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ASPECTS ON THE DYNAMIC ISOLATION FOR BUILDINGS USING PENDULUM SYSTEMS

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Abstract: *This paper briefly presents an analytical research regarding to the complex influences of passive elastic elements about changing dynamic characteristics of the vibration absorbers using pendulum devices. In the paper is developed a system model which contains a base subsystem that must be isolated and an additional subsystem of a dynamic restricted pendulum absorber type. The studied case considers elastic elements and disregard damping elements. The author has proposed a set of interest parameters and for these parameters variation it was made an analyze of those behaviour on the overall system.*
Keywords: *pendulum device, vibration isolation, dynamic restricted pendulum absorber.*

1. INTRODUCTION

The anti-vibratory isolation study shows which are the constructive measures through that can achieve the followed purpose. Also, this study establishes the parameters through that can be measured the insulation efficiency, avoids the wrong solutions and guides the designer towards the most economic methods into solving the problem [1], [3], [10], [11].

Currently are known and widely used two methods for anti-seismic isolation and protection against vibrations [12], [13]:

- the base isolation using elements with elastic and dissipative characteristics fitted between the structure and the foundation such that the movement components, which can be vibrations or seismic waves, to be transmitted in a lesser degree from the foundation to the insulated structure. The method is generally used for rigid structures or compact structures that not allow additional structural elements;
- the change of the dynamic characteristics of the isolated structure when acts disturbing actions such as dynamic vibrations or seismic waves, through the installation of additional dissipative elastic structural elements [6], [7]. This method is mainly used for elastic structures that allow the installation of additional structural elements.

The process that changes the dynamic characteristics of the structure "benefits" of the following types of implementations:

❖ the use of elastic bearing elements

❖ the use of dissipative elements or systems

❖ the use of composed elements / devices / systems with elastic and dissipative characteristics [8], [9].

The considered and analyzed system from this work falls into the category of dynamic characteristics changes of the isolated structure through the use of some elements with a predominant elastic feature [2], [4], [5].

It had to be mentioned that this study takes part from a large research concerning the analysis, optimization and improvement of dynamic vibration absorbers in order to obtain two major objectives as follows:

- reducing structural complexity in terms of geometrical dimensions, respectively ensemble weight
- performant controlling of dynamic characteristics in order to obtain the widening of the dynamic isolation domain [14], [15].

2. THEORETICAL APPROACHES

Physical and mathematical model developed in this paper is a system model with two parts: a rigid body of a trolley type which has a horizontally translational moving and an dynamic pendulum absorber, hinged in the gravity center of the subsystem to be isolated.

The mass of the body which is in translational moving was denoted by m_1 and his displacement into horizontal direction was denoted by x. The pendulum absorber consists of an additional mass that contains a bar whose mass was noted with m_2 and the length with l and a concentrated mass, which is on the end of the bar, denoted by m_3. The angle made by the lever 2 with the vertical direction was noted with θ, the bar executes a plan parallel movement. The considered system has two degrees of freedom (a OX axis translational and a OZ axis rotational, denoted by θ). For the mathematical modeling of the differential equations that define the system motion, were taken into account all three masses which form the system.

The studied case considers elastic elements and disregard damping elements. Given that the mass m_1 simulates the behavior of a structure type system, it can be considered that it is double restricted (on both sides). In this regard, the elastic stiffness of the two elements were noted with k_1, respectively k_2. The restriction of the pendulum movement of the absorber was simulated through the insertion of a torsion elastic element in the pin point, denoted in Fig. 1 by k_3.

For the mathematical modeling of the motion equations of the system were noted with C_2, respectively C_3, the center of gravity of the 2 body (the bar) and 3 body (the concentrated mass). A computational hypothesis, which was the basis for the modelling of the system equations, says that it is used a plane motion and this workplane is XOY, as shown in Figure 1.

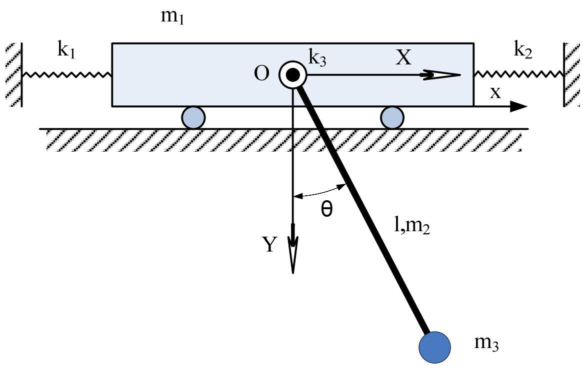

Figure 1. System with two degrees of freedom consists of a pendulum dynamic absorber and a body in translational movement

$$\begin{cases} x_{C_2} = x + \dfrac{l}{2}\sin\theta \\ y_{C_2} = \dfrac{l}{2}\cos\theta \end{cases} \tag{1}$$

$$\begin{cases} x_{C_3} = x + l\sin\theta \\ y_{C_3} = l\cos\theta \end{cases} \tag{2}$$

$$E = E_1 + E_2 + E_3 \tag{3}$$

$$E = \frac{1}{2}(m_1 + m_2 + m_3)\dot{x}^2 + + \frac{1}{2}l^2\left(\frac{m_2}{3} + m_3\right)\dot{\theta}^2 + l\left(\frac{m_2}{2} + m_3\right)\dot{x}\dot{\theta} \tag{4}$$

$$V = V_1 + V_2 + V_3 + V_{k_1} + V_{k_2} + V_{k_3} \tag{5}$$

$$V = \frac{1}{2}(k_1 + k_2)x^2 + + \frac{1}{2}\left(\frac{1}{2}m_2gl + m_3gl + k_3\right)\theta^2 \tag{6}$$

The differential equation system in this case is written as follows:

$$\begin{cases} (m_1 + m_2 + m_3)\ddot{x} + l\left(\dfrac{m_2}{2} + m_3\right)\ddot{\theta} + \\ + (k_1 + k_2)x = 0 \\ l\left(\dfrac{m_2}{2} + m_3\right)\ddot{x} + l^2\left(\dfrac{m_2}{3} + m_3\right)\ddot{\theta} + \\ + \left(\dfrac{1}{2}m_2gl + m_3gl + k_3\right)\theta = 0 \end{cases} \tag{7}$$

In the (7) expression were made the following notations:

$$\begin{cases} C_1 = m_1 + m_2 + m_3 \\ C_2 = \dfrac{m_2}{2} + m_3 \\ C_3 = k_1 + k_2 \\ C_4 = \dfrac{m_2}{3} + m_3 \\ C_5 = \dfrac{1}{2} m_2 gl + m_3 gl + k_3 \end{cases} \tag{8}$$

The system solutions are written in the following form:

$$\begin{cases} x = A \sin pt \\ \theta = B \sin pt \end{cases} \tag{9}$$

The differential equation system becomes:

$$\begin{cases} \left(-p^2 C_1 + C_3\right)A - p^2 l C_2 B = 0 \\ -p^2 l C_2 A + \left(-p^2 l^2 C_4 + C_5\right)B = 0 \end{cases} \tag{10}$$

To obtain unique solution must be satisfied the condition:

$$\left(-p^2 C_1 + C_3\right)\left(-p^2 l^2 C_4 + C_5\right) - \\ - p^4 l^2 C_2^2 = 0 \tag{11}$$

After solving the equation we obtain the natural pulsations:

$$p_1 = \sqrt{\frac{\left(C_1 C_5 + l^2 C_3 C_4\right) + \\ + \sqrt{\left(C_1 C_5 + l^2 C_3 C_4\right)^2 - \\ - 4 C_3 C_5 \left(C_1 C_4 l^2 - l^2 C_2^2\right)}}{2\left(C_1 C_4 l^2 - l^2 C_2^2\right)}} \tag{12}$$

$$p_2 = \sqrt{\frac{\left(C_1 C_5 + l^2 C_3 C_4\right) - \\ - \sqrt{\left(C_1 C_5 + l^2 C_3 C_4\right)^2 - \\ - 4 C_3 C_5 \left(C_1 C_4 l^2 - l^2 C_2^2\right)}}{2\left(C_1 C_4 l^2 - l^2 C_2^2\right)}} \tag{13}$$

Besides the natural pulsations calculation, for the proposed system is interesting to obtain the amplitudes corresponding of the two degrees of freedom. In this regard it is considered that on the system acts a harmonic excitation whose amplitude is F_0. Given these considerations we obtain the amplitudes:

$$A = \frac{-p^2 l^2 C_4 + C_5}{p^4 l^2 \left(C_1 C_4 - C_2^2\right) - \\ - p^2 \left(l^2 C_3 C_4 + C_1 C_5\right) + C_3 C_5} F_0 \tag{14}$$

$$B = \frac{p^2 l C_2}{p^4 l^2 \left(C_1 C_4 - C_2^2\right) - \\ - p^2 \left(l^2 C_3 C_4 + C_1 C_5\right) + C_3 C_5} F_0 \tag{15}$$

3. BEHAVIOUR ANALYSIS OF THE RESTRICTED PENDULUM DYNAMIC ABSORBER

For the previously adopted system are proposed to be considered as interest parameters the length l of the bar which is part of the pendulum absorber, the concentrated mass m_3, located on the end of the bar (and she is a component of the subsystem named absorber) and the elastic torsion element located in the pinned point, whose stiffness was denoted by k_3. Given this, it was proposed to analyze the effect of the variation of these parameters on the overall system behavior. The proposed analysis was made considering the reference pulsation denoted by p_{ref} and the reference length, denoted by l_{ref}.

For the evaluation of the l_{ref} parameter, it will be analyzed the expression of the A amplitude numerator. Through the cancelling of this numerator it is obtained the pulsation value for which the amplitude becomes zero. The mathematical expression that highlights the above assertions is:

$$-p^2 l^2 C_4 + C_5 = 0 \tag{16}$$

where C_4 and C_5 are the notations previously used.

Giving a physical interpretation of the mathematical calculation above, we can say that the subsystem which corresponds to this degree of freedom, ie the mass m_1 that needs to be isolated, it remains at rest (just as long as the above mentioned condition remains valid).

With the condition above, we have:

$$p^2 = \frac{C_5}{l^2 C_4} \tag{17}$$

Substituting C_4 and C_5 with their expressions, we obtain:

$$p^2 = \frac{\frac{1}{2} m_2 gl + m_3 gl + k_3}{l^2 \left(\frac{m_2}{3} + m_3 \right)} \tag{18}$$

or:

$$p^2 = \frac{\frac{1}{2} m_2 gl + m_3 gl}{l^2 \left(\frac{m_2}{3} + m_3 \right)} + \frac{k_3}{l^2 \left(\frac{m_2}{3} + m_3 \right)} \tag{19}$$

Analyzing the obtained expression is observed that the pulsation has two terms: a base term (reference), corresponding to the unrestricted pendulum system (through m_2, m_3, l parameters) and an adjustment term, corresponding to the restriction (through the elastic torsion element k_3).

To assess the reference parameters originally proposed to be monitored, it uses the following assumption: it is considered zero the additional effect of the adjustment ensemble. In a physical sense, this means to neglect the effect given by the elastic characteristic or, mathematically, annulment of the k_3 parameter. If there are no elastic element in the pinned point ($k_3 = 0$), then, between the two subsystems remains only a mechanical pinned point, without restrictions (rotation kinematic $C5$ coupling).

In these conditions, in the pulsation expression appear the two parameters, p_{ref} and l_{ref}, and the expression becomes:

$$p_{ref}^2 = \frac{1}{l_{ref}} \frac{g\left(\frac{1}{2} m_2 + m_3 \right)}{\frac{m_2}{3} + m_3} \tag{20}$$

or, after reordering the terms:

$$l_{ref} = \frac{1}{p_{ref}^2} \frac{g\left(\frac{1}{2} m_2 + m_3 \right)}{\frac{m_2}{3} + m_3} \tag{21}$$

The evaluation of two reference parameters involved in the above expression requires a priori knowledge of one parameter and the determination of the other one. Given the practical application that led to getting this mathematical expression, can be considered as a minimum abstraction level the initial system that requires dynamic isolation against vibration action. Thus, the reference pulsation is given by the pulsation of a system with one degree of freedom consisting of mass m_1 and two elastic elements k_1, respectively k_2. In these conditions the expression of the reference pulsation is:

$$p_{ref}^2 = \frac{k_1 + k_2}{m_1} \tag{22}$$

Another interpretation of the assumptions stated above is due to the fact that the reference parameters are those essential parameters which correspond to the initial system (or base system), no additional components or devices that allow additional functionalities compared to the base system. In other words, the reference pulsation is the pulsation of the system which must to be isolated and the reference length is the length of the classic pendulum system, unrestricted. Regarding the other two interest parameters imposed in this study from the beginning - the reference stiffness and the reference mass - it mentions that they correspond to the auxiliary pendulum subsystem, which is an additionally ensemble to the base system. Both m_3, and k_3 parameters representing reference stiffness, respectively reference mass, will be analyzed as a percentage variations from the values of the corresponding base system parameters m_1, respectively k_1.

To analyze the pulsation evolution depending on the three parameters previously proposed - l, m_3, k_3 - was used the graphic representation in relative coordinates, each evaluated parameter being reported to the corresponding reference values. For Figures 2...4 the notations significance from the legend is: p_1, p_2 are the natural pulsations of the system, p_a is the annulment pulsation.

4. CONCLUSIONS

The evolution of the relative pulsation in relation with the normalized length (see Figure 2) highlights the widening of the tuning spectral band by reducing the pendulum length in range $(10...100)\%$ relative to the reference length.

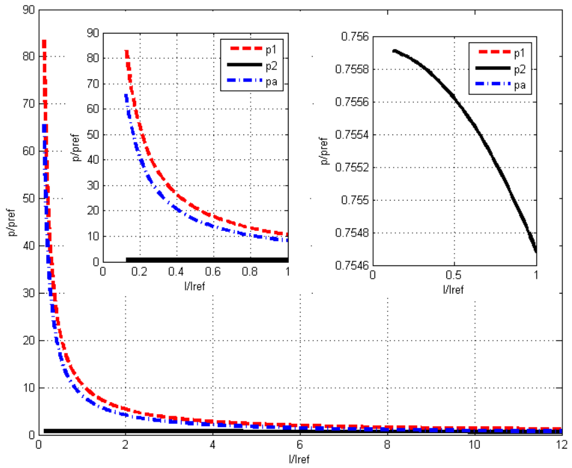

Figure 2. Evolution of relative pulsations depending on relative length

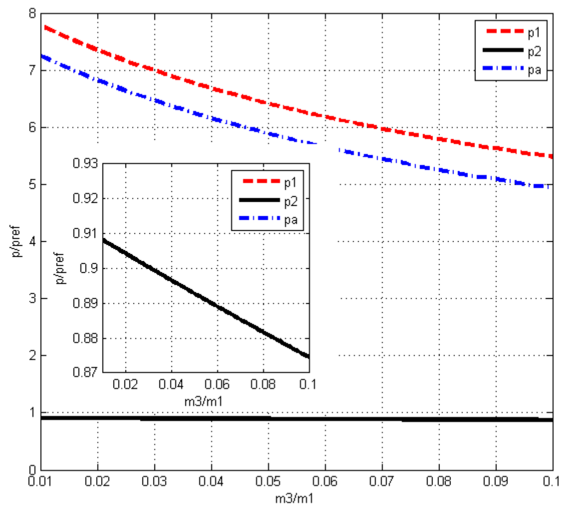

Figure 3. Evolution of relative pulsations depending on relative mass

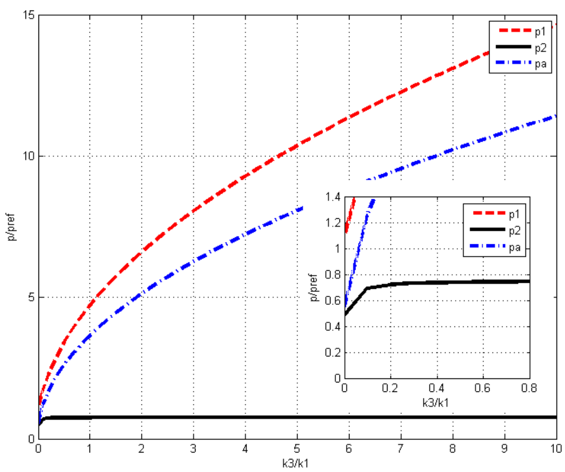

Figure 4. Evolution of relative pulsations depending on relative stiffness

The widening of the tuning spectral band (TSB) is more evident in the case of the relative pulsation variation versus the pendulum mass (see Fig. 3), respectively versus the functional restriction of the pendulum system (in this case the stiffness k_3 - see Fig. 4). Thus, the reducing of the pendulum mass implies a pronounced increase of TSB, significant and useful values are registered for the normalized mass in the $0.02 - 0.03$ domain. Regarding the evolution of the relative pulsation versus the normalized stiffness is seen from Fig. 4 that, with the occurrence and increase of the functional restriction - elastic type appears a pronounced widening of TSB.

It can observe that, generally, the p_2 minimal pulsation variations get values below 1% for all three analyzed cases. In the same time, the p_1 maximal pulsation has considerable variations whose maximum value varies depending on the studied parameter.

Taking into account the previous conclusive remarks results two main conclusions of this analysis as follows:

- ❖ the whole set of parameters initially considered assure the TSB widening, which means the fulfilling of one objective of this research;
- ❖ the introduction of the functional restriction - elastic type provides the means and the additional conditions for a continuous tuning with the excitation signal in terms of spectral composition.

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ASPECTE ASUPRA IZOLĂRII DINAMICE A CLĂDIRILOR FOLOSIND SISTEME PENDULARE

***Rezumat**: Lucrarea prezinta pe scurt un studiu analitic in privinta influentelor complexe ale elementelor pasive elastice asupra schimbarii caracteristicilor dinamice ale absorbitorilor de vibratii folosind dispozitive pendulare. În lucrare este dezvoltat un model de sistem care conține un subsistem de bază care trebuie izolat şi un sistem adițional de tip absorbitor dinamic pendular restricționat. Cazul studiat ia în considerare elementele elastice şi nu ia în considerare pe cele de amortizare. Autorul a propus un set de parametri de interes şi pentru variația acestor parametri a fost făcută o analiză a comportamentului acestora asupra întregului sistem.*

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