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GEOMETRIC AND NUMERICAL MODELING OF HELICAL SURFACES

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Abstract: *In this paper I proposed to present theoretical and practical issues concerning the representation of the helical surface. The first part presents how to make a cylindrical propeller, then expand the representation to a propeller cone, and finally is presented theoretically and practically how to generate an Archimedical helicoidally surfaces. The program, who generates points necessary representation is AutoLISP, and AutoCAD is used to display graphics.*

Key words: *cylindrical propeller, cone propeller, surface, helical surface, parametric equations.*

1. INTRODUCTION

To make a geometric study of a helicoidally surface is needed mathematical and numerical model of the surface.

Helical surface we have proposed to do in this paper is a flank surface of a Archimedical worm. It is known that the axial section profile of a Archimedical worm form is rectilinear. Points axial profile section we will print a screw motion about the axis OZ if we consider an orthogonal reference system as OXYZ. Thus, every point will describe a propeller, the first phase will be a cylindrical propeller, after which we will expand to a cone propeller.

2. PARAMETRIC EQUATIONS OF A CYLINDRICAL PROPELLER

We consider a curve in space (C) and a right orthogonal reference system as OXYZ with axes versors note with i, j, k (Fig. 1.) Orthogonal system is right because its axes are mutually perpendicular. Vector equation of the curve is written as:

$$r = r(t) = x(t)i + y(t)j + z(t)k \qquad (1)$$

where t is called the parameter curve.

This equation appears as a vector with point application in origin of the coordinate system and the peak in the current point on the curve. The represented curve has a sense, sense given by the direction of travel of the point M when the parameter t increases.

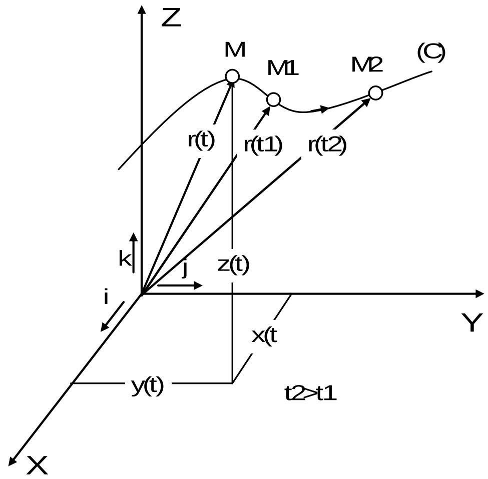


Fig. 1. Curve reported at an orthogonal system

Equation (1) we can write in a parametric form, like a system of three scalar equations:

$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \qquad (2)$$

The helical movement is composed of a rotation around the axis OZ simultaneously with a motion of translation parallel to that axis.

The curve that materialized helical trajectory is called propeller**.**

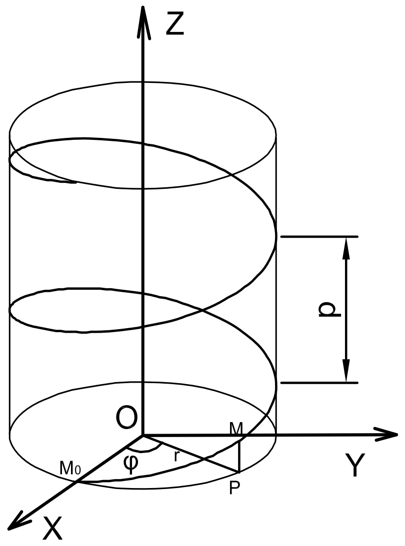


Fig. 2. Cylindrical propeller

The parametric equations of cylindrical propeller (Fig. 2.) are:

$$\begin{cases} x = x_0 + r\cos\varphi \\ y = y_0 \pm r\sin\varphi \\ z = z_0 + \dfrac{p}{2\pi}\varphi \end{cases} \qquad (3)$$

where:

- r is the radius of the cylinder which is wrapped the propeller,
- φ is the propeller parameter,
- p is the propeller pitch,
- sign ± control the development propeller left or right.

3. PARAMETRIC EQUATIONS OF A CONE PROPELLER

Based on the cylindrical propeller equations, we can write parametric equations of the propeller cone (Fig. 3.) so:

$$\begin{cases} x = x_0 + r_1\cos\varphi \\ y = y_0 \pm r_1\sin\varphi \\ z = z_0 + \dfrac{p}{2\pi}\varphi \end{cases} \qquad (4)$$

where:

- r is the radius of cone base on which the propeller wrap,
- r_1 is the distance between the center of the cone base and the projection of a point M1 situated on the cone propeller on the plan in witch is located base of cone (in our case xOy)
- φ is the propeller parameter,
- p is the propeller pitch,
- sign ± control the development propeller left or right.

In ΔABM_1 with angle B of 90°

$$tg\beta = \frac{AB}{BM_1} \qquad (5)$$

AB we can write as

$$AB = r_1 - r \qquad (6)$$

and

$$BM_1 = \frac{p}{2\pi}\varphi \qquad (7)$$

Substituting equation (6) and (7) in (5) we obtain:

$$r_1 = r + \frac{p}{2\pi}\varphi \times tg\beta \qquad (8)$$

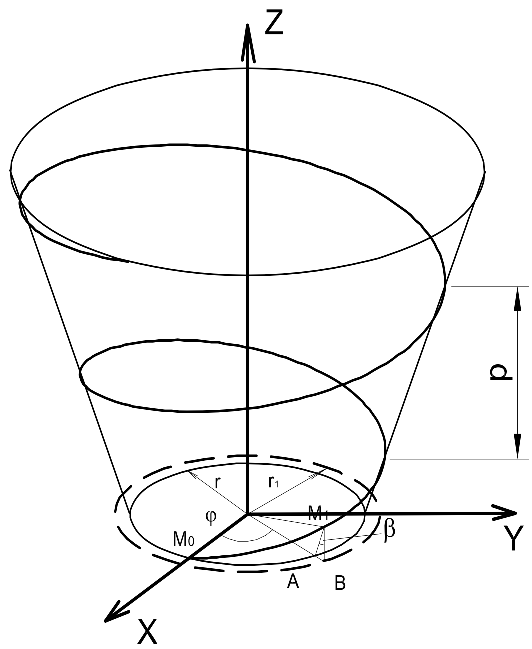


Fig. 3. Cone propeller

Substituting equation (8) in (4) we get the parametric equations of the cone propeller:

$$\begin{cases} x = x_0 + (r + \frac{p}{2\pi}\varphi \times tg\beta)\cos\varphi \\ y = y_0 \pm (r + \frac{p}{2\pi}\varphi \times tg\beta)\sin\varphi \\ z = z_0 + \frac{p}{2\pi}\varphi \end{cases} \quad (9)$$

In addition to our cylindrical propeller we have the angle β which is the cone angle.

4. BLOCK DIAGRAM OF A DRAWING PROGRAM FOR CONICAL OR CYLINDRICAL PROPELLER

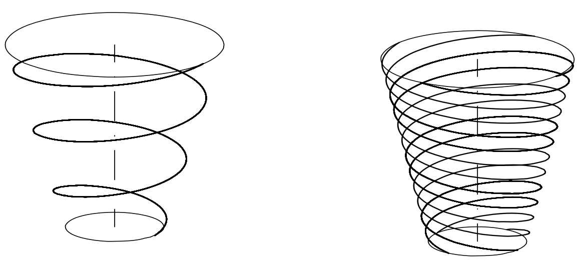
Below is represented the block diagram (Fig.4.) of program executed in AutoLISP, program that will provide the points coordinates needed to drawing in AutoCAD a cylindrical or conical propeller.

I present a simulation of a propeller (Fig.5.a, b) whose inputs are presented in the following table (Table 1).

Table 1

The input data for representation of propellers

	Elicea 1 (a)	Elicea 2 (b)
razai [mm]	20	20
beta [grade]	15	12
pasul [mm]	30	30
spire	3	3
sens [S/D]	S	D
inc [1...4]	1	4



a. *b.*

Fig. 5. Simulated propeller

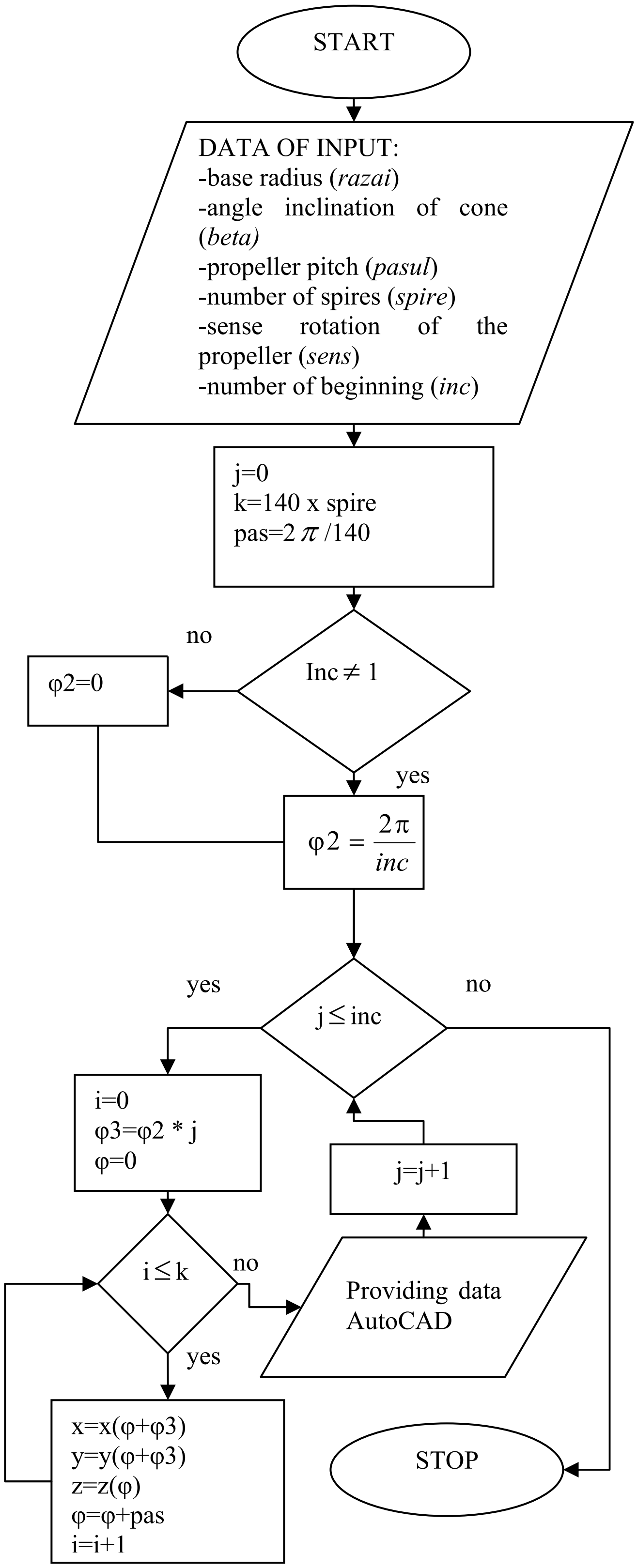


Fig. 4. Block diagram representation of a propeller

5. PARAMETRIC EQUATIONS OF A SURFACE

We consider a surface (S) and an orthogonal reference right system OXYZ with unit vectors axes note with i, j, k (Fig.6).

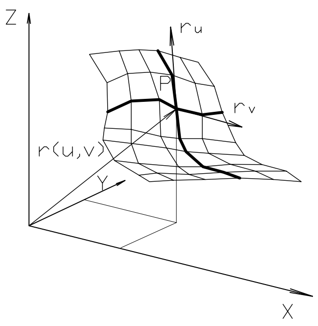


Fig. 6. Surface (S) and its coordinate curves

Vector equation of the surface is written as:

$$r = r(u,v) = x(u,v)i + y(u,v)j + z(u,v)k \quad (10)$$

where u and v parameters are also called curvilinear coordinates of the surface.

Equation (10) can be written parametrically as a system of three scalar equations:

$$\begin{cases} x = x(u,v) \\ y = y(u,v) \\ z = z(u,v) \end{cases} \quad (11)$$

For $u = u_0 = ct.$, point P moves along a curved of a surface (v), called a coordinated curve.

$$r = r(u_0,v) = x(u_0,v)i + y(u_0,v)j + z(u_0,v)k \quad (12)$$

Using parametric equations of the propeller cone (equation (9)), parametric equations of a helical surfaces can be written:

$$\begin{cases} x = x_0 + \left[r + \frac{p}{2\pi}\varphi \times tg\beta + s(t) \right]\cos\varphi \\ y = y_0 \pm \left[r + \frac{p}{2\pi}\varphi \times tg\beta + s(t) \right]\sin\varphi \\ z = z_0 + t + \frac{p}{2\pi}\varphi \end{cases} \quad (13)$$

where:

- φ and t are surface parameters,
- s(t) represents equation of generating curve.

6. PARAMETRIC EQUATIONS OF A ARCHIMEDICALAL HELICOIDALLY SURFACES

The generated surface is actually a flank of a Archimedical worm, therefore we make the program so that we can represent 1 - 4 surfaces.

We know that cutting with a axial plan (XOZ) an Archimedical helicoidally surface (Fig.7.) the resulting curve is a line (P_0P).

We can write parametric equations of the Archimedical helical surface:

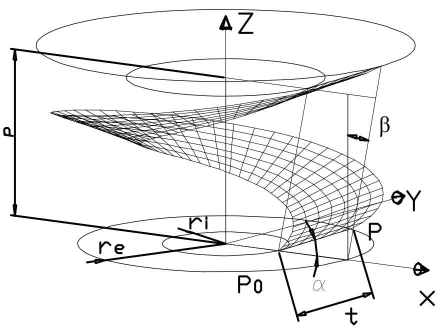


Fig. 7. Archimedical helical surface

$$\begin{cases} x = x_0 + \left[r_i + \frac{p}{2\pi}\varphi \times tg\beta + t\cos\alpha \right]\cos\varphi \\ y = y_0 \pm \left[r_i + \frac{p}{2\pi}\varphi \times tg\beta + t\cos\alpha \right]\sin\varphi \\ z = z_0 + t\sin\alpha + \frac{p}{2\pi}\varphi \end{cases} \quad (14)$$

where:

- x_0, y_0, z_0 are the coordinates of the center base,
- r_i , r_e define the study domain of the surface,
- φ is the propeller parameter,
- p is the propeller pitch,
- sign ± control the development propeller left or right,
- α tilt angle of the generators line,
- β tilt angle of the cone.

To calculate length t of the right P_0P we will represent the triangle formed between the right P_0P and the axis OX in plain XOZ (Fig.8.):

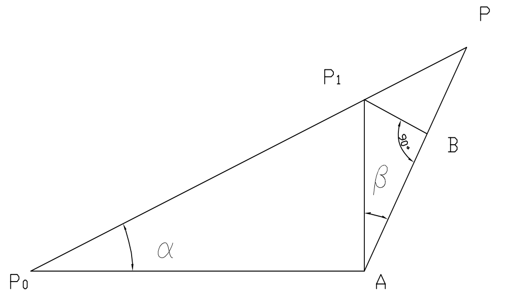


Fig.8. The scheme for calculation length of generating

Where we know the value of α, β and
$P_0A = r_e - r_i$.

$$P_0P = P_0P_1 + P_1P \quad (15)$$

In ΔP_0AP_1 with angle A of 90° ,

$$P_0P_1 = \frac{r_e - r_i}{\cos\alpha} \quad (16)$$

for $\cos\alpha \neq 0$.

$$AP_1 = (r_e - r_i)tg\alpha \quad (17)$$

In ΔP_0AP angle $P = \pi - \alpha - \left(\frac{\pi}{2} + \beta\right)$

$$P = \frac{\pi}{2} - (\alpha + \beta) \quad (18)$$

In ΔABP_1 with angle B of 90° ,

$$P_1B = AP_1 \sin\beta \quad (19)$$

Substituting equation (19) in (21) we obtain:

$$P_1B = (r_e - r_i)tg\alpha \sin\beta \quad (20)$$

In ΔP_1BP with angle B of 90° ,

$$P_1P = P_1B / \sin\left(\frac{\pi}{2} - (\alpha + \beta)\right) \quad (21)$$

Substituting equation (20) in (21) we obtain:

$$P_1P = \frac{(r_e - r_i)\sin\alpha\sin\beta}{\cos\alpha\cos(\alpha + \beta)} \quad (22)$$

for $\cos\alpha \neq 0$ and $\cos(\alpha + \beta) \neq 0$.

Substituting equation (16) and (22) in (15) and obtain:

$$P_0P = \frac{(r_e - r_i)}{\cos\alpha}\left(1 + \frac{\sin\alpha\sin\beta}{\cos(\alpha + \beta)}\right) \quad (23)$$

for $\cos\alpha \neq 0$ and $\cos(\alpha + \beta) \neq 0$.

Practically, to create a surface we have need two curves, a directory curve and a generating curve. In our case the guiding curve is a conic helix that I presented it in the first part of work and the generating curve is a line (P_0P), inclined angle to the axis OX with an angle α .

To represent a surface in AutoCAD, we will use "3dmesh" command that is addressed in general programs.

7. BLOCK DIAGRAM OF THE PROGRAM FOR PLOTTING HELICAL SURFACES

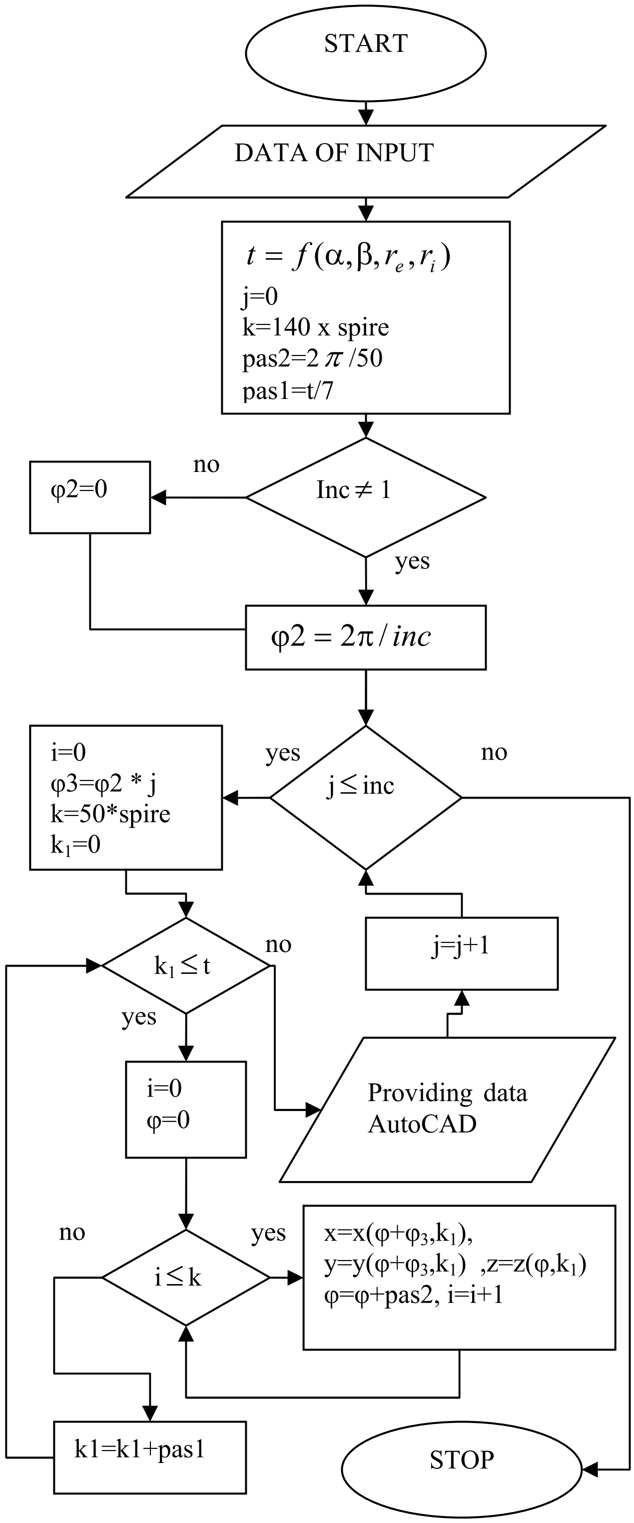


Fig.9. Logic diagram for representing an Archimedical helicoidally surfaces

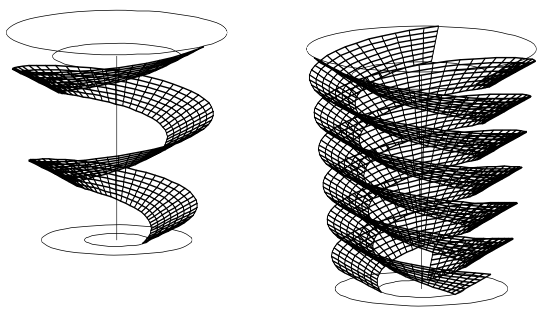
For generate helical surface, we move a point on the generators P_0P and for each point on the right we will generate points which belong to a helical curve. We will create a cloud of points which will be sent to AutoCAD to view the resulting surface. Block diagram of the program who is executed in AutoLISP is follows (Fig.9.).

Data entry can be seen from the table below (Table 2), where are presented two examples of calculating the required points for generating helical surfaces (Fig.10):

Table 2

The input data for representation of helical surfaces

	Surface 1 (a)	Surface 2 (b)
razai(r_i) [mm]	15	20
razae(r_e) [mm]	35	40
alfa1 (α)[grade]	20	22
beta (β) [grade]	14	10
pasul (p) [mm]	30	35
spire	2	2
sens	S	D
inc	1	3



a. *b.*

Fig.10. Helical surfaces generated

8. CONCLUSION

This paper includes issues relating to the generation of helical surfaces with the scope to prepare the necessary knowledge base for the execution of cylindrical and conical worms on numerically controlled machine tools.

Thus, using the AutoLISP which is a programming language for AutoCAD, we can calculate the coordinates of points and on their based we can generate any kind of helical surfaces where can determine the mathematical model.

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MODELAREA GEOMETRICĂ ŞI NUMERICĂ A SUPRAFEŢELOR ELICOIDALE

Rezumat: În prezenta lucrare am prezentat aspecte teoretice şi practice cu privire la modul de reprezentare a suprafetelor elicoidale. În prima parte este prezentat modul de realizare a unei elice cilindrice, apoi se extinde reprezentarea la elice conică, iar la final este prezentat teoretic şi practic modul de generare a unei suprafete elicoidale arhimedice. Programul care generează punctele necesare reprezentărilor este AutoLISP, iar pentru afişarea grafică se foloseşte AutoCAD.

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