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A NEW PROOF OF RULE OF THUMB REGARDING CLEARANCE VARIATION WITH TEMPERATURE IN TWO TAPERED ROLLER BEARINGS IN O-ARRANGEMENT

Florina RUSU, Cristina TUDOSE, Lucian TUDOSE

Abstract: *In this paper a new and more rigorous mathematical proof of rule of thumb regarding the total internal clearance variation when shaft thermal expansion occurs in two tapered roller bearing in O-arrangement is presented. It has also been proven that the mentioned rule of thumb is valid even when shaft and housing have slight different thermal expansion coefficients.*
Key words: *tapered roller bearings, back-to-back arrangement, thermal expansion.*

1. INTRODUCTION

The total amount of displacement between the rings and the rolling elements of a bearing is defined as internal clearance. When either the inner ring or the outer ring is fixed and the other ring is free to move, displacement can take place in either axial or radial direction. The amount of displacement, depending on the direction, is called the *radial internal clearance* or the *axial internal clearance*.

Internal clearance is desirable in applications where a certain allowance must be provided for thermal expansion of components, for optimum load distribution, for slight shaft-bearing housing misalignment accommodation, for free rotation of rolling element, or for other application requirements. The internal clearance may also influence noise, vibration, heat build-up and fatigue life of the bearing.

Due to the fact that bearings hold the rotating parts of a mechanism in proper position across the entire performance envelope of an application, selecting and setting the correct internal clearance becomes extremely important.

2. RULE OF THUMB

When setting the axial internal clearance during the bearing installation it is very important to consider the thermal expansions of the shaft, housing, and bearing parts during the operation. In the case of a bearing arrangement consisting in two adjusted tapered roller bearings in face-to-face configuration the clearance inside the bearings will increase with the increase of the temperature. Unlike this arrangement, in the case of two adjusted tapered roller bearings in back-to-back configuration, one cannot assert, for sure, that the clearance inside the bearings will increase.

The things are a little bit more complicated for this bearing arrangement case and a well-known rule of thumb is used in bearing units design [1]: *Due to the temperature difference between the shaft and housing during the operation, for two adjusted tapered roller bearings in O-arrangement (back-to-back arrangement) one can assert that:*

- *the bearing clearance increases with the increase of the bearing operating temperature, if the bearing roller cones do not meet* (Fig. 1 a);
- *the bearing clearance remains the same with the increase of the bearing operating temperature, if the bearing roller cones apexes coincide* (Fig. 1 b);
- *the bearing clearance decreases with the increase of the bearing operating temperature, if the bearing roller cones overlap* (Fig. 1 c).

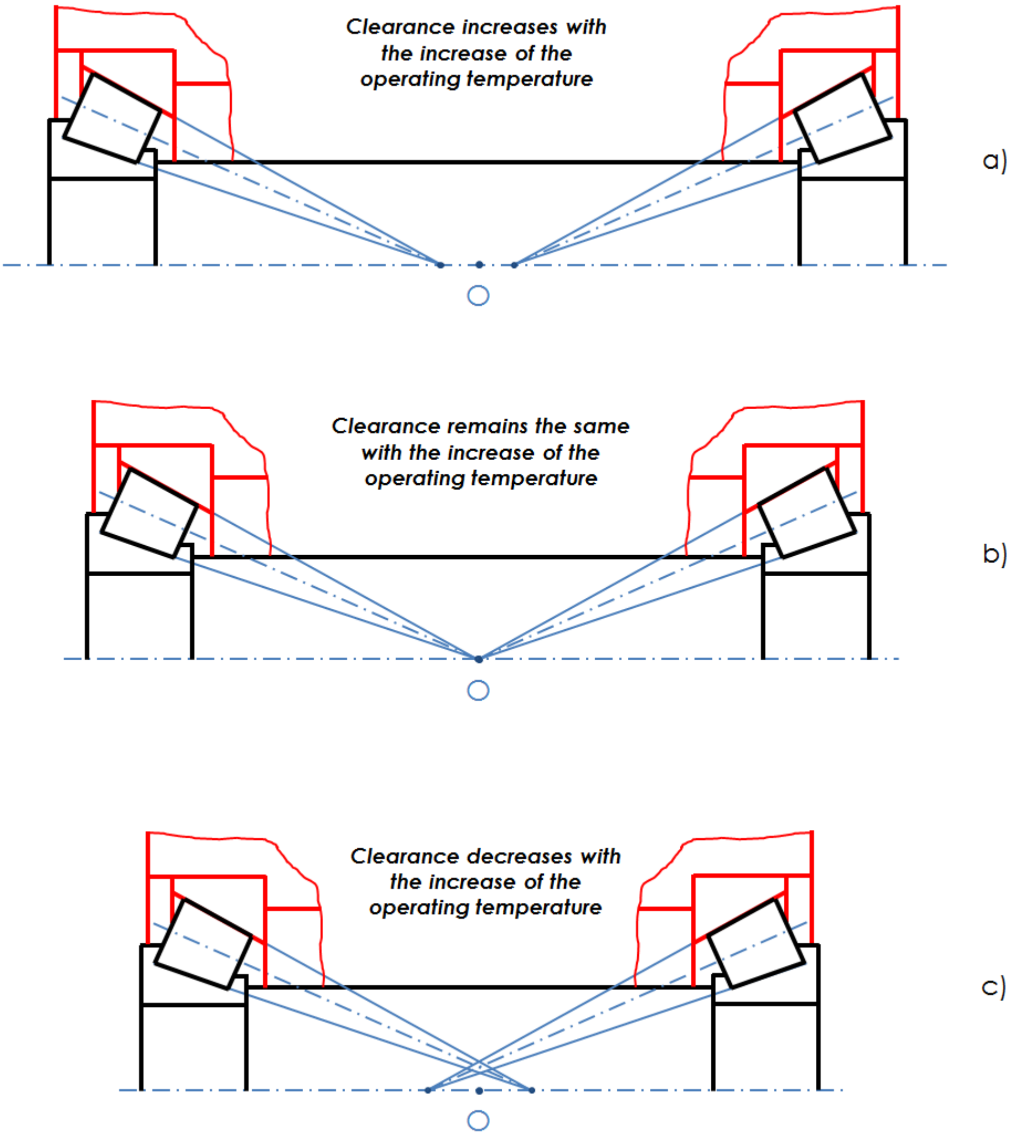

Fig. 1 Clearance variation in a two tapered roller bearings in back-to-back configuration: the roller cones a) do not meet; b) apexes coincide; c) overlap

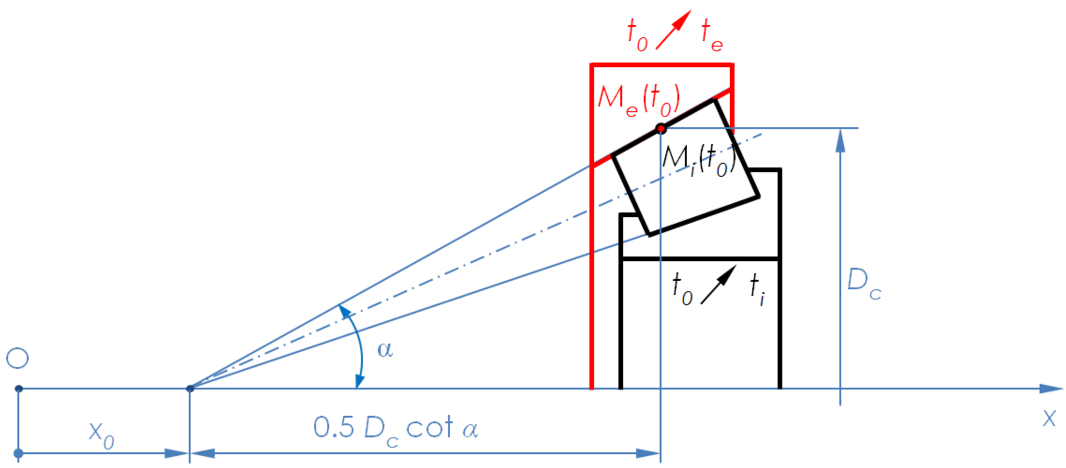

Fig. 2 Tapered roller bearing with zero mounting clearance before heating

According to many bearing manufacturers (e.g. [2], [3]) this rule of thumb is valid in the following conditions:

- shaft and housing of same isotropic material (same coefficient of thermal expansion, α_t, in both radial and axial direction);
- inner ring and shaft operate at same temperature (t_i);
- outer ring and entire housing at identical working temperature (t_e).

we will prove that the rule remains the same even if the above mentioned coefficients of thermal expansion are slight different.

3. PROOF

In Fig. 2 a tapered roller bearing with zero mounting clearance is presented. The distance between the roller cone apex and the midpoint O of the distance between the bearings seats (same as in Fig. 2 is denoted by x_0 which is positive, zero, or negative as the actual arrangement is in one of the cases presented in Fig. 1 a, b, and c, respectively. The bearing mounting temperature is t_0 and D_c is the diameter of the circle on which the contact point between roller and outer ring raceway (at the middle of the roller) is situated.

Consider a contact point situated at the middle the roller generatrix. At the mounting temperature this point is denoted by $M_i(t_0)$, if it is considered belonging to the roller, and by $M_e(t_0)$, if the same point is considered on the outer ring raceway. During the bearing operation roller temperature increases from t_0 to t_i (bearing cage, inner ring, and shaft temperatures also), and the outer ring (and housing also) temperature increases from t_0 to t_e. As result, due to thermal expansions of the parts, the points $M_i(t_0)$ and $M_e(t_0)$ reach the positions $M_i(t_i)$ and $M_e(t_e)$, respectively.

It can be proven that the roller generatrix after bearing heating is parallel to the roller generatrix at mounting temperature. This is obviously true for the outer ring raceway generatrix, also. Consider two points, $A_1(x_1, y_1)$ and $A_2(x_2, y_2)$, on the straight line d representing the roller generatrix or the outer ring raceway (Fig. 3). After the temperature increases by Δt, they move to $A_1'(x_1', y_1')$ and $A_2'(x_2', y_2')$, where:

$$x_1' = x_1 + \delta_{x_1} = x_1 + \propto_t x_1 \Delta t$$
$$y_1' = y_1 + \delta_{y_1} = y_1 + \propto_t y_1 \Delta t$$
$$x_2' = x_2 + \delta_{x_2} = x_2 + \propto_t x_2 \Delta t$$
$$y_2' = y_2 + \delta_{y_2} = y_2 + \propto_t y_2 \Delta t$$

Note that in Fig. 3 the position of the Cartesian coordinate system is the same with the position of the Cartesian coordinate system used in Fig. 2.

Now, the slope of the straight line determined by A_1' and A_2' can be calculated:

$$\frac{y_2' - y_1'}{x_2' - x_1'} = \frac{(y_2 - y_1)\cdot(1 + \propto_t \Delta t)}{(x_2 - x_1)\cdot(1 + \propto_t \Delta t)} = \frac{y_2 - y_1}{x_2 - x_1}$$

As it can be seen, the slope of the straight line determined by A_1' and A_2' equals the one of the straight line determined by A_1 and A_2. This means that, after a temperature increase by Δt, the roller generatrix (and the outer ring raceway generatrix, also) make an angle with the positive direction of the x-axis equal to the bearing nominal contact angle α.

Now we will prove that after the thermal expansion the points $M_i(t_0) = M_e(t_0)$, $M_i(t_i)$, and $M_e(t_e)$ lie on the same straight line (i.e. they are collinear). In this purpose we will calculate and compare the values of the slopes of the straight lines $M_i(t_0)M_i(t_i)$ and $M_e(t_0)\, M_e(t_e)$.

The coordinates of $M_i(t_i)$ are given by:

$$\delta x_i = \alpha_{ti} \cdot \left(x_0 + \frac{D_c}{2\tan\alpha}\right) \cdot (t_i - t_0)$$

and

$$\delta r_i = \alpha_{ti} \cdot \frac{D_c}{2} \cdot (t_i - t_0)$$

Analogously, the coordinates of $M_e(t_e)$ yield from the following equations:

$$\delta x_e = \alpha_{te} \cdot \left(x_0 + \frac{D_c}{2\tan\alpha}\right) \cdot (t_e - t_0)$$

and

$$\delta r_e = \alpha_{te} \cdot \frac{D_c}{2} \cdot (t_e - t_0)$$

where α_{ti} and α_{te} are thermal expansion coefficients of shaft (and inner ring, cage rollers and) and housing (and outer ring), respectively.

Calculating the tangent of the angle between $M_i(t_0)M_i(t_i)$ and the bearing axis the following equation is obtained:

$$\frac{\delta r_i}{\delta x_i} = \frac{\alpha_{ti} \cdot \frac{D_c}{2} \cdot (t_i - t_0)}{\alpha_{ti} \cdot \left(x_0 + \frac{D_c}{2\tan\alpha}\right) \cdot (t_i - t_0)} = \frac{\frac{D_c}{2}}{x_0 + \frac{D_c}{2\tan\alpha}}$$

Analogously, the tangent of the angle between the straight line determined by $M_e(t_0)$ and $M_e(t_e)$ and the bearing axis equals:

$$\frac{\delta r_e}{\delta x_e} = \frac{\alpha_{te} \cdot \frac{D_c}{2} \cdot (t_e - t_0)}{\alpha_{te} \cdot \left(x_0 + \frac{D_c}{2\tan\alpha}\right) \cdot (t_e - t_0)} = \frac{\frac{D_c}{2}}{x_0 + \frac{D_c}{2\tan\alpha}}$$

As it can be observed

$$\frac{\delta r_i}{\delta x_i} = \frac{\delta r_e}{\delta x_e}$$

which means that the points $M_i(t_0) = M_e(t_0)$, $M_i(t_i)$, and $M_e(t_e)$ are collinear.

Therefore we denoted

$$\frac{\delta r_i}{\delta x_i} = \frac{\delta r_e}{\delta x_e} = \tan\beta_t$$

when the line they lie on is inclined to the bearing axis by the angle β_t (Fig. 4).

Depending on the value of this angle and its relationship with the nominal contact angle α, the relative position of roller and outer ring raceway results in

- clearance, if $\alpha > \beta_t$ (Fig. 4 a);
- zero clearance, if $\alpha = \beta_t$;
- interference, if $\alpha < \beta_t$ (Fig. 4 b).

On the other hand between the two angles β_t and α (whatever the relationship between them) there exists a simple geometric relationship given by the equation:

$$\tan\alpha - \tan\beta_t = \frac{x_0 \tan\alpha}{x_0 + 0.5\, D_c \cot\alpha}$$

where:

- D_c – diameter of the circle on which the contact point between roller and outer ring raceway (at the middle of the roller) is situated, mm;
- x_0 – distance between the roller cone apex and the midpoint O of the distance between the bearings seats (same as in Fig.1), mm;
- α – nominal contact angle, rad or deg;
- β_t – inclination angle (from the bearing axis), of the straight line connecting the points $M_i(t_0) = M_e(t_0)$, $M_i(t_i)$ and $M_e(t_e)$, rad or deg.

Note that this equation is independent of the temperatures t_0, t_i, and t_e and the denominator of the fraction is always positive, even at the limit when x_0 is negative and bearings touch each other. Equation () is also independent of the thermal expansion coefficients α_{ti} and α_{te}.

From the above equation immediately results:

- $\alpha > \beta_t$, if and only if $x_0 > 0$;
- $\alpha = \beta_t$, if and only if $x_0 = 0$;
- $\alpha < \beta_t$, if $x_0 < 0$ and it do not exceed the limit when the bearings touch each other.

Hence, the relative position of roller and outer ring raceway results in

- clearance, if and only if $x_0 > 0$;
- zero clearance, if and only if $x_0 = 0$;
- interference, if $x_0 < 0$ and it do not exceed the limit when the bearings touch each other.

If the initial bearing has not zero-clearance but a certain internal clearance and if the temperature of shaft increases relative to the housing temperature all above became:

- the bearing clearance increases, if and only if $x_0 > 0$;
- the bearing clearance remains the same, if and only if $x_0 = 0$;
- the bearing clearance decreases, if $x_0 < 0$ and it do not exceed the limit when the bearings touch each other.

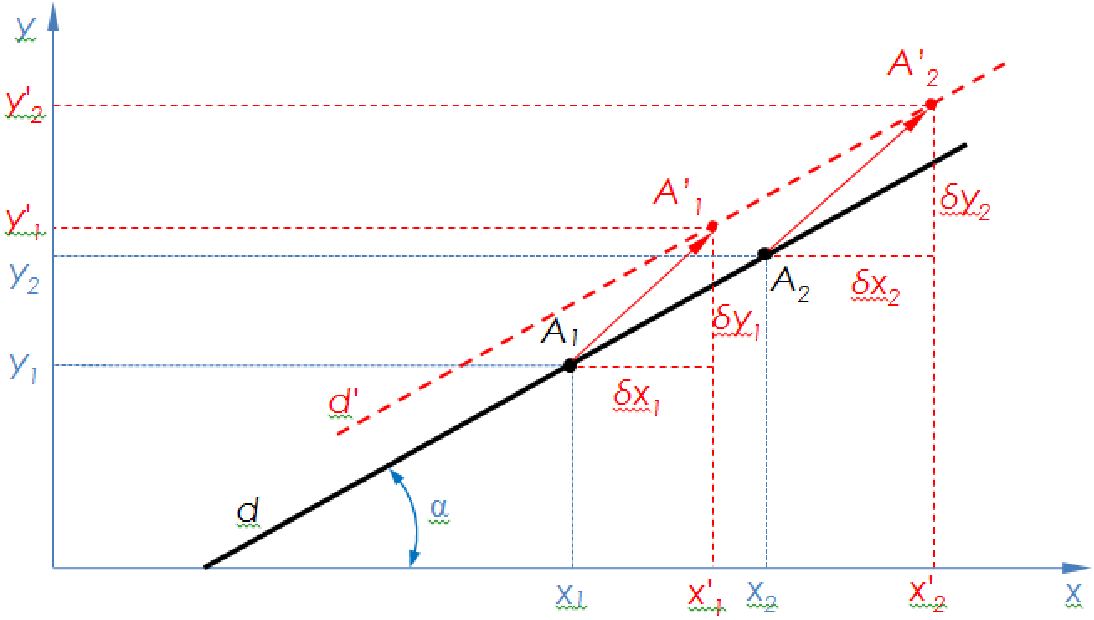

Fig. 3 Relative position of two points after thermal expansion

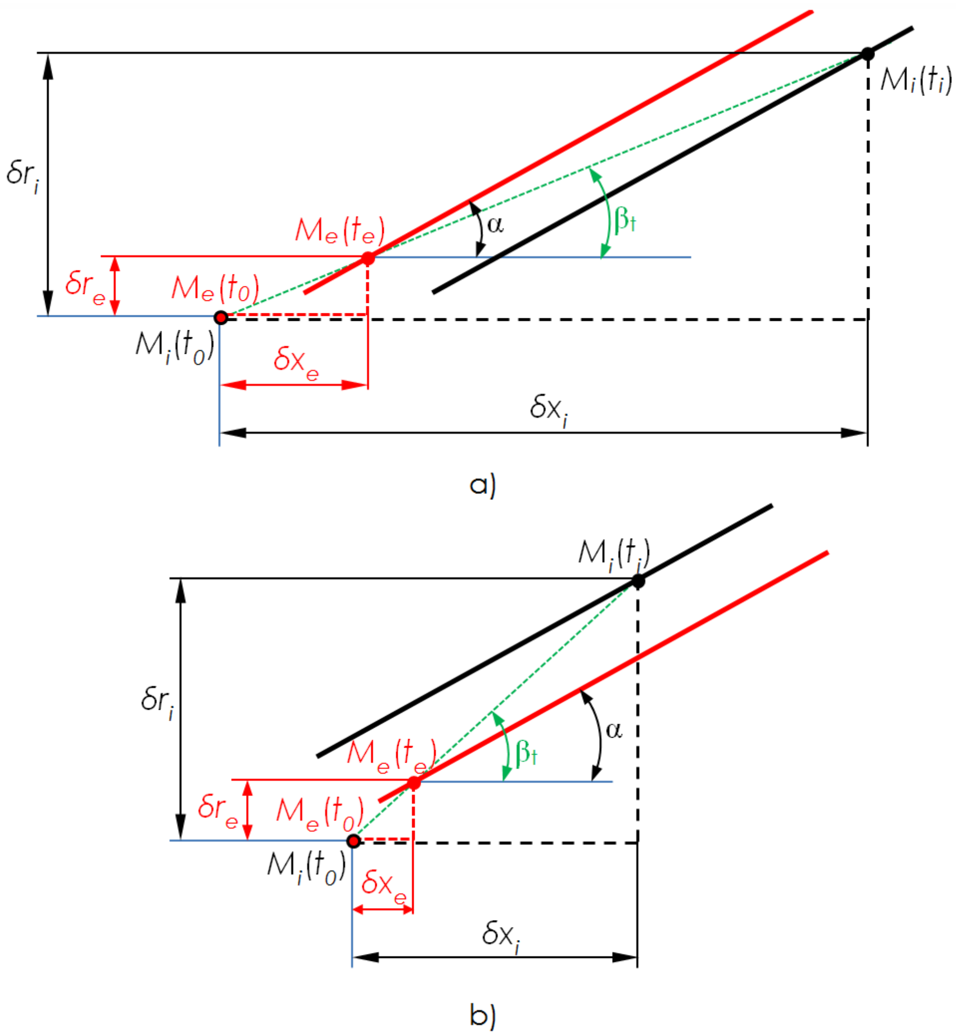

Fig. 4 Relative position of roller and outer ring raceway after bearing heating during operation:

a) clearance; b) interference

With all these, the proof of the rule of thumb regarding the bearing clearance variation in a bearing arrangement consisting of two tapered roller bearings in O-arrangement (back-to-back arrangement) according to the relative position of the roller contact cones of the two bearings (presented in Fig. 1) is complete.

4. CONCLUSIONS

Some conclusions can be immediately drawn from above proof:

- rule of thumb regarding total internal clearance variation when shaft thermal expansion occurs in two tapered roller bearing in O-arrangement (back-to-back arrangement) has solid mathematical and physical basis.
- the mentioned rule of thumb is valid even when shaft and housing have slight different thermal expansion coefficients.

The encouraging results of this work pave the way for more precise investigation of the variation of the clearance considering the fact that quite often, housing, shaft and bearing parts have thermal expansion coefficients significantly different. In addition, in real world, the temperature of the outer ring and housing are different and the shaft temperature differs from the temperatures of inner ring and rollers. It is interesting to see if the above rule of thumb will be still valid in this complex but real situation.

ACKNOWLEDGMENTS

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5. REFERENCES

[1] Pop, D., Tudose, L., Haragas, S., *Lagare cu rulmenti. Proiectare*, Todesco, Cluj-Napoca, 2006 (in Romanian).

[2] *Rolling bearings. Ball bearings. Roller bearings. Needle roller bearings. Track rollers. Bearing for screw drives. Insert bearings, housing units. Bearing housings. Accessories*, Schaeffler KG, 2008.

[3] *SKF General Catlague (Catalogue 6000/I EN – June 2008)*, SKF Group, 2008.

O NOUĂ DEMONSTRATIE A REGULII EMPIRICE PRIVIND VARIATIA JOCULUI ÎN FUNCTIE DE TEMEPRATURĂ ÎN DOI RULMENTI RADIAL-AXIALI CU ROLE CONICE MONTATI ÎN O

Rezumat: În acest articol a fost prezentată o nouă demonstratie matematică riguroasă a regulii empirice privind variatia jocului intern total atunci când intervine expansiunea termică a arborelui în doi rulmenti radial-axiali cu role conice de tip spate-în-spate. S-a dovedit de asemenea că regula mentionată este adevărată si atunci când arborele si carcasa au coeficienti de expansiune termică diferiti.

Florina RUSU, PhD student, Technical University of Cluj-Napoca, Mechanical System Engineering Department, Florina.Rusu@omt.utcluj.ro

Cristina TUDOSE, Assist. Prof. Dr. Eng., Technical University of Cluj-Napoca, Mechanical System Engineering Department, Cristina.Stanescu@omt.utcluj.ro, AC&O Dept. RKB Bearing Industries, Switzerland

Lucian TUDOSE, Prof. Dr. Eng., Technical University of Cluj-Napoca, Mechanical System Engineering Department, Lucian.Tudose@omt.utcluj.ro, Head of AC&O Dept. RKB Bearing Industries, Switzerland