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THE KINEMATICAL MODEL OF THE NEW 4P - 2S – 2TCP - 2R PARALLEL KINEMATIC MECHANISM

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Abstract: *The 4P-2S–2TCP-2R is a spatial four degrees of freedom Parallel Kinematic Mechanism. The secondary platforms are guided in space by two arms. Each arm consists of a linear actuators. The actuators are fixed to the ground. The first actueter is linked to the secondary platform by Conetip –Plane joint and the second one by Spherical joint. The secondary platforms are connected to the main platform through revolute joints. This paper deals with the mechanical structure of a new four degrees of freedom spatial mechanism. This paper also deals with the kinematical model of the mechanism.* ***Key words:*** *P - Prismatic joint, R - Revolute joint, TCP – Conetip –Plane joint, PKM - Parallel Kinematic Mechanism, DOF - Degrees of Freedom, main platform, secondary platform.*

1. INTRODUCTION

A Parallel Kinematic Mechanism is a spatial multi branch and degrees of freedom mechanical system in which the end - effector (platform) is guided by many open or closed kinematic chains. This closed topology shows an increased rigidity of the structure, platform positioning accuracy, a reduced total structural weight and thus good dynamical behaviour. The PKMs are able to perform heavy duty tasks at high speeds and accelerations up to 20g. On the other PKMs possess a restricted work space. The singularity positions are also a major drawback in establishing of the kinematical command. The beginning of the researches in the field of PKMs goes back to the 1960-es. Stewart presented a structure with 6 DOF for flight simulator, commonly known as the Stewart platform [16]. A similar machine for tire testing was developed earlier by Gough [5]. Many designs have been based on the Stewart structure. Examples include designs by McCallion, who applied a Stewart platform for robotized assembly operations [10], by Fichter [4], Clavel DELTA [2], the Pierrot Hexa robot [12] etc. PKM structures with 3 DOF are presented in [11] and [9]. Examples of 4 DOF architectures were proposed by Kerekes [6], [7], by Pierrot, Salgado [13], [15]. Also 4 DOF structures as flight simulator were proposed also by Fattah and Kasaei [3]. Coeloho [1] proposed a 4 DOF PKM for the prototype of a CNC milling machine. Five DOF architectures were proposed by Austad [9], by Pollard for a spraying machine [14] and Kerekes [8]. In this paper the structure of a new 4 degrees of freedom Parallel Kinematic Mechanism is presented. The mathematical model of the mechanism will be presented in this work.

2. THE STRUCTURE AND THE MATHEMATICAL MODEL OF THE 4P-2S–2TCP-2R PKM

The 4P-2S–2TCP-2R (Fig.1) is a spatial four degrees of freedom Parallel Kinematic Mechanism. The secondary platforms are guided in space by two arms. Each arm consists of a linear actuators. The actuators are fixed to the ground. The first actuator is linked to the secondary platform by Conetip – Plane joint and the second one by Spherical joint. The secondary platforms are connected to the main platform through revolute joints. The revolute joints are placed in the points C_k, the Spherical in B_k and the Conetip – Plane joints in the points A_k.

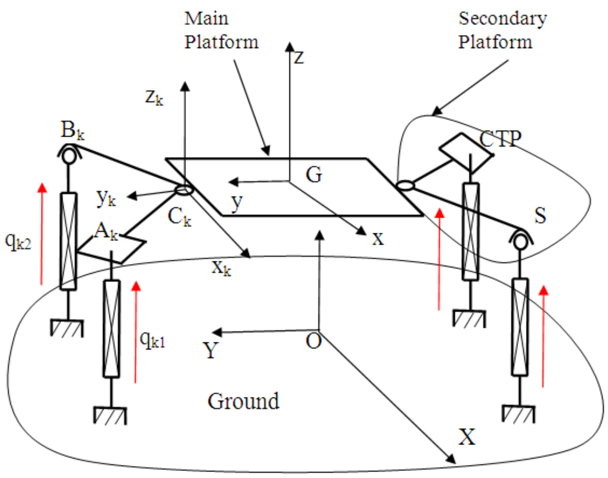

Fig.1. The kinematic scheme of the mechanism

Notations:

- OXYZ- the fixed frame;
- Gxyz - the mobile frame attached to the main platform;
- $C_k x_k y_k z_k, k = 1..2,$ the mobile frame attached to the secondary platform;
- $q_{k1}, k = 1..2$ - the length of the arm, which actuates the Conetip - Plane joint;
- $q_{k2}, k = 1..2$ - the length of the arm, which actuates the Spherical joint;
- $S_k(x_0, y_0, 0)$ an arbitrary point on the secondary platform;
- The coordinates $(x_0, y_0, 0)$ are expressed in the frame $C_k x_k y_k z_k, k = 1..2,$
- $GC_1 = GC_2 = a,$ the coordinates of C_1 are (a,0,0) and coordinates C_2 are(-a,0,0) in the Gxyz - the mobile frame attached to the main platform;
- The coordinates of the points B_k are $(u_k, v_k, 0)$ in the $C_k x_k y_k z_k, k = 1..2,$ - mobile frame attached to the secondary platform. The coordinates $(u_k, v_k, 0)$ are known parameters;
- The coordinates of the points B_k are $(\xi_k, \eta_k, 0)$ and in the $C_k x_k y_k z_k, k = 1..2,$ - mobile frame attached to the secondary platform. The coordinates $(\xi_k, \eta_k, 0)$ are unknown parameters;
- X, Y, Z - the coordinates of the point G in the fixed frame;
- δ_k - the rotational angles in the revolute joints placed in the platform points C_k;
- ψ, θ, φ - The pitch, yaw and roll angles of the platform;
- $c_m, m = 1..5$ - the number of m class joints;
- N - the degrees of freedom;
- M - the number of the elements;
- Rot(x,u), Rot(y,v), Rot(z,w)- the rotational matrixes;

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- $\mathrm{Rot}(z,w) = \begin{bmatrix} cw & -sw & 0 \\ sw & cw & 0 \\ 0 & 0 & 1 \end{bmatrix};$
- [r]- the rotational matrix of Gxyz to OXYZ;
- $s\lambda = \sin\lambda$, $c\lambda = \cos\lambda$;

The 4P-2S–2TCP-2R PKM mechanism posses the following structure:

$$M = 7; c_1 = 2; c_2 = 0; c_3 = 2; c_3 = 0; c_5 = 6 \qquad (1)$$

According to Artobolevski – Gruebler - Kutzbach criteria the degree of freedom for the 4P-2S–2TCP-2R PKM is given by:

$$\ldots \quad \ldots N = 6M - \sum_{m=1}^{5} m c_m = 4 \ldots\ldots \quad \ldots(2)$$

Let us suppose an arbitrary $S(x_0, y_0, 0)$ an arbitrary point on the secondary platform. After the rotation of the secondary platform with the angle δ_k around the $C_k z_k$ the coordinates of the point S in the frame linked to the main platform can be expressed by:

$$[x_s, y_s, z_s]^T = [a,0,0]^T + \mathrm{Rot}(z, \delta_k).[x_0, y_0, 0]^T \qquad (3)$$

In the fixed frame the coordinates of the point S after the transformation of the of frame linked to the main platform can be expressed by:

$$[X_s, Y_s, Z_s]^T = [X, Y, Z]^T + [r].[x_s, y_s, z_s]^T \quad (4)$$

According to Fig.1 the coordinates of the point S in the fixed frame can also be expressed by:

$$[X_s, Y_s, Z_s]^T = [a + x_0, y_0, q_k]^T \quad (5)$$

By particularising S = A_k and S = B_k and from Eqn. (3), (4) and (5) the following relations can be derived:

$$a - X = (A_{1k} - 1)\xi_k + B_{1k}\eta_k; \quad (6)$$

$$-Y = A_{2k}\xi_k + (B_{1k} - 1)\eta_k; \quad (7)$$

$$q_{1k} = Z + A_{3k}\xi_k + B_{3k}\eta_k; \quad (8)$$

$$a + u_k - X = p_{11k}c\lambda_k + p_{12k}s\lambda_k; \quad (9)$$

$$v_k - Y = p_{21k}c\lambda_k + p_{22k}s\lambda_k; \quad (10)$$

$$q_{3k} - Z = p_{31k}c\lambda_k + p_{32k}s\lambda_k; \quad (11)$$

$$A_{ik} = r_{i1}(c\lambda_k + s\lambda_k), B_{ik} = r_{i2}(c\lambda_k - s\lambda_k),$$
$$p_{i1k} = u_k r_{i1} + v_k r_{i2}, p_{i2k} = u_k r_{i1} - v_k r_{i2},$$
$$i = 1..3, k = 1..2; \quad (12)$$

From Eqn. (9) and (10) the following relations can be derived:

$$c\lambda_k = \frac{1}{d_k}[-(a + u_k - X)p_{21k} + (v_k - Y)p_{22k}],$$
$$s\lambda_k = \frac{1}{d_k}[(a + u_k - X)p_{11k} - (v_k - Y)p_{12k}],$$
$$d_k = p_{12k}p_{21k} - p_{11k}p_{22k}; \quad (13)$$

From Eqn. (6) and (7) the following relations can be derived:

$$\xi_k = \frac{1}{t_k}[(a - X)(B_{2k} - 1) + YB_{1k}],$$
$$\eta_k = \frac{1}{t_k}[(a - X)A_{2k} + Y(A_{1k} - 1)],$$
$$t_k = (B_{2k} - 1)(A_{1k} - 1) - B_{1k}A_{2k},$$
$$q_{2k} = Z + \sqrt{(M_k)^2 - (a + u_k - X)^2 - (v_k - Y)^2}$$

(14)

For the inverse kinematic problem when four platform parameters are given Z, ψ, θ, φ the determination of X and Y according to the others can be made by using the relation $(sx)^2+(cx)^2=1$ with $x=\lambda_k$ k=1..2. thus the following system of Eqn. in unknown X and Y can be derived:

$$H_1(X,Y) = 0, H_2(X,Y) = 0,$$
$$H_k(X,Y) = E_k + F_k + G_k;$$
$$E_k = (-(a + u_k - X)p_{22k} + (v_k - Y)p_{21k})^2,$$
$$F_k = ((a + u_k - X)p_{21k} - (v_k - Y)p_{11k})^2,$$
$$G_k = (d_k)^2; \quad (15)$$

3. CONCLUSIONS

In this paper the structure of a new 4 DOF PKM was presented. Its degrees of freedom were verified by means of the Artobolevski-Gruebler-Kutzbach criteria. This work also deals with the mathematical model of the mechanism. Thus the relations, which describe the interdependency between the generalized coordinates and the platform parameters, were derived. Considerations regarding the inverse and direct kinematics of the proposed new 4 DOF PKM will be the topic of future works.

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STRUCTURA ŞI MODELUL MATEMATIC AL MECANISMULUI PARALEL 4P-2S–2TCP-2R

Mecanismul paralel 4P-2S–2TCP-2R este un mecanism spatial cu patru grade de libertate. Cele doua platforme secundare sunt ghidate de doua brate constituind motoarele lineare de actionare. Unul dintre brate se leaga de platforma printr - o cupla sferica iar cealalt printr – o cupla virf de con plan. Platformele secundare se leaga de platforma principala prin cuple de rotatie. În acest articol este prezentat modelul cinematic al mecanismului. Gradul de libertate al mecanismului se va determina utilizînd criteriul Artobolevski – Gruebler – Kutzbach. Se va deduce deasemenea modelul matematic al mecanismului.

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