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INFLUENCE OF APPROPIATE CALCULATION OF SHAFT SUPPORT REACTIONS ON BEARING LIFE PREDICTION

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Abstract: *The comparison between the results of the bearing rating life calculation obtained using the schematization of the shaft as a statically undetermined beam and these obtained using the simplified schematization of the shaft as a statically determined beam, for the same bearing arrangement, is presented in this paper. In order to calculate the real radial reactions (bearing loads), deflections, and slopes for a bearing arrangement considered as a statically undetermined beam, the original software belonging to RKB Bearing Industries was used.*
Key words: *bearing rating life, statically determined beam, statically undetermined beam*

1. INTRODUCTION

The prediction accuracy of rating life depends on the calculation accuracy of the loads on the bearings. It is a current practice to consider the shaft on which the bearings are mounted as a statically loaded beam with the external loads originated from the machine elements mounted on the shaft.

Obviously, the supports of this beam are the bearings mounted on the shaft. The problem of support reactions calculation is elementary when the shaft is supported by only two bearings because the beam is statically determined. Solutions for this problem can be found in any machine elements book.

Whenever the shaft is supported on more than two bearings, the problem becomes more complicated because the beam becomes statically undetermined. In this case, the beam deflections have to be taken into consideration. This complicates the problem when the beam (i.e. the shaft) has sections with variable dimensions and different stiffness. The more these sections are, the harder the problem becomes to solve. Traditional approaches that can also be found in technical books are hard to implement for mechanical application shafts precisely because of the shaft geometric complexity. An instrument that proved to be useful in structures analysis from civil engineering is *slope and deflection method* ([1], [3], [4], [5]). Even though it was published more than a century ago [6], this method has found its utility lately. The slope-deflection method focuses on the unknown displacements and rotations of the structure rather than unknown forces. In this method, also referred as *displacement method*, the individual equations are relatively easy to be obtained regardless of the number of unknowns.

An important characteristic of the slope-deflection method is that the higher the degree of indeterminacy the more advantageous the slope-deflection method becomes. Based on this method the RKB Technical Unit has developed an original software in order to calculate the radial reactions in the supports of statically (un)determined shafts, moment reactions, shaft deflections, and slopes.

This paper goal is to prove how much reaction calculations influences bearing life rating, at least in the real case presented below.

2. PROBLEM DESCRIPTION

Fig. 1 is a sketch representation of a cross-section through a bevel pinion shaft supported by a bearing arrangement consisting of two tapered roller bearings in back-to-back arrangement (as locating bearing) and a cylindrical roller bearing (as non-locating bearing). On the shaft, between the bearing units, is mounted a spur gear also.

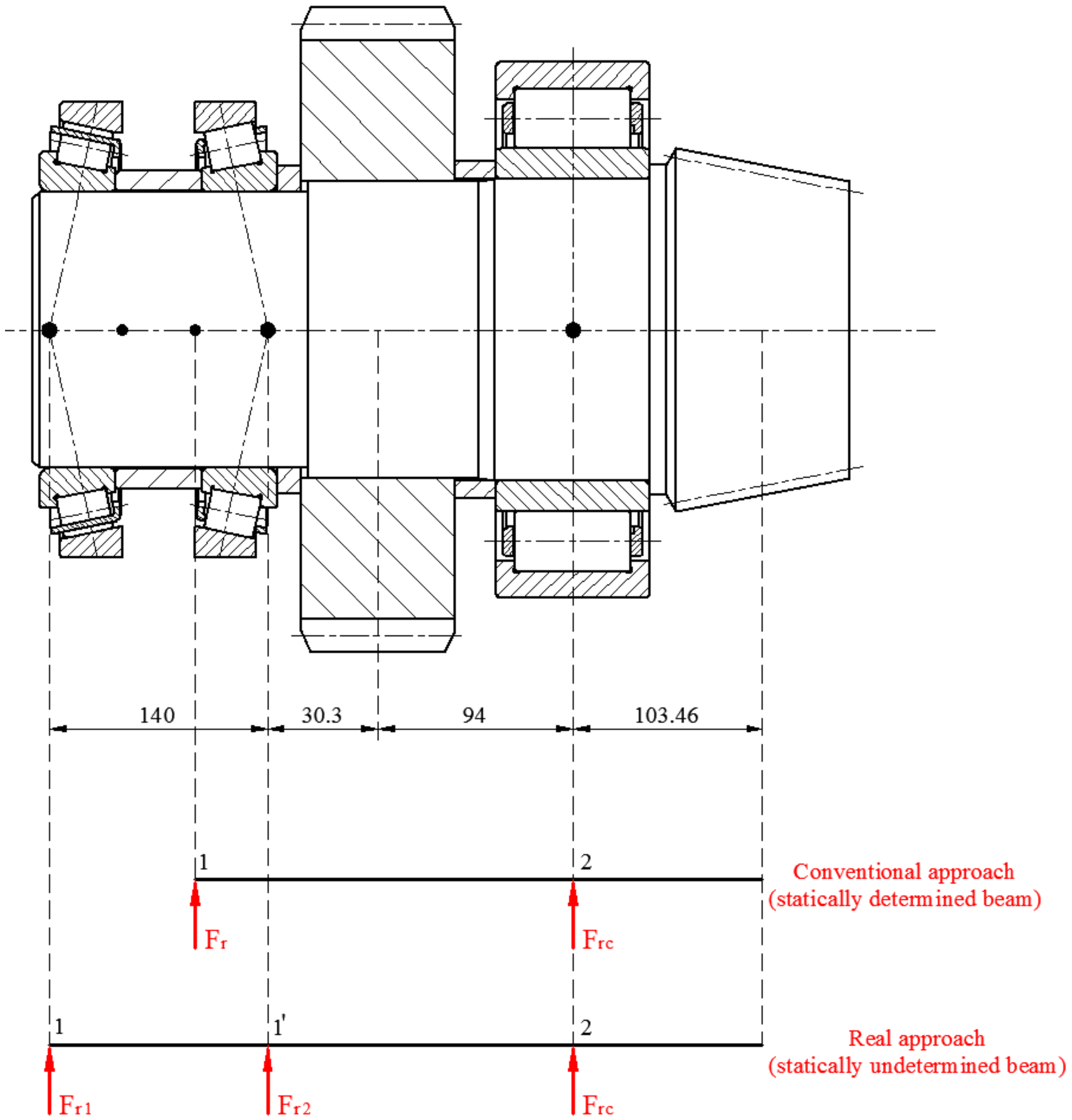

Fig. 1 Shaft geometry and schematizations used for reaction calculation

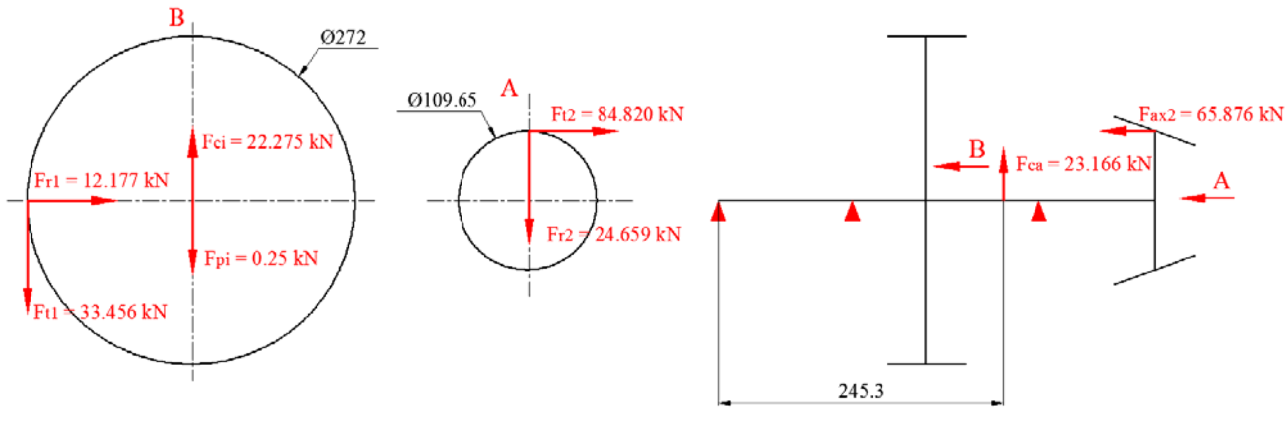

Fig. 2 Loads acting on the shaft

SHAFT CALCULATION

Project:

Input data

Material data

Modulus of elasticity *E*, MPa 210000

Radial and axial loads, and bending moments

Plane	Node	1	2	3	4	5	6	7	8	9	10	11	12
H	Radial load [kN]				12.177								84.820
	Axial load [kN]												
	Moment [kN mm]												
V	Radial load [kN]				-11.431		23.116						-24.659
	Axial load [kN]												-65.876
	Moment [kN mm]												-3611.7
	Support type	A		A				A					

Shaft geometry

Segment	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Length [mm]	136.8	3.2	30.3	42	33	19	45	10.42	19.2	19.2	9.64
Outer diameter [mm]	95	108	108	108	110	110	110	130	125.92	117.76	109.65
Inner diameter [mm]	0	0	0	0	0	0	0	0	0	0	0

Fig. 3 RKB software – Input data

Output data

Radial reactions, moment reactions, deflections, and slopes

Plane	Node	1	2	3	4	5	6	7	8	9	10	11	12
H	Reaction [kN]	-10.18	-	83.04	-	-	-	-169.85	-	-	-	-	-
	Moment [kN mm]	-	-	-	-	-	-	-	-	0.0	0.0	0.0	-
	Deflection [10^{-3} mm]	-	0.2	-	-2.6	-4.8	-3.2	-	14.9	19.4	28.1	37.2	41.8
	Slope [10^{-6} -]	39.6	-73.9	-77.1	-84.0	-7.1	118.7	219.1	423.8	439.8	463.9	479.7	482.4
V	Reaction [kN]	-1.35	-	16.52	-	-	-	-2.19	-	-	-	-	-
	Moment [kN mm]	-	-	-	-	-	-	-	-	-	-	-	-
	Deflection [10^{-3} mm]	-	0.0	-	-0.3	-0.5	-0.3	-	1.9	2.7	4.4	6.6	7.9
	Slope [10^{-6} -]	5.3	-9.8	-10.2	-9.4	1.1	11.7	21.9	70.0	78.2	97.9	128.3	150.9
Res	Reaction [kN]	10.270	-	84.664	-	-	-	169.868	-	-	-	-	-
	Moment [kN mm]	-	-	-	-	-	-	-	-	0.0	0.0	0.0	-
	Deflection [10^{-3} mm]	-	0.2	-	2.6	4.9	3.2	-	15.0	19.6	28.4	37.7	42.6
	Slope [10^{-6} -]	39.9	-74.5	-77.8	-84.6	-7.2	119.3	220.1	429.6	446.7	474.2	496.6	505.4
	Support type	A		A				A					

Axial forces

Plane	Axial force [kN]
H	0
V	-65.876
Res	-65.876

Fig. 4 RKB software – Output data

The main problem that occurs in this real world application is that the bearings did not reach the forecasted life and the left tapered roller bearing was destroyed very quickly.

The loads acting on the shaft of this application are presented in Fig. 2.

3. RESULTS

In not very important applications, it is acceptable to simplify the problem and to transform the real statically indeterminate beam in a statically determinate one: the pair of supports 1 and 1' are substituted artificially by a single support situated at the third of the distance between the supports 1 and 1' and closest to the bevel gear load (Fig. 1). The reaction in this support, obtained after a simple calculation, will be considered the radial load on the "double-row tapered roller bearing" (in fact the pair of the two tapered roller bearings). The calculation of the bearing loads is very simple but the results could be far from their actual values.

Following this widely used simplification and recommended by the most of the bearing manufacturer catalogues and handbooks, after calculation, a dynamic equivalent radial load of 110.08 kN (Table 1) should be considered in the life calculation of the "double-row tapered roller bearing".

Table 1

Loads on the "double-row tapered roller bearing"

F_r [kN]	F_a [kN]	P_r [kN]
45.72	65.88	110.08

But, in fact, the real supporting of the shaft (Fig. 1) consists in three supports, the shaft has to be considered as a statically undetermined beam and the calculation of the real reactions in supports (i.e. the real forces that load the bearings) is difficult. The new RKB software (Fig. 3 and Fig. 4) dedicated to the calculation of the reactions in the supports of statically undetermined beams, based on the slope deflection method, was used. Running the software the real loads acting on the two tapered roller bearings are obtained. The results are presented Table 2.

Table 2

Loads in the two tapered roller bearings

F_{r1} [kN]	F_{r2} [kN]	F_{a1} [kN]	F_{a2} [kN]	P_{r1} [kN]	P_{r2} [kN]
10.27	84.66	7.13	73.01	10.27	89.80

As one can easily observe the most loaded bearing is the right one, which experiences a dynamic equivalent radial load of 89.80 kN, about 9 times higher than the one of the left bearing. If the dynamic load rating of one of the two identical tapered roller bearings is denoted by *C*, the basic rating life of the paired bearings (calculated as for the double-row roller bearings) is given by the following equation :

$$L_{10\_paired\_bearings} = \left(\frac{2^{7/9} \cdot C}{P_{r\_paired\_bearings}} \right)^{10/3}$$

and the basic rating life of the most loaded bearing (right one) is obtained from the equation:

$$L_{10\_most\_loaded\_bearing} =$$

$$= \left(\frac{C}{P_{r\_most\_loaded\_bearing}} \right)^{10/3}$$

The ratio between the two above basic rating lives is:

$$\frac{L_{10\_most\_loaded\_bearing}}{L_{10\_paired\_bearings}} =$$

$$= \left(\frac{1}{2^{7/9}} \cdot \frac{P_{r\_paired\_bearings}}{P_{r\_most\_loaded\_bearing}} \right)^{10/3} =$$

$$= \left(\frac{1}{1.714} \cdot \frac{110.08}{89.80} \right)^{10/3} = 0.326$$

4. CONCLUSIONS

Some conclusions can be immediately drawn from the above calculation:

- Considering the life of the paired tapered roller bearings as the time until one of the two bearings (the most loaded one) will collapse, the expected life of the tapered roller bearings calculated according to the real supporting situation of the shaft is about 3 times less than that calculated following the common simplification. This ratio for the corresponded modified rating lives will be almost the same.
- The left tapered roller bearing is about 9 times less loaded than the right one and the level of the axial load is low. This fact leads to the conclusion that without an appropriate preload it will be rapidly damaged.
- It is obvious that the real schematization of the shaft supporting system will lead to the correct values of the bearing loads. The damage on the left tapered roller bearing is more pregnant for back-to-back arrangement than for the face to face arrangement, due to the considerable reduced distance between the pressure centers of the two bearings.
- The new RKB software provides the values of node slopes and deformations. This means that the values of the angles of rotation of the shaft in supports (due to the loads and shaft elastic deformations) are known and can be used in bearing live calculation.

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INFLUENTA CALCULĂRII CORESPUNZĂTOARE A REACTIUNILOR ÎN REAZĂMELE ARBORELUI ASUPRA PREZICERII DURABILITĂTII RULMENTILOR

Rezumat: În această lucrare este prezentată comparatia dintre rezultatele calculării durabilitătii unui rulment obtinut schematizând un arbore ca o grindă static nedeterminată si rezultatele obtinute schematizând arborele ca o grindă static determinată, pentru acelasi aranjament de rulmenti. Pentru calculul fortelor radiale reale si al deformărilor pentru un aranjament de rulmenti considerat a fi o grindă static nedeterminată s-a folosit un soft original apartinând de RKB Bearing Industries.

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