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THE INVERSE PROBLEM OF THE "3KTK SPATIAL PARALLEL" MANIPULATOR'S POSITIONS

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Abstract: *This paper presents in detail the structure of the 3KTK manipulator with 3 degrees of freedom in translation and the inverse problem of the positions is solved analytically by using the input - output equations.*

Key words: *parallel manipulator, the inverse problem, manipulator's positions.*

1.INTRODUCTION

Figure 1 shows the kinematic diagram of the 3KTK spatial parallel manipulator having three degrees of freedom in translation and three identical kinematic chains [7]. The symbolic notation of such a mechanism is linked to its characteristics: 3 - number of degrees of freedom (in translation); KTK - subsequent joints type of kinematic chain from the base to the final element (K – cardan joint, T - translation).

Cardan passive joints of base and mobile platform (the couplings centers have been noted with B_i and A_i) are placed in the tops of two equilateral triangles: $B_1B_2B_3$ and $A_1A_2A_3$ respectively, having the edges 'b" and "a". In each of the three kinematic chains, the first axis of the cardan joint from the base and the last axis of cardan joint on the mobile platform are placed in the plane of the base and in the plane of the platform respectively, perpendicular to the bisectors of the two triangles' angles. For each of the three kinematic chains, the second axis of the cardan joint from the base is parallel to the first axis of the cardan coupling from the mobile platform through a carriage (q_i length), on each carriage being placed a translational joint (Fig.1). Such an arrangement of the joints in the kinematics chains leads to a spatial parallel mechanism with three degrees of freedom in translation (Fig. 1).

According to some Korean researchers [8], the clearances of the cardan (universal) joints must be less than $0{,}05^0$.

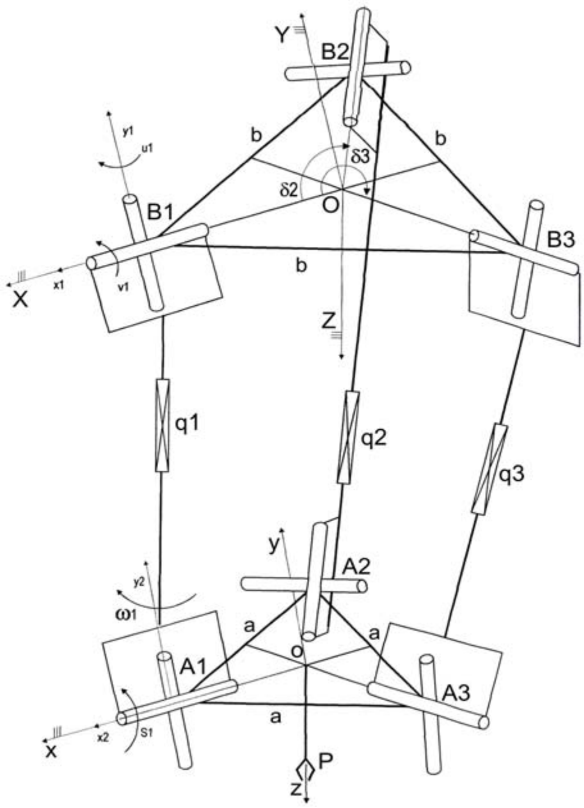

Fig. 1 The kinematic scheme of the manipulator

In order to study the manipulator it was considered a system of fixed axes OXYZ with “O” origin in the center of the circumscribed circle of the triangle on the base platform (whose radius is denoted by "R"). The OXY plane is such chosen in order to contain the fixed platform and the OX axis to contain the point B_1. The angles between the OX and OB_i are denoted by δ_i (i=1, 2, 3) (see Figure 1). It was also chosen a mobile reference system, OXYZ, linked to the mobile platform, and having the "O" origin in the center of the circumscribed circle of the triangle on the mobile platform (its radius is denoted by "r").

OXY plane is chosen in such a way so it contains the mobile platform, and for the Ox axis to contain the point A_1. The angles formed by the Ox and OA_i have been noted with $\delta_i^{'}$ (i=1, 2, 3).

Generalized coordinates of the manipulator are: qi - linear displacement from the motor joints, i = 1, 2, 3.

Generalized coordinates of the mobile platform are: X_P, Y_P, Z_P - cartesian coordinates of the point P in the center of the gripping device are connected to the OXYZ fixed system.

By varying the coordinates q_i, i = 1, 2, 3, the manipulated object in the space can be positioned depending on the phases of the handling operation.

2. THE INPUT - OUTPUT EQUATIONS SYSTEM

The first stage of the research of a parallel mechanism is the determination of its positions at a time, which is possible only if we know in advance the input-output equations [2]. [3], [4], [5], [6]. These equations contain two types of parameters: *kinematics*-generalized coordinates of the mechanism and of the handling object and *geometrical* - constructive dimensions of the mechanism.

The analytical method of determining the input-output system of equations for the 3KTK manipulator involves the following stages:

a) The deduction of the analytical expressions of the Cartesian coordinates for the guided points A_i (X_i, Y_i, Z_i) i = 1,2,3 relative to the fixed system (OXYZ), depending on the generalized coordinates of the mechanism - q_i, curvilinear coordinates of the guided points A_i and structural-geometric elements of the manipulator.

b) The deduction of the analytical expressions of the Cartesian coordinates for the guided points A_i (X_i, Y_i, Z_i), i = 1,2,3 relative to the same fixed system (OXYZ), but expressed according to the generalized coordinates of the manipulated object (X_P, Y_P, Z_P) and structural-geometric elements of the manipulator, based on the relationship between the points A_i, P, O.

c) Equaling the expressions obtained in points a) and b) leads to a system of equations from which it must be removed (by mathematical artifices) *the relative curvilinear coordinates* to finally yield *the input-output system of equations.*

3. THE INVERSE PROBLEM OF THE “3KTK”MANIPULATOR’S POSITIONS

The inverse geometric model consists in determining the articulated coordinates q_i (i = 1, 2, 3) of the manipulator when the manipulated object position and the generalized coordinates: X_P, Y_P, Z_P are known.

For solving the problem it is presented a method based on obtaining input-output equations of the mechanism, specifically the relationship between articulated coordinates (q_i, i= 1, 2, 3) and the generalized coordinates of the manipulated object (X_P, Y_P, Z_P). For this purpose, each element of each kinematic chain is attached to a mobile reference system with the origin in the center of each kinematic chain joint, as shown in fig. 1.

The coordinates of the points A_i shall be expressed in the fixed reference system OXYZ linked to the base, according to the vectorial relation (Fig. 1):

$$\overrightarrow{OA_i} = \overrightarrow{OB_i} + R(OZ,\delta_i)R(OY,u_i)R(OX,v_i)\cdot \overrightarrow{B_iA_i}$$

The matrices of rotation have the following expressions:

$$R(OZ,\delta_i) = \begin{bmatrix} \cos\delta_i & -\sin\delta_i & 0 \\ \sin\delta_i & \cos\delta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(OY,u_i)=\begin{bmatrix}\cos u_i & 0 & -\sin u_i\\ 0 & 1 & 0\\ \sin u_i & 0 & \cos u_i\end{bmatrix}$$

$$R(OX,v_i)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos v_i & -\sin v_i\\ 0 & \sin v_i & \cos v_i\end{bmatrix}$$

Finally, the vectorial relation leads to the system (1):

$$\begin{cases}X_{A_i}=X_{B_i}+(\sin\delta_i\sin v_i-\cos\delta_i\sin u_i\cos v_i)q_i\\ Y_{A_i}=Y_{B_i}-(\cos\delta_i\sin v_i+\sin\delta_i\sin u_i\cos v_i)q_i\\ Z_{A_i}=Z_{B_i}+\cos u_i\cos v_i q_i\end{cases}\quad(1)$$

where:
$X_{B_i}=R\cdot\cos\delta_i, Y_{B_i}=R\cdot\sin\delta_i, Z_{B_i}=0,\ i=1-3$
are known.

Squaring each equation of the system (1) and adding member to member, after intermediate calculations the equation (2) is obtained,

$$\left(X_{A_i}-X_{B_i}\right)^2+\left(Y_{A_i}-Y_{B_i}\right)^2+\left(Z_{A_i}-Z_{B_i}\right)^2=q_i^2\ (2)$$

which confirms on the one hand that B_iA_i segment's length is q_i and on the other hand the accuracy of the calculations performed by now.

From equation (2) it follows immediately:

$$q_i=\pm\sqrt{\left(X_{A_i}-R\cos\delta_i\right)^2+\left(Y_{A_i}-R\sin\delta_i\right)^2+Z_{A_i}^2}\quad(3)$$

where unknowns are q_i, X_{A_i}, Y_{A_i}, Z_{A_i}, i=1, 2, 3.

The coordinates of points A_i shall be expressed in the fixed reference system OXYZ of the base, starting from the mobile platform, according to the vectorial relation:

$$\overrightarrow{OA_i}=\overrightarrow{OP}+R\cdot\overrightarrow{PA_i}\quad i=1,2,3$$

which written in vector form becomes:

$$\begin{bmatrix}X_{A_i}\\ Y_{A_i}\\ Z_{A_i}\end{bmatrix}=\begin{bmatrix}X_P\\ Y_P\\ Z_P\end{bmatrix}+\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_{A_i}-x_P\\ y_{A_i}-y_P\\ z_{A_i}-z_P\end{bmatrix}$$

Finally system (4) is obtained:

$$\begin{cases}X_{A_i}=X_P+r\cos\delta_i'\\ Y_{A_i}=Y_P+r\sin\delta_i'\\ Z_{A_i}=Z_P-h\end{cases}\quad(4)$$

Introducing the relations (4) into equations (2) and considering coordinates' of the Bi points expressions there are obtained the input-output equations (5):

$$\left(X_P+r\cos\delta_i'-R\cos\delta_i\right)^2+\left(Y_P+r\sin\delta_i'-R\sin\delta_i\right)^2+\left(Z_P-h\right)^2=q_i^2\equiv F(q_i,X_P,Y_P,Z_P)\quad(5)$$

From the input-output equations (5), (considering that $\delta_i=\delta_i'\quad i=1,2,3$), the analytical solutions of the inverse problem of the 3KTK parallel manipulator positions which result immediately are made in the following form:

$$q_i=\pm\sqrt{\left[X_P+(r-R)\cos\delta_i\right]^2+\left[Y_P+(r-R)\sin\delta_i\right]^2+(Z_P-h)^2}\quad(6)$$

It will be always chosen as solution the positive value of q_i.

Based on the proposed mathematical model, it was developed and implemented a program for the numerical solution of the inverse problem positions by computer, program which runs a large number of data, some of which being presented below.

For the 3KTK mechanism, having the following configuration:

$$\begin{cases}b=400mm;\\ a=200mm;\\ h=50mm;\end{cases}\qquad\begin{cases}\delta_1=\delta_1'=0^0;\\ \delta_2=\delta_2'=120^0;\\ \delta_2=\delta_2'=240^0;\end{cases}$$

there are illustrated in Table 1 some of the numerical results of solving the inverse problem of the position.

Table 1

Some of the numerical results of solving the inverse problem of the position.

Input data			Output Data		
X_P	Y_P	Z_P	q_1	q_2	q_3
-150	0	200	304.92	202.52	202.52
70	-150	200	216.95	318.30	203.26
70	-140	200	210.16	310.51	201.04
70	140	200	210.16	201.04	310.51
70	150	200	216.95	203.26	318.30
80	-150	200	215.08	322.45	209.69
80	-140	200	208.23	314.76	207.54
80	-130	200	201.64	307.20	205.84
-150	-40	220	317.76	238.77	202.52
-150	-30	220	316.66	233.05	205.70
-150	-20	220	315.87	227.62	209.32
-150	-10	220	315.40	222.51	213.34
-150	0	220	315.24	217.74	217.74
-150	10	220	315.40	213.34	222.51
-150	-110	300	380.89	339.28	266.67
-150	-100	300	378.12	333.19	266.48
-150	-90	300	375.60	327.28	266.67
-150	-80	300	373.33	321.58	267.23
-150	-70	300	371.31	316.09	268.17
-150	-60	300	369.56	310.83	269.47
-10	20	420	391.21	381.55	391.89
-10	30	420	391.85	379.58	395.07
-10	40	420	392.74	377.86	398.47
0	-40	420	389.66	399.79	379.25
0	-30	420	388.76	396.40	380.96
0	-20	420	388.12	393.23	382.93
40	10	420	377.75	393.13	398.19
50	0	420	375.75	398.13	398.13

4. CONCLUSIONS

- The paper presents meticulously the algorithm for calculating the inverse problem for 3KTK manipulator positions based on the input - output equations. The conclusions are :
- The existence of the translatory actuators in the 3KTK manipulator structure makes the inverse problem of the positions to be relatively simple, easily finding analytical solution;
- The analytical solutions to the inverse problem of the 3KTK parallel manipulator positions will always positive values of the equation (6)
- Conceiving the program for solving the inverse problem is not problematic and the algorithm is relatively simple.
- The analytical solutions of the inverse problem of the positions will be the basis for determining the workspace of the manipulator.

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Problema inversă a pozițiilor manipulatorului paralel spațial 3KTK

Abstract: *În acest articol se prezintă detaliat structura manipulatorului 3KTK care are 3 grade de libertate în translație şi se rezolvă analitic problema inversă a pozițiilor, cu ajutorul ecuațiilor de intrare – ieşire.* ***Key words:*** *manipulator paralel, problema inversă a pozițiilor manipulatorului, platformă mobilă, ecuații de intrare – ieşire, grade de libertate, coordonate generalizate, coordonate curbilinii relative.*

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