TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 56, Issue IV, November, 2013

OPTIMIZATION OF MEMS STRUCTURES USING CUCKOO SEARCH ALGORITHM

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Abstract: *The results of MEMS structures optimizations using the meta-heuristic algorithm called Cuckoo Search are presented in this paper. Two optimizations were conducted: one on a microbridge and the other one on a microcantilever. The aim of the optimizations was to obtain the optimal geometric dimensions of the considered MEMS structures in order to minimize the maximum bending stress.*

Key words: *optimization, Cuckoo Search algorithm, microbridge, microcantilever*

1. INTRODUCTION

Micro-Electro-Mechanical Systems(MEMS) are small integrated devices or systems that combine electrical and mechanical components. MEMS technology is employed to produce a wide range of common products. As the existing technology is applied to the miniaturization and integration of conventional devices, new applications for MEMS are emerging.

MEMS are not only about miniaturization of mechanical systems, they are also new patterns for designing mechanical devices and systems. Taking into account the application where the device is used, there are specific design requirements.

Optimizations of MEMS dimensions for a microbridge and a microcantilever were conducted in this paper in order to obtain a maximum deflection of 0.3 µm in each case.

2. CUCKOO SEARCH ALGORITHM

Most modern meta-heuristics imitate the biological systems evolved from natural selection over millions of years. The two main versions of Cuckoo Search algorithm, standard version [2] and Lèvy flights version [3], are both inspired by the lifestyle of a bird family called cuckoo. Specific egg laying and breeding of cuckoos are the basis of this optimization algorithm.

The Cuckoo Search algorithm is useful for design optimization, a domain that has become an integrated part of designing any new products in engineering and industry [4].

The optimizations presented in this paper use the Lèvy flights version of the algorithm taking into account the following rules:

- Each cuckoo lays one egg at a time, and dump its egg in a nest chosen using Lèvy flights;
- The number of available host nests is fixed, and the egg laid by a cuckoo is discovered by the host bird with a certain probability.

The algorithm uses an initial random generation of cuckoos that lays eggs. Each egg from a nest represents a possible solution for the optimization problem. Until a maximum number of generations is reached, new generations are generated using Lèvy flights in order to replace the old ones. This replacement is partial, the high quality eggs being carried over to the next generations so that the optimal solution is not overlooked.

3. OPTIMIZATION

3.1 Microbrigde case

For microbridge (Fig. 1) the bending moment M_{by} will have two different equations one for each of the intervals (2-1) and (1-3), namely:

$$M_{by} = \begin{cases} F_{2z} \cdot x - M_{2y}, & 0 \le x \le l_1 \\ F_{2z} \cdot x - F_{1z}(x - l_1) - M_{2y}, & l_1 \le x \le l \end{cases} \quad (1)$$

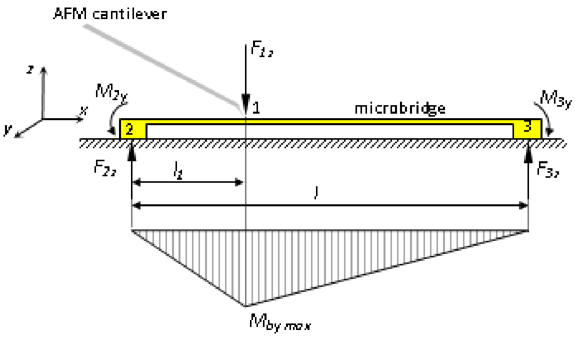

Fig. 1. Schematic representation of microbridge.

The deflection and rotation can be written, using the Castigliano's displacement theorem, as [1]:

$$u_z = \int_l \frac{M_{by}}{EI_y} \cdot \frac{\partial M_{by}}{\partial F_z} dx$$
$$\theta_y = \int_l \frac{M_{by}}{EI_y} \cdot \frac{\partial M_{by}}{\partial M_y} dx \quad (2)$$

The factors used in equation (2) are: E is Young's modulus; M_{by} -bending moments, $I_y = wt^3/12$ -moment of inertia; w- width, t- thickness.

After applying Castigliano's displacement theorem at point 2 (Fig.1) and performing the necessary calculations, it is found that the force F_{2z} and bending moment M_{2y} are:

$$F_{2z} = \frac{l^3 + 2l_1^3 - 3l_1^2 l}{l^3} F_{1z}$$
$$M_{2y} = \frac{l_1^3 - 2l_1^2 l + l_1 l^2}{l^2} F_{1z} \quad (3)$$

The bending moment is maximum at the point where the force is applied and its value is:

$$M_{by\max} = 2F_{1z}\left(\frac{l_1}{l}\right)^3 (l - l_1)^2 \quad (4)$$

The maximum bending stress at that location can be calculated as:

$$\sigma_{by\max} = 12F_{1z}\frac{l_1^2 (l - l_1)^2}{wt^2 l^3} \quad (5)$$

Taking into account that the load can be calculated using the deflection of the microbridge and its stiffness, for a load applied at midpoint of microbridge, the maximum bending stress becomes:

$$\sigma_{by\max} = \frac{192Ezt}{16l^2} \quad (6)$$

where z is deflection.

The purpose of the conducted optimization on a microbridge manufactured from different materials was determining its geometric dimensions so that the bending stress should be minimum in the folowing conditions:

- The load is applied at microbridge midpoint;
- The microbridge width is 55 µm;
- The maximum deflection is 0.3 µm.

The objective function used in the Cuckoo Search optimization program was the function that returns the maximum bending stress given by equation (6). Each cuckoo had two dimensions, one representing the length of the microbridge and the other representing the thickness. The program performed a search for the optimum solution in the following intervals (in µm): [300, 450] for microbridge length and [1, 3] for microbridge thickness. Optimisation results are presented in Table 1.

Table 1

Optimization results for different microbridges

Material	E (GPa)	T (µm)	l (µm)	Maximum bending stress (MPa)
Gold	72	1	450	1.28
Aluminum	68	1	450	1.21
Polysilicon	150	1	450	2.67

One can see that the optimum microbridge length obtained is the upper limit of the search interval, while the optimum microbridge thickness obtained is the lower limit of the search interval. It is due to the fact that the increase of the microbridge length causes the decrease of its stiffness. A decrease of the necessary load is achieved in order to obtain a maximum deflection of 0.3 µm. The decrease of the load causes a decrease of bending stress. The calculated values of the stiffness and the loads necessary to obtain a maximum deflection of 0.3 µm for each microbridge obtained after optimization are presented in Table 2.

Table 2
Stiffnesses and loads for optimum microbridges

Material	E (GPa)	Stiffness (N/m)	Load (nN)	Maximum bending stress (MPa)
Gold	72	0.6953	208.59	1.28
Aluminum	68	0.6567	197.02	1.21
Polysilicon	150	1.4486	434.58	2.67

3.2 Microcantilever case

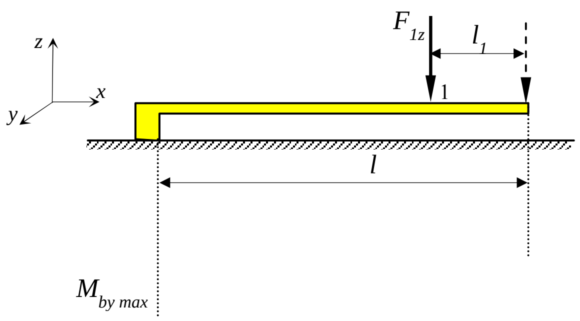

Fig. 2. Schematic representation of microcantilever.

The schematic representation of microcantilever is shown in Fig. 2. The deflection at point 1 is [1]:

$$u_{1z} = \frac{\partial U}{\partial F_{1z}} = \int_0^l \left(\frac{M_{by}}{EI_y} \cdot \frac{\partial M_{by}}{\partial F_{1z}} \right) dx \qquad (7)$$

The bending moment M_{by} for microcantilever can be written as:

$$M_{by} = F_{1z}(l - l_1) \qquad (8)$$

After performing the necessary calculations, the dependence between Young's modulus and geometric dimensions is found. If the force is applied at the (l-l_1) distance from the fixed boundary the bending stress, at the point where the load is applied, can be calculated as:

$$\sigma_{by} = 6F_{1z} \frac{(l - l_1)}{wt^2} \qquad (9)$$

The maximum bending stress is obtained if the load is applied at the free end of microcantilever, and it is computed as follows:

$$\sigma_{by\max} = 6F_{1z} \frac{l}{wt^2} \qquad (10)$$

Taking into account that the load can be calculated using microbridge deflection and microbridge stiffness, for a load applied at microbridge midpoint, the maximum bending stress becomes:

$$\sigma_{by\max} = \frac{3Ezt}{2l^2} \qquad (11)$$

where z is deflection.

The purpose of the conducted optimization on a microcantilever manufactured from different materials was to determine its geometric dimensions so that the bending stress is minimum in the folowing conditions:

- The load is applied at microcantilever free end;
- The microcantilever width is 55 µm;
- The maximum deflection is 0.3 µm.

The objective function used in the Cuckoo Search optimization program was the function that returns the maximum bending stress given by equation (11). Each cuckoo had two dimensions, one representing the length of the microcantilever and the other representing its thickness. The program performed a search for the optimum solution in the following intervals (in μ m): [300, 450] for microcantilever length and [1, 3] for microcantilever thickness. Optimisation results are presented in Table 3.

Table 3
Optimization results for different microcantilevers

Material	E (GPa)	t (μ m)	l (μ m)	Maximum bending stress (MPa)
Gold	72	1	450	0.16

Aluminum	68	1	450	0.15
Polysilicon	150	1	450	0.33

One can see that the optimum microcantilever length obtained is the upper limit of the search interval, while the optimum microcantilever thickness obtained is the lower limit of the search interval. It is due to the fact that the increase of the microcantilever length causes the decrease of its stiffness. A decrease of the necessary load is achieved in order to obtain a maximum deflection of 0.3 μm. The decrease of the load causes a decrease of bending stress. The calculated values of the stiffness and the loads necessary to obtain a maximum deflection of 0.3 μm for each microcantilever obtained after optimization are presented in Table 4.

Table 4

Stiffnesses and loads for optimum microcantilever

Material	E (GPa)	Stiffness (N/m)	Load (nN)	Maximum bending stress (MPa)
Gold	72	0.6953	208.59	0.16
Aluminum	68	0.6567	197.02	0.15
Polysilicon	150	1.4486	434.58	0.33

4. CONCLUSIONS

Based on the optimizations described in this paper one can draw the following conclusions:

- In order to minimize the maximum bending stress, for a fixed maximum deflection, a long and thin beam is obtained regardless of the type of beam (microcantilever or microbridge).
- When a fixed maximum beam deflection is taken into account in the optimization, the beam material does not influence the geometric dimensions of the optimum beam, but it does influence the value of the maximum bending stress of the optimum beam.

ACKNOWLEDGMENTS

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-RU-TE-2011-3-0106.

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OPTIMIZAREA STRUCTURILOR MEMS FOLOSIND ALGORITMUL CUCKOO SEARCH

Rezumat: În această lucrare sunt prezentate rezultatele optimizărilor structurilor MEMS folosind algoritmul meta-euristic numit Cuckoo Search. S-au realizat două optimizări: una pentru o grindă incastrată la ambele capete si cealaltă pentru o grindă incastrată la un capăt. Scopul optimizărilor a fost acela de a obtine dimensiunile geometrice optime ale structurilor MEMS considerate, astfel incat să se minimizeze tensiunea maximă de incovoiere.

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