect profund şi anume proprietățile mecanice ale articulației cotului. Cartilajul cotului având o grosime foarte mică acest aspect se neglijează în marea majoritate a lucrărilor. Lucrarea îşi propune un studiu bazat pe o platformă teoretică.
În primul rând, abordează studiului liniar al cartilajelor de la cot, deoarece un studiu neliniare ar implica introducerea unor multitudini de factori interni şi externi, care ar trebui să se ia în calcul şi cercetare teoretică ar fi deosebit de dificil, dacă nu imposibilă. În plus, în primul caz (caz 1) de studiu, fie el liniar, nu a fost abordat de literatura de specialitate şi este deosebit de important să se cunoască comportamentul mecanic al cartilajului la solicitări. Scopul final al lucrării este de a determina, din perspectiva liniară, proprietăți mecanice ca alungire şi rezistența cartilajului hialin din punct de vedere al ruperii, în ceea ce priveşte articulația cotului. În cazul 2 de studiu, se analizează evoluția neliniară, cu privire la alungire si rezistenta la rupere plecându-se de la experimente de laborator, anterioare, pe țesut hialin al cartilajelor.
Abordarea teoretică la această problemă, din punctul de vedere al modele matematice elaborate, ele încearcă să respecte exact modelul real al sistemului mână-braț, s-a demonstrat că proprietățile mecanice (rezistenta şi alungirea) în articulatia cotului nu sunt de neglijat. **Cuvinte cheie:** cartilaj, încheietura cotului, caracteristicile mecanice, rezistența la rupere şi alungirea.

Aurora-Felicia CRISTEA, Lecturer PhD. Dipl. Eng., Technical University of Cluj-Napoca, Mechanical Engineering System Department, no.103-105 B-dul Muncii, Cluj-Napoca, ROMANIA, e-mail: cristea_fa@yahoo.de.