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THE POWER LOST BY FRICTION, BETWEEN THE TEETH FLANKS, FOR CYLINDRICAL SPUR GEARS, AT THE POINTS WHERE THE MESHING STARS AND ENDS

Tiberiu Alexandru ANTAL

***Abstract**: The paper presents the computation of the power lost by friction, between the teeth's flanks, during the meshing of the teeth. Several methods for determining the specific addendum modifications at cylindrical spur gears can be found in the technical literature however these don't consider the friction coefficient. If equalization of the power lost by friction succeeds, at the points where the meshing starts and ends, a new criterion can be found to determine the specific addendum modification taking into account the friction coefficient between teeth flanks.*

***Key words:** Power lost by friction, Friction coefficient, Specific addendum modification, Power equalization.*

1. INTRODUCTION

When determining the geometric dimensions of cylindrical spur gears, most often, we use the specific addendum modification. Their purpose may be to obtain a well defined distance between the axes at the same time ensuring the correct meshing of teeth flanks over a longer time. The x_1 and x_2 specific addendum modification can be chosen according different recommendations from the technical literature [1] – [5]. This paper describes a new method for choosing the specific addendum modifications x_1 and x_2 which takes into account the friction coefficient μ between the meshing teeth flanks.

2. POWER LOSSES BY FRICTION BETWEEN THE TOOTH FLANKS

It is known that at the meshing of the involute tooth profiles, during the transmission of movements and loads, there are slides on the meshing line and therefore frictions too. Because of the sliding the teeth wear especially at the head and base. The wearing can be controlled if the specific addendum modification x_1 and x_2 are computed by taking into account the μ friction coefficient as follows. The power lost by friction, between the teeth flanks, can be written in the general case as:

$$P_{vz} = \mu_z F_n V_g \,. \qquad (1)$$

where μ_z is the friction coefficient between the teeth flanks, F_n the normal force between the teeth and V_g the sliding velocity of the profiles (teeth flanks).

As seen in Figure 1 the power lost by friction varies along the segment AE gear. For the A point, where the tooth meshing begins, the power lost by friction can be determined by the following relationship:

$$P_{vzA} = \mu_A F_{nA} V_{gA} \,. \qquad (2)$$

where μ_A is the friction coefficient between the teeth flanks at the A point, F_{nA} the normal force between the teeth at the A point and V_{gA} the sliding velocity of the profiles at point A.

The normal force between the teeth, from the A point, is given by the relationship:

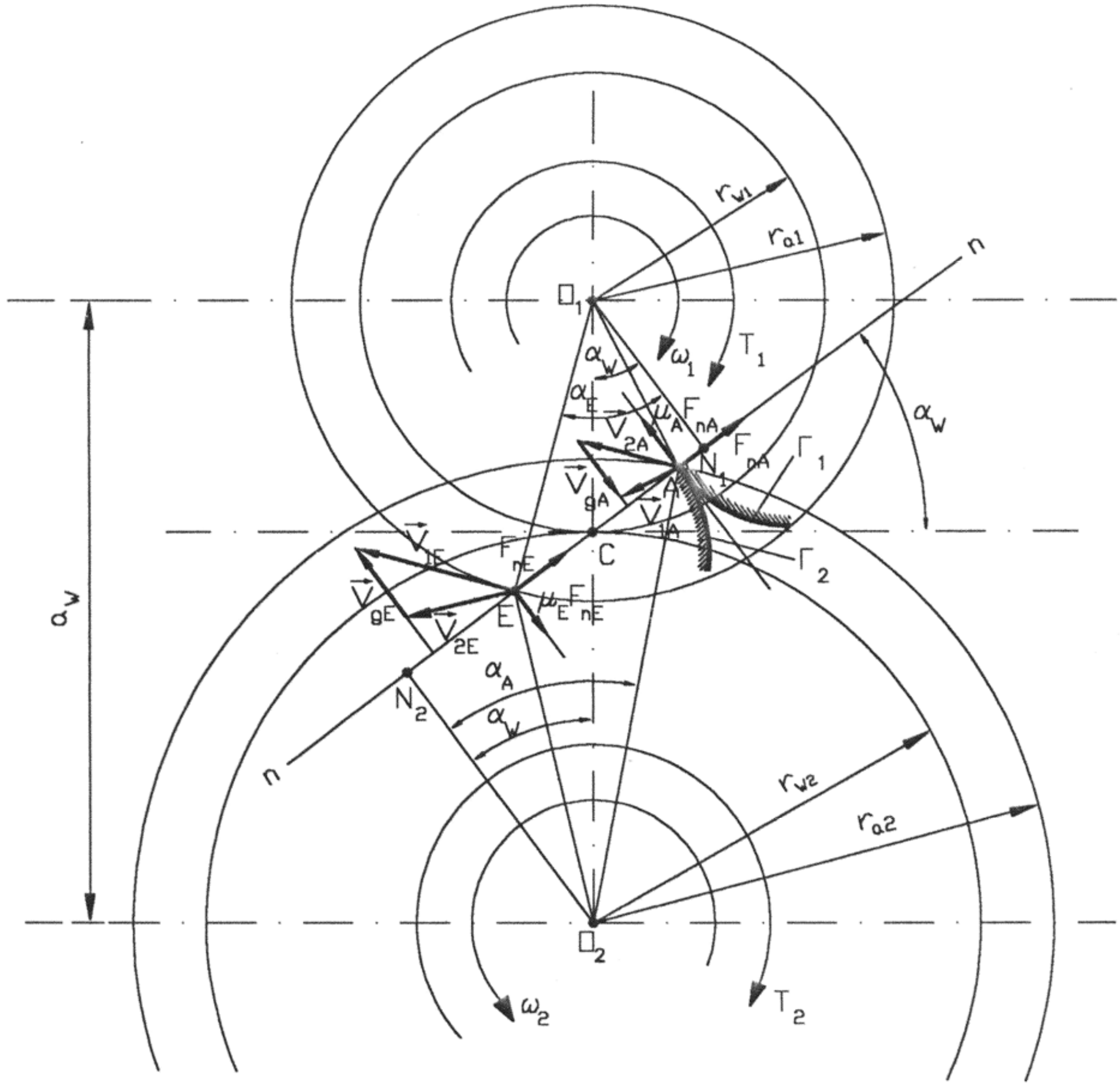

Fig. 1. Teeth meshing during the mechanical power transmission.

$$F_{nA} = \frac{T_1}{r_{b1} - \mu_A \rho_{1A}}. \tag{3}$$

where T_1 is the torque on the driving wheel, r_{b1} the base circle radius of the driving wheel, ρ_{1A} the radius of curvature of the Γ_1 involute profile at point A.

Taking Figure 1 into account the followings can be written:

$$\begin{aligned}\rho_{1A} &= AN_1 = r_{b1}\tan(\alpha_w) - AC = \\ &= r_{b1}\tan(\alpha_w) - e_A\end{aligned}. \tag{4}$$

and

$$V_{gA} = AC(\omega_1 + \omega_2) = e_A \omega_1 (1 + i_{21}). \tag{5}$$

Replacing relations (3), (4) and (5) in relation (2) the following expression is obtained:

$$P_{vzA} = \mu_A \, P_1 \, \frac{e_A (1 + i_{21})}{r_{b1}(1 - \mu_A \tan(\alpha_w)) + \mu_A \, e_A}. \tag{6}$$

where $P_1 = T_1 \, \omega_1$ is the power on the driving wheel, α_w the meshing angle and $i_{21}=\omega_2/\omega_1$ the gear ratio.

The power lost by friction at point E, where the meshing ends, is determined by the expression:

$$P_{vzE} = \mu_E F_{nE} V_{gE} . \quad (7)$$

where μ_E is the friction coefficient between the teeth flanks at the E point F_{nE} the normal force between the teeth at the E point and V_{gE} the sliding velocity of the profiles at point E

The normal force between the teeth, from the A point, is given by the relationship:

$$F_{nE} = \frac{T_1}{r_{b1} + \mu_E \rho_{1E}} . \quad (8)$$

where ρ_{1E} the radius of curvature of the Γ_2 involute profile at the E point.

From Figure 1 the followings can be written:

$$\begin{aligned} \rho_{1E} &= EN_1 = r_{b1}\tan(\alpha_w) + EC = \\ &= r_{b1}\tan(\alpha_w) + e_E \end{aligned} . \quad (9)$$

and

$$V_{gE} = EC(\omega_1 + \omega_2) = e_E \omega_1 (1 + i_{21}) . \quad (10)$$

Replacing relations (8), (9) and (10) in relation (7) the following expression is obtained:

$$\mathrm{P_{vzE}} = \mu_{\mathrm{E}}\, \mathrm{P_1} \frac{\mathrm{e_E}(1 + \mathrm{i_{21}})}{\mathrm{r_{b1}}(1 + \mu_{\mathrm{E}} \tan(\alpha_{\mathrm{w}})) + \mu_{\mathrm{E}}\, \mathrm{e_E}} . \quad (11)$$

3. GRAPHICAL REPRESENTATION OF THE POWER LOSSES BY FRICTION BETWEEN THE TEETH FLANKS

In order to obtain the graphical representation of the surfaces from (6) and (11) the following input data were considered: $\mu_A = \mu_E = \mu = 0.05$, P_1=200W, r_{b1} =20mm. The x_1 and x_2 the values are in the range [-1, 1] and the z_1 and z_2 values are fixed for each surface representation. The following formulae were used to compute the (6) expression (the expression used to compute (11) can also be obtained from Figure 1):

$$y = \left(\frac{z_1 + z_2}{2}\right) \times \left(\frac{\cos(\alpha_0)}{\cos(\alpha_w)} - 1\right) . \quad (12)$$

$$k = x_1 + x_2 - y . \quad (13)$$

$$r_2 / r_{a2} = \frac{z_2}{z_2 + 2h_a + 2x_2 - 2k} . \quad (14)$$

$$\cos(\alpha_A) = r_2 / r_{a2} \times \cos(\alpha_0) . \quad (15)$$

$$e_A = r_{b1} \frac{z_2}{z_1} \left(\tan(\alpha_A) - \tan(\alpha_w)\right) . \quad (16)$$

where α_w is computed from the following equation:

$$\mathrm{x_1} + \mathrm{x_2} - \frac{\mathrm{inv}(\alpha_\mathrm{w}) - \mathrm{inv}(\alpha_0)}{2\tan(\alpha_0)} \times (\mathrm{z_1} + \mathrm{z_2}) = 0 . \quad (17)$$

The following Matlab code was used to obtain the results:

```
x1min=-1.0;
x1max=1.0;
nx1=41;
pasx1=(x1max-x1min)/(nx1-1);
x2min=-1.0;
x2max=1.0;
nx2=nx1;
pasx2=(x2max-x2min)/(nx2-1);
for i=1:nx2
    x2=x2min+(i-1)*pasx2;
    VX2(i)=x2;
    for j=1:nx1
        x1=x1min+(j-1)*pasx1;
        VX1(j)=x1; a_w = (17)
        Za(i,j) = (6)
        Ze(i,j) = (11)
    end
end
set(gcf,'Render','zbuffer');
xlabel('x_1');
ylabel('x_2');
zlabel('P_{vzA} (red), P_{vzE}
(yellow)');
surface(VX1,VX2,Za,'FaceColor',[1 0
0]); hold on;
surface(VX1,VX2,Ze,'FaceColor',[1 1
0]); view([1,1,1]);
zdiff=Ze-Za;
C = contour(VX1,VX2,zdiff, [0 0]);
xi = C(1, 2:end); yi = C(2, 2:end);
zi = interp2(VX1, VX2, Ze, xi, yi);
line(xi, yi, zi, 'Color', 'k',
'LineWidth', 3);
```

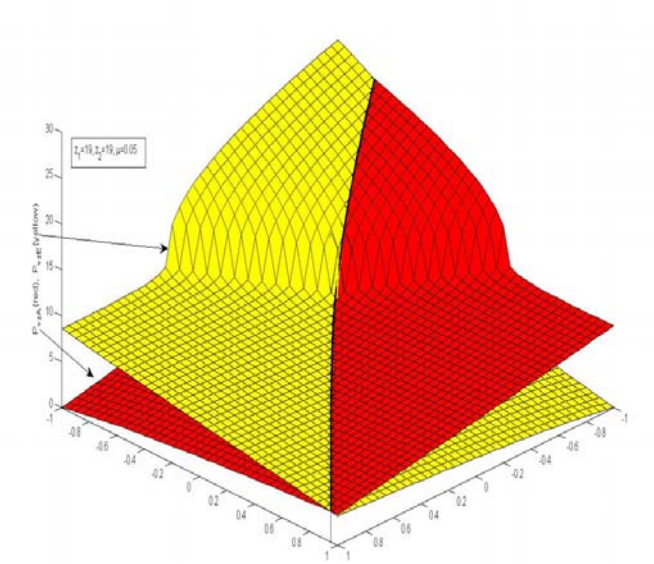

Fig. 2. Intersection of the (6) and (11) surfaces for z_1=19, z_2=19 and μ=0.05.

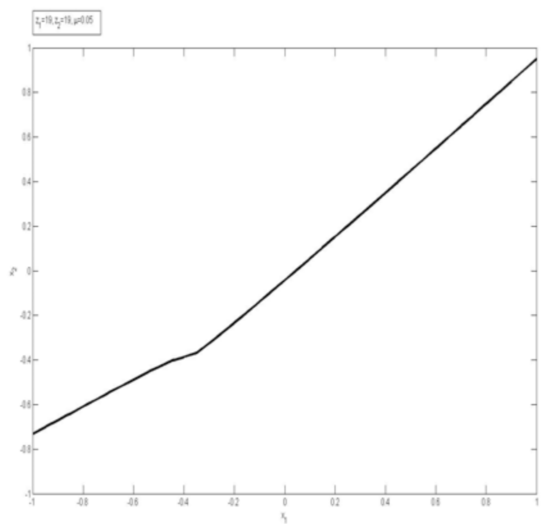

Fig. 3. The intersection curve of the two surfaces for z_1=19, z_2=19 and μ=0.05.

The intersection of the surfaces from Figure 2 is given in Figure 3. The curve is obtained by the Matlab code automatically. First the isoline of the intersection is obtained, and then the curve is obtained by interpolation on the second surface (11).

4. CONCLUSION

As seen in Figure 2 and Figure 3 the two surfaces intersect, so the equalization of the powers lost by friction, at the A and E points, have solutions. These can be obtained with a better approximation, by using suitable numerical methods, instead of extracting the intersection by Matlab facilities.

REFERENCES

[1] Erney Gy., *Fogaskerekek.* Müszaki Könyvkiadó, Budapest, 1983.

[2] Niemann G., Winter H., *Maschinenelemente – Band II.* Spinger Verlag, Berlin, 1983.

[3] Linke H., *Stirnradverzahnung. Berechnumg. Werstoffe. Fertigung*, Carl Hanser Verlag, München-Berlin, 1996.

[4] Antal T. A., *A new algorithm for helical gear design with addendum modification*, Mechanika, 3(77), p. 53-57, 2009.

[5] Antal T. A., *Efficiency of crossed helical gears with equalized sliding coefficients*, Annals for DAAAM - Proceedings of 20th DAAAM International Symposium,Volume 20 No.1, p. 1017-1018, Vienna, 2009.

DETEMINAREA PUTERILOR PIERDUTE PRIN FRECARE, ÎN CAZUL ANGRENAJELOR CILINDRICE CU DINȚI DREPȚI, ÎN PUNTELE DE ÎNCEPUT ŞI DE SFÂRŞIT ALE ANGRENĂRII

Rezumat: Lucrarea îşi propune determinarea expresiilor puterilor pierdute prin frecare, în punctele în care angrenarea începe (A) şi se termină (E). Apoi, se reprezintă grafic suprafețele corespunzătoare, în funcție de deplasările specifice de profil şi se verifică existența intersecției dintre cele două suprafețe. Rezultatele numerice din Matlab confirmă posibilitatea egalizării deci şi a determinării deplasărilor specifice de profil în condițiile celor două puteri egalizate.

Tiberiu Alexandru ANTAL, Professor, dr. eng., Technical University of Cluj-Napoca, Department of Mechanical System Engineering, antaljr@bavaria.utcluj.ro, 0264-401667, B-dul Muncii, Nr. 103-105, Cluj-Napoca, ROMANIA.