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ON THE VEHICLE PLANAR MOTION

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Abstract: *Our aim is to present the experimental data obtained using a VBOX Skid device mounted on a Dacia LOGAN 1,6 V, in order to study its stability on a planar circular path. Were performed the tests for the cases of the speeds of the automotive of 25km=h and 30km=h on a circular path with a 15m radius. Based on these experimental results, we propose a mathematical model which might govern the planar circular motion of the mass center of the automotive.*

Key words: *vehicle dynamics, circular path, experimental test, stability automotive, mathematical modeling.*

1. INTRODUCTION

For the current appreciation of the cars performance, among the terms commonly used on engine performance (maximum power consumption zone or specific consumption, etc.) or of the whole car (acceleration, maximum speed, time and space for start-up, braking time and space, arranging and composing type brakes, transmissions, steering and suspension, fuel consumption per 100km, facilities and equipment used, etc.) are rarely found information or feedback regarding the limits of stability, that the car is able to meet in certain travel arrangements.

We appreciate that it is difficult to specify such criteria and limit performance, considering the diversity of traveling ways that a car may have throughout his life. Given, however, the importance of preserving the stability of the car in all conditions of displacement, for the safety of the passengers are in the interior and to other participants in traffic, for the safety of transported goods, taking into account the speed of movement of the car, such criteria are becoming increasingly necessary to be established and specified in the phase of conception and disclosed to any purchaser of automobiles, for those interested to know how their driving way and safe use. Determining the stability limits of any automobile with all the implications that arise, when these are exceeded on the occurrence of accidents, it becomes an extremely serious problem in the design process or during exploitation.

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It is harmful and uneconomic for a car not to meet the technical standards, for some of its components not to meet or not to maintain the operating standards as specified by the manufacturer, but it is extremely dangerous to loose stability, even for little time, regardless of the generating causes, because in such situations, it looses control of the car, with all the following consequences.

Considering the multitude of conditions of travel that one car can provide, as well as the multitude of situations encountered in the practice of driving on the road, we cannot always provide the limits of stability or the conduct of the car at the limit for all regimes. However certain predominant regimes encountered can be analyzed; can determine the limit conditions at which it can lose stability. So we will analyze next the conditions of stability of a turning car, on a path running horizontally, with constant speed.

2. VEHICLE MODELING FOR THE CIRCULAR PATH CASE

To achieve a vehicle easy model but able to describe yet a lot of kinematics characteristics for it, we must fix some working hypotheses, from which we list some: the vehicle is a rigid body; the vehicle is moving on a level road, comparable as a geometrical plane; without suspension system; the longitudinal velocity is steady-time, only for this model; the weight of the wheels does not have an influence regarding the weight of the vehicle, so the position of the center of gravity does not change; etc. In short, all this hypotheses allow as to adopt the kinematics framework for the material point case.

As is known, the kinematic properties of a material point along the circular path γ_0, equation $x^2+y^2=R_0^2$, written in the absolute inertial system $O_w xy$ (Fig. 1.), assuming constant forward speed module $U=\left|\vec{U}\right|$, are governed by the following equations [3], [4]

$$\begin{cases} \dot{x}=U\cos\theta, \\ \dot{y}=U\cos\theta, \end{cases}$$

where the law of motion is considered to be given by the function $\theta=\theta(t)$, $\forall t>0$. Obviously, if we consider now (x,y) as the Cartesian coordinates of the mass centre of the car, the kinematic equations above would describe its ”ideal” movement along a circular paths, i.e. without taking into account the contact properties of the tire with the road, or the variation of the turning angle, etc.

Taking into account the results of the tests presented in paragraph 3, we consider the general case of the real γ path for the mass center of the vehicle, whose equation is written in polar coordinates as $R=R(\theta)$. Therefore, at any time $t>0$, its position is given by

$$\begin{cases} x=R(\theta)\cos\theta, \\ y=R(\theta)\sin\theta, \end{cases} \tag{1}$$

where the polar angle $\theta=\theta(t)$ is a variable function of the time $t\in[0,+\infty)$, a priori unknown. Assuming that $\theta\in C^2([0,+\infty),R)$, the projections of the speed $\vec{V}$ for the mass center on the axis of the absolute system $O_w xy$ are written as

$$\begin{cases} \dot{x}=\dot{R}\cos\theta-R\dot{\theta}\sin\theta, \\ \dot{y}=\dot{R}\sin\theta+R\dot{\theta}\cos\theta, \end{cases} \tag{2}$$

where the polar radius is given by

$$R(\theta)=R_0+r(\theta), \tag{3}$$

and $r=r(\theta)$, being the distance from the position of the vehicle on the γ path to the point of intersection of $\vec{R}$ with γ_0. We assume that $r\in C^2([0,+\infty),[-1,+1])$, that comes to the fact that the car path γ belongs to the symmetric neighborhood centered in γ_0 with an unitary radius.

Furthermore, this hypothesis rests on the comparison of two synchronous movements: one on the γ_0 and the other on the γ.

Next, consider the system joint to the vehicle, with the vectors $(\vec{\tau}_s,\vec{\nu}_s)$, where $\vec{\nu}_s=\vec{R}/R$ and $\vec{\tau}_s\perp\vec{\nu}_s$ (Fig. 1.). Then, we get the expression for the mass center speed given as

$$\vec{V} = \dot{R}\vec{v}_s + R\dot{\theta}\,\vec{\tau}_s \tag{4}$$

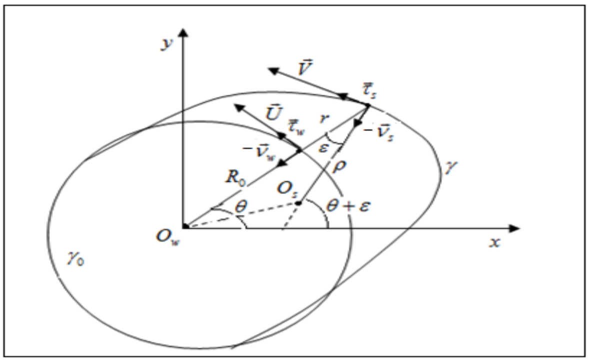

Fig. 1. The geometrical representation of the automobile motion in the plan.

And, its acceleration is

$$\vec{a} = \left(\ddot{R} - R\dot{\theta}^2\right)\vec{v}_s + \left(2\dot{R}\dot{\theta} + R\ddot{\theta}\right)\vec{\tau}_s\,. \tag{5}$$

From equations (4) and (5) we get that the longitudinal speed and the lateral one are

$$V_v = \dot{R} = \dot{r},\quad V_{\tau_s} = R\dot{\theta}\,, \tag{6}$$

and, accordingly, for the longitudinal acceleration and the lateral one we get

$$a_{v_s} = \ddot{R} - R\dot{\theta}^2\,,\quad a_{\tau_s} = 2\dot{R}\dot{\theta} + R\ddot{\theta}\,. \tag{7}$$

We remark that in Fig. 1., we have used the obvious notation conventions that lead to $\left(\vec{\tau}_s, -\vec{v}_s\right) \approx \left(\vec{\tau}_w, -\vec{v}_w\right)$, the difference between the two vectors systems being given only by the point where they are considered.

In the experimental tests hypothesis, about maintaining constant speed is given, without changing gear, i.e.

$$\left|\vec{U}\right| = \left|\vec{V}\right| = const., \tag{8}$$

and from the recorded results for the longitudinal and lateral accelerations, we can write the following differential equation system

$$\begin{cases} \ddot{R} - R\dot{\theta}^2 = k_1, \\ 2\dot{R}\dot{\theta} + R\ddot{\theta} = k_2, \end{cases} \tag{9}$$

which govern the dynamic of the mass center on γ. Obviously, from the variations of the accelerations, presented in Figs. 2.-4., we consider $k_1 \prec O(1)$ and $k_2 \prec O(0)$. Therefore, we can write the equation that governs the dynamic of the mass center of the vehicle on the γ path as

$$\ddot{R} - \omega(t)^2 R = 0\,, \tag{10}$$

where $\dot{\theta} = \omega(t)$ is the variation law for the rotation speed of the car in relation with the O_w pole, a priori unknown. If we assuming that $\omega = \omega(t)$ is periodical, then (10) is a Mathieu equation [7], which if we were to re-write it for the unknown r it would look like that

$$\ddot{r} - \omega(t)^2 r = \omega(t)^2 R_0$$

The initial natural conditions we need to attach to the Eq. (10) are

$$R(0) = R_0,\quad \dot{R}(0) = 0\,,$$

meaning that at the initial moment $t_0 = 0$, the mass center was on γ_0 and its lateral speed was equals zero.

On the other hand, if we adopt the optimal control model of a nonholonomic vehicle [1], [6], than we have following motion equation for the mass centre on the γ

$$\begin{cases} \dot{x} = V\cos\theta, \\ \dot{y} = V\sin\theta, \\ \dot{\theta} = \omega, \end{cases}$$

where the control law is the variation of the angle speed, which will obviously depend the ε angle between the vector radius $\vec{R}(\theta)$ and the curve radius $\vec{\rho}(\theta)$ (Fig. 1.), with

$$\rho = \frac{\ddot{R}\dot{\theta} - \ddot{\theta}\dot{R} - R^2\dot{\theta}^3 - 2\dot{R}^2\dot{\theta}}{\left(\dot{R}^2 + R^2\dot{\theta}\right)^{3/2}}\,,$$

then the control law must be re-written also considering that angle too.

The α angle between $\vec{R}$ and $\vec{\tau}_s$ is given by

$$tg\ \alpha = \frac{R(\theta)}{R'(\theta)} = \frac{\dot{\theta}R}{\dot{R}} = \frac{V}{\dot{R}}\,.$$

Therefore

$$tg\ \varepsilon = -\frac{1}{tg\ \alpha} = -\frac{\dot{R}}{V}\,.$$

In the assumed hypothesis, we have $\varepsilon \prec O\ (0)$, which corresponds to the physical experiment, i.e.

$$\frac{\dot{R}}{V} \prec O\ (0)\,.$$

For the Eq. (10) we need a deep approach of the function $\omega(t)^2$, like in [2], [5], [8]. We note that as far as we know, the literature does not paying attention to the mathematical modeling of the automotive stability on a circular path, so no to the equation (10), in which to consider some of the parameters that influence the movement (the tyres adherence coefficient, the variation of the force on the wheels when turning, etc). This remains one of our future research objectives.

3. EXPERIMENTAL DETERMINATIONS

Using the *VBOX Skid* device mounted on a *Dacia LOGAN 1,6V* experimental determinations were made by moving with constant speed of *25km/h* to *30km/h*, on a circular path with a *15* meters radius, on a concrete runway. He chose such a trail for lack of another bigger to make determinations.

In figure 2 we present the real samples recorded using the *VBOX Mini*, by turning the *Dacia LOGAN* on a *15* meters radius circle. There are also presented the graphs for speed changing, those of lateral and longitudinal accelerations as well as the circular form of the given route.

When the vehicle moves at a steady speed in curves with small radius or, especially, if it turns in the small radius circles, Fig. 3. ($V = 25km/h, R = 15m$) the following is found:

a) it is difficult to maintain a constant speed on the circle, because there is a tendency to skid sideways, for which records must be made after 1-2 spins on the circle for car driver to get used with these conditions;

b) registration accuracy of the *VBOX Mini* is $0.01m/h$ and, consequently, are registered all deviations at a speed of $25km/h$, desired to be kept constant (Figs. 4., 5.) within the measuring range;

c) the lateral acceleration values varies in the range $(-0.350g; 0.24g)$, actually felt by the experimenter as close to reaching the limit of dizziness behind the wheel, and the modifications of the values of lateral acceleration are justified by accelerating more or less intense by the experimenter during the trial. The maximum lateral acceleration values occur when the car speed increases in the curve.

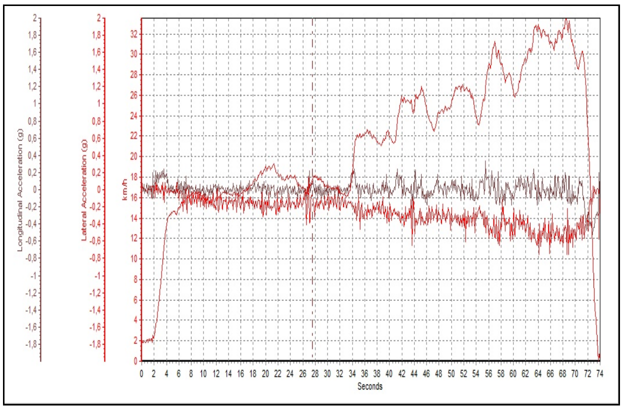

Fig. 2. Graphs for speed and longitudinal and lateral acceleration modifications.

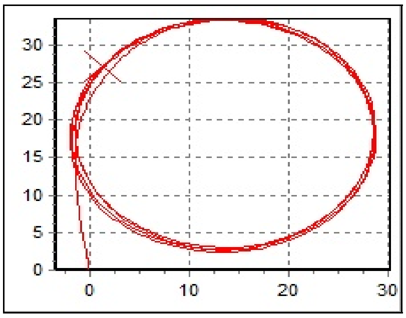

Fig. 3. The real shape of the trial trajectory of the automobile.

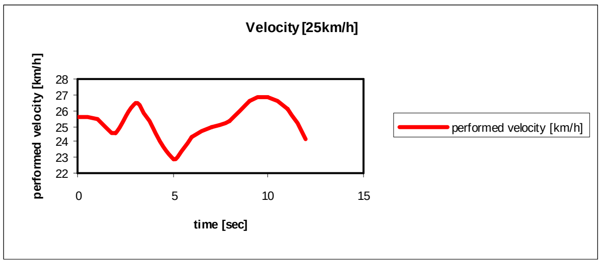

Fig. 4. The performed automotive speed.

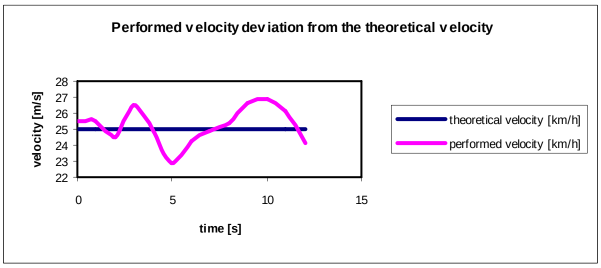

Fig. 5. Performed speed deviation from the theoretical speed.

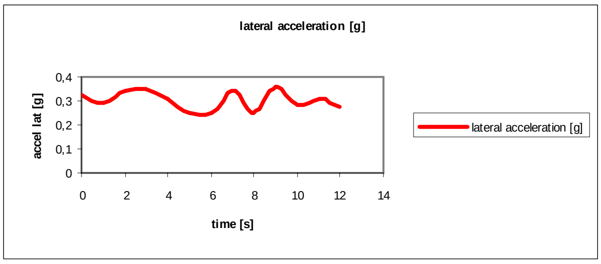

Fig. 6. The variation of the automotive lateral acceleration.

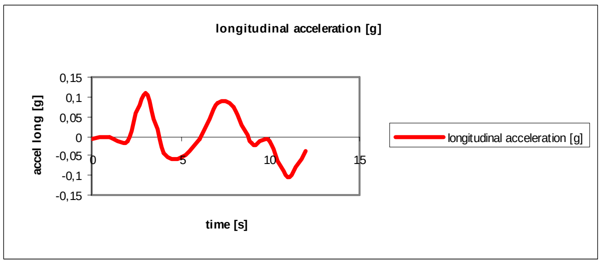

Fig. 7. The variation of the automotive longitudinal acceleration.

d) when accelerating may find that increasing the speed of motion on the circle grow both lateral acceleration and longitudinal components (Figs. 6., 7.).
e) as the lateral acceleration decreases, lowering the speed of motion on the circle, we can observe a decrease of the longitudinal acceleration (Fig. 7.).

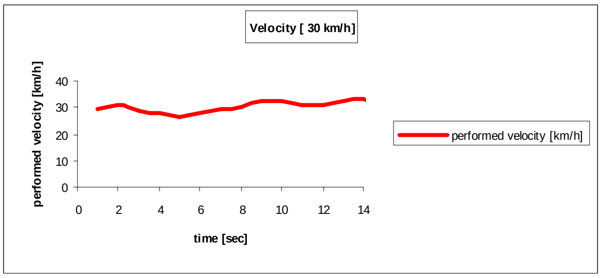

Fig. 8. The performed automotive speed**.**

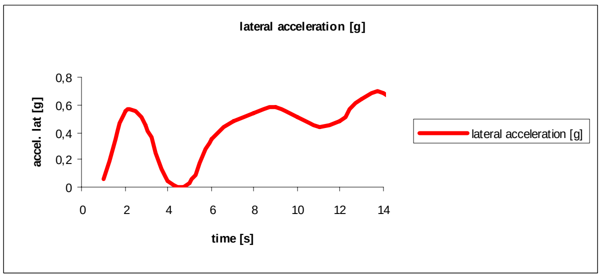

Fig. 9. The variation of the automotive lateral acceleration.

Figs. 8. and 9. show important changes in the values of the lateral acceleration, $(0.3g;0.675g)$, which is explained by the difficulty of maintaining a constant speed of $V = 30km/h$ when turning on the circle of radius $R = 15m$. By comparing the changes in the speed of movement, in the same time interval, in the figure 7 we can see that the speed of movement on the circle, 30km/h, has reduced changes of $0.3kh/h$, plus or minus.

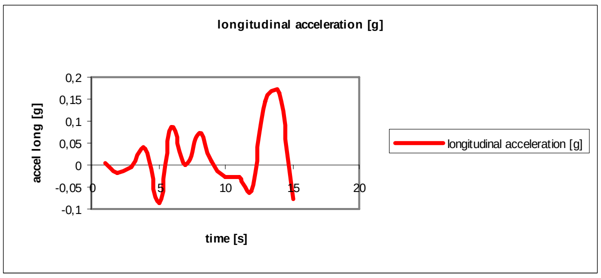

Fig. 10. The variation of the automotive longitudinal acceleration.

The positive or negative values of the longitudinal acceleration presented in figure 10 are explained by the fact that when reaching the lateral stability limit, the acceleration of the car has been sensibly reduced. This is the result of the driver's reflex to take his foot off the acceleration pedal or press on it.

By comparing the values of the lateral and longitudinal accelerations, we can observe that in the time interval from second 13 to 14, these values reach their maximum level. This is explained by the fact that the value of the turning speed on the circle reaches maximum values in this time interval (Fig. 7.), the car being accelerated (longitudinal acceleration is positive defined).

4. CONCLUSION

This paper presents a study of the motion of the automotive during cornering, at the limit of adherence, on known radius circles, starting from the motion of the material point on the circle and ending with experimental measurements made using *Racelogic Vbox* unit.

The calculation and modification of important parameters – speed, longitudinal acceleration, transversal/lateral acceleration, turning angle – which can help to analysis the circular motion of the car, show that turning on a curve, at the adherence limit, is not constant, although it was intended. The reasons are multiple and are linked to the quality of the motion path, the driver's experience, the automotive quality (*Dacia LOGAN 1,6V*).

The paper showed a constant change of the measured values, which will be useful for determining a range of allowable values for each measured parameter, to ensure the safe turning of the car, without lateral sliding and the appearance of insecurity in motion.

With the development of experimental research on the properties of stability of the automotive on a flat circular path, we will need to expand the study for the differential equation 10, in the dynamical systems theory.

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Asupra miscării circulare plane a unui autovehicul

În lucrare sunt prezentate datele experimentale înregistrate cu ajutorul dispozitivului Mini VBOX montat pe o Dacia Logan 1,6 V, în cadrul testelor de stabiltatea acestuia pe o traiectorie circulară plană. Au fost efectuate teste pentru cazurile de viteze de înaintare de 25kh/h şi de 30km/h, pentru o traiectorie circulară de rază de 15m. Pe baza rezultatelor experimentale obţinute, este propus un model matematic care să guverneze miscarea centrului de greutate al autovegicului de-a lungul traiectoriei plane circulare.

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