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ANALYTICAL CALCULUS OF THE OUTPUT ANGLE DEPENDING ON INPUT

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Abstract: *The converter is a mechanism designed so that all functional parts, apart from the primary and secondary shafts oscillate around certain average positions with the same harmonic frequency motion. All parts of this type are subject to internal forces which are also harmonically, and have the same frequency.*
Key words: *converter, the output angle, angle of entry, speed, harmonic motion*

1. INTRODUCTION

By definition and construction the converter is a device that divides the harmonic motion obtained from a primary continuously rotating shaft, in two parts. A component is applied to a load, while the other is applied to a pair of unidirectional motion mechanisms that convert harmonic movement and harmonic forces into rotating intermittent unidirectional pulses. The simplest mechanism by which a constant rotary motion is converted into simple harmonic motion is a crank and a connecting rod of appropriate length. Theoretically a simple harmonic motion is obtained only if the bar is very long, but for practical purposes a rod length of about two times the race is considered to be appropriate. In any case, the movement can be expressed by the fundamental frequency and a second harmonic equal to twice the frequency. In the analysis that follows, for simplicity, the second harmonic term was not taken into account, its influence not being important, given that enough attention is accorded to designing the unidirectional and that of the couplings for the primary and secondary axis and especially for the elastic couplings.

2. INFORMATION

The speed of the free end of the connection bar that runs along a line which passes through the rotation axis of a primary crank can be expressed by a relationship as the above:

$$v = V \cdot \sin(\omega t + \psi) \quad (1)$$

Speed is divided into two components inside the converter, a component applied to oscillate a mass, while the other powers the unidirectional or as the author suggestively called them "mechanical diodes".

The function of these diodes is to convert harmonic forces and harmonic angular movements applied to them, in an medium torsion point *M_1*, and an angular medium speed *ω_1*, so that medium harmonic torsion point *M_1* is proportional to the harmonic forces amplitude, while the medium angular speed *ω_1* is proportional to the amplitude of harmonic angular movements applicable to them.. Another condition for an ideal diode is that the harmonic force applied to them and the corresponding angular velocity is always in phase. This means that if the harmonic force applied is:

$$f_1 = F_1 \cdot \sin \omega t \quad (2)$$

The corresponding velocity v_1 of the application point of this force will be:

$$v_1 = V_1 \cdot \sin \omega t \quad (3)$$

That is equivalent to:

$$\frac{f_1}{F_1} = \frac{v_1}{V_1} = \sin \omega t \qquad (4)$$

The explanation for this condition is the following: lets imagine that instead of relationship (3) there is: $v_1 = V_1 \cdot \sin(\omega t + \psi)$

The average energy per second E transmitted through the force f_1 to the diode can be expressed through:

$$E = \frac{1}{T} \cdot \int_0^T f_1 \cdot v_1 \cdot dt =$$

$$\frac{F_1 \cdot V_1}{T} \cdot \int_0^T \sin \omega t \cdot \sin(\omega t + \psi) \cdot dt \qquad (5)$$

T being the time of a full period, $T = \frac{2\pi}{\omega}$.

From above, results:

$$E = \frac{F_1 \cdot V_1}{2} \cdot \cos \psi \qquad (6)$$

On the other hand, assuming for simplicity that there is no energy loss from the diodes, power transmission will be collected on the secondary shaft and therefore the energy equation can be expressed:

$$E = M_1 \cdot \omega_1 \qquad (7)$$

$$M_1 \to M_m$$

From equation *(8)*:

$$M_1 \frac{v_1}{r_1} = \frac{F_1 v_1}{2} \cos \psi \qquad (8)$$

Or

$$M_1 = \frac{F_1 r_1}{2} \cos \psi$$

$$v_1 = \omega_1 r_1 \Rightarrow \omega_1 = \frac{v_1}{r_1} \qquad (9)$$

From the build of the unidirectional:

$$\psi = 0 \Rightarrow \cos \psi = 1 \Rightarrow M_1 = \frac{r_1}{2} F_1 \qquad (10)$$

$$f = d_1 f_1 = d_2 f_2 \qquad (11)$$

$$v = \omega r = a_1 \omega_1 r_1 + a_2 \omega_2 r_2 \qquad (12)$$

$$f_2 = m_2 \frac{dv_2}{dt} \qquad (13)$$

From (11) results:

$$f_1 = f_2 \frac{d_2}{d_1} \qquad (14)$$

Using relationships (3) *(2)*:

$$v_1 = V_1 \cdot \sin \omega t$$

$$f_1 = F_1 \cdot \sin \omega t \qquad (15)$$

And expressing $\frac{dv_2}{dt}$ from equation *(13)* we get $\frac{dv_2}{dt} = \frac{f_2}{m_2}$ but from relationship *(14)* results the following:

$$f_2 = \frac{d_1 f_1}{d_2} \qquad (16)$$

Thus:

$$\frac{dv_2}{dt} = \frac{d_1 f_1}{d_2 m_2}$$

But

$$f_1 = F_1 \cdot \sin \omega t$$

So:

$$\frac{dv_2}{dt} = F_1 \frac{d_1}{d_2 m_2} \sin \omega t \Rightarrow$$

$$v_2 = \frac{F_1 d_1}{d_2 m_2} \int \sin \omega t dt = -\frac{F_1 d_1 \cos \omega t}{d_2 m_2 \omega} \qquad (17)$$

As a result:

$$v_2 = -\frac{F_1 d_1 \cos \omega t}{d_2 m_2 \omega} \qquad (18)$$

Relationship (12) can be written as:

$$v = a_1 v_1 + a_2 v_2$$

Using relationship (15):

$$v = V \cdot \sin \omega t$$

$$v_1 = V_1 \cdot \sin \omega t$$

$$v_2 = V_2 \cdot \sin(\omega t + \beta)$$

From the above equation we can further obtain, after obtaining relationship *(18)*, the following:

$$v \sin(\omega t + \psi) = a_1 v_1 \sin \omega t - \frac{F_1 d_1 \cos \omega t}{d_2 m_2 \omega} a_2 \qquad (19)$$

From (12) one can obtain:

$$v = \omega r$$

$$v_1 = a_1 \cdot \omega_1 r_1$$

Thus (19) becomes:

$$\omega r \sin \omega t = a_1 \omega_1 r_1 \sin \omega t - \frac{d_1 a_2 \cos \omega t}{d_2 m_2 \omega} F_1$$

From *(10)* we obtain:

$$F_1 = \frac{2 M_1}{r_1}$$

That transforms the above equation in:

$$\omega r \sin(\omega t + \psi) = a_1 \omega_1 r_1 \sin \omega t - \frac{F_1 d_1 a_2 \cos \omega t}{r_1 d_2 m_2 \omega} M_1 \qquad (20)$$

If relationship *(20)is further developed we obtain*:

$$\omega r [(\sin \omega t \cos \psi + \sin \psi \cos \omega t)] =$$

$$a_1 \omega_1 r_1 \sin \omega t - \frac{F_1 d_1 a_2 \cos \omega t}{r_1 d_2 m_2} M_1$$

So for this equation to be satisfied for any values of t, we must independently have:

$$\omega r \cos\psi - a_1\omega_1 r_1 = 0$$

$$\omega r \sin\psi + \frac{2d_1 a_2 M_1}{r_1 d_2 m_2 \omega} = 0$$

Or

$$\psi = -\frac{2d_1 a_2 M_1}{r r_1 \omega^2 m_2} = K_1\frac{M_1}{\omega^2}$$

where

$$K_1 = \frac{2d_1\omega_2}{r r_1 m_{2\ 1}}$$

So:

$$\sin\psi = K_1\frac{M_1}{\omega^2}$$

$$\cos\psi = \frac{a_1\omega_1 r_1}{\omega r} = K_2\frac{\omega_1}{a_1}$$

Where

$$K_2 = a_1\frac{r_1}{r}$$

So:

$$\cos\psi = K_2\frac{\omega_1}{\omega}$$

$$tg\,\psi = \frac{K_1\frac{M_1}{\omega^2}}{K_2\frac{\omega_1}{\omega}} = \frac{K_1}{K_2}\frac{M_1}{\omega\omega_1} = K_3\frac{M_1}{\omega\omega_1} \tag{21}$$

Where

$$K_3 = \frac{K_1}{K_2}$$

Results that:

$$tg\,\psi = K_3\frac{M_1}{\omega\omega_1}$$

From $\sin^2\psi + \cos^2\psi = 1$ one obtains:

$$K_1^2\left(\frac{M_1}{\omega^2}\right)^2 + K_2^2\left(\frac{\omega_1}{\omega}\right)^2 = 1 \tag{22}$$

$$K_1^2 M_1^2 + K_2^2(\omega\omega_1)^2 = \omega^4 \tag{23}$$

Special cases

Primary engine torque is maintained consistently (M=ct)

Energy equation:

$$E_t = M\cdot\omega = M_1\cdot\omega_1$$

for M=ct=K_4 becomes:

$$\omega = \frac{M_1}{M}\omega_1 = \frac{1}{K_4}M_1\omega_1 = K_5 M_1\omega_1$$

where

$$\frac{1}{K_4} = K_5$$

Introducing this value to *(23)* it becomes:

$$K_1^2 M_1^2 (K_5 M_1\omega_1^2)^2 = K_5^4 M_1^4\omega_1^4$$

$$\frac{K_1^2}{\omega_1^4} + K_2^2 K_5^2 = K_5^4 M_1^2 \Rightarrow$$

$$M_1 = \frac{1}{K_5^2}\sqrt{K_3^2 K_5^2 + \frac{K_1^2}{\omega_1^4}}$$

We note:

$$\frac{1}{K_5^2} = K_6$$

$$K_2^2 K_5^2 = K_7 \Rightarrow M_1 = K_6\sqrt{K_7 + \frac{K_1^2}{\omega_1^4}}$$

From $E_t = M\cdot\omega = M_1\cdot\omega_1$ after replacing M_1 with the above expression we obtain:

$$E = K_6\omega_1\sqrt{K_7\omega_1^2 + \frac{K_1^2}{\omega_1^4}} = K_6\sqrt{K_7\omega_1^2 + \frac{K_1^2}{\omega_1^2}}$$

so:

$$E = K_6\sqrt{K_7\omega_1^2 + \frac{K_1^2}{\omega_1^4}}$$

M_1 and E *relationship* can be written in the following form:

$$M_1 = K_6\sqrt{K_7}\sqrt{1 + \frac{K_1^2}{K_7}\frac{1}{\omega_1^4}}$$

$$E = K_6\sqrt{K_7}\sqrt{\omega_1^2 + \frac{K_1^2}{K_7}\frac{1}{\omega_1^4}}$$

We note

$$K_6\sqrt{K_7} = K_8$$

$$\frac{K_1^2}{K_7} = K_9$$

We obtain the expressions for M_1 and E:

$$M_1 = K_8\sqrt{1 + \frac{K_9}{\omega_1^4}} \tag{24}$$

$$E = K_8\sqrt{\omega_1^2 + \frac{K_9}{\omega_1^2}} \tag{25}$$

For high values of the secondary torque ω_1 a continuously rising speed in the primary torque ω is produced and of the energy E, cases in which starting from the energy's equation

$$E = M\cdot\omega = M_1\cdot\omega_1 = K_4\omega = ct$$

from *(24)* for

$$\omega_1 \to \infty \Rightarrow M_1 \approx K_8 = M_n = constant$$

From *(25)* for $\omega_1 \to \infty \Rightarrow E = K_8\cdot\omega_1$

So

$$E = K_4\omega \approx K_8\omega_1$$

$$M_1 = i_t M_n \tag{26}$$

We consider

$$a = \frac{\omega}{\omega_{max}} \text{ si } a_1 = \frac{\omega_1}{\omega_{max}}$$

Which we meet in *(24)*:

$$M_1 = K_8\sqrt{1 + \frac{K_9}{a_1^4\omega_{max}^4}} = K_8\sqrt{1 + \frac{K_{10}}{a_1^4}}$$

where

$$K_{10} = \frac{K_9}{\omega_{max}^4}$$

But it was proven that $M_l = M_n = K_8$, so M_l becomes

$$M_1 = M_n\sqrt{1 + \frac{K_{10}}{a_1^4}} \quad (27)$$

$$\omega\sqrt{2} = \sqrt{\omega_1^2 + \frac{1}{\omega_1^2}} \quad (28)$$

The curves in Fig. 2.1 show that the speed of the mayor is going through a minimum for a certain value of the secondary velocity. If this minimum particular value is considered as a measurement unit for the primary velocity, and at the same the secondary particular velocity as a mass measurement unit, when the primary goes through the minimal value we represent through ω_M și ω_1 the numbers that measure ω and ω_1 in the same units. Formulas *(24)* and *(25)* show the simple numerical form.

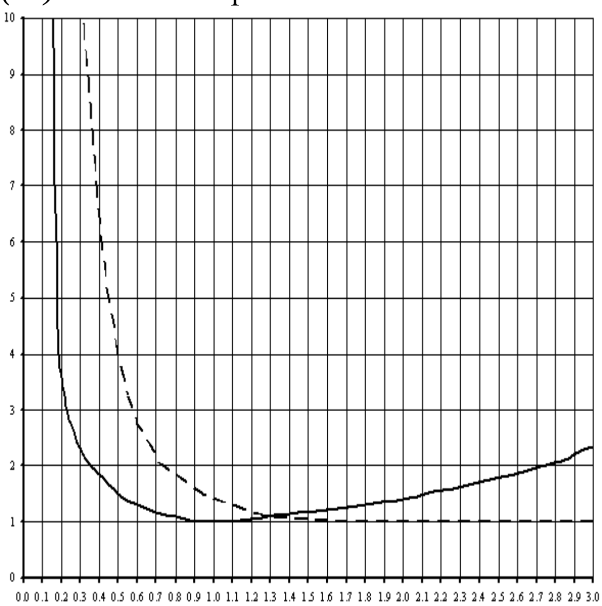

Fig. 2.1 The converters feature for the case when primary engine torque is constant; relative values: M_l – the torque to the secondary drive shaft; ω - primary shaft torque; ω_l – secondary shaft torque

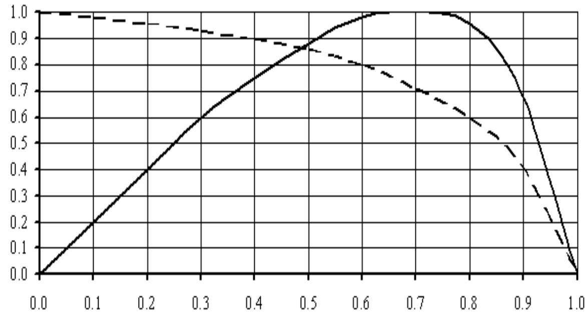

Fig.2.2 The converters feature when the primary engines torque is constant; relative values: M_l – the secondary shafts torque; M – the primary shaft torque; ω_l - secondary shaft rotation speed

The main engines torqueω , is constant

In the energy equation $E = M \cdot \omega = M_1 \cdot \omega_1$ if $\omega = c = ct.$, the fundamental equation *(23)* then becomes:

$$K_1^2 M_1^2 - K_2^2 c^2 \omega_1^2 = c^4$$

$$\frac{K_1^2}{c^4} M_1^2 - \frac{K_2^2}{c^2}\omega_1^2 = 1 \Rightarrow$$

$$M_1^2 = \frac{c^4}{K_1^2}\left(1 - \frac{K_2^2}{c^2}\omega_1^2\right) \text{ or}$$

$$M_1 = \frac{c^2}{K_1}\sqrt{1 - \frac{K_2^2}{c^2}\omega_1^2}$$

Replacing M_l in the energy equation, one obtains:

$$E = Mc = M_1\omega_1 \Rightarrow M = \frac{M_1\omega_1}{c}$$

So

$$M = \frac{c}{K_1}\omega_1\sqrt{1 - \frac{K_2^2}{c^2}\omega_1^2}$$

We note

$$\frac{c}{K_1} = c_1 \;; \quad \frac{K_2^2}{c^2} = c_2 \;; \quad cc_1 = c_2$$

Thus

$$M_1 = c_3\sqrt{1 - c_2\omega_1^2} \quad (29)$$

$$M = c_1\omega_1\sqrt{1 - c_2\omega_1^2} \quad (30)$$

This equation shows that the torque from main shaft M, as well as the secondary M_l, disappear when ω_1 reaches a maximum value represented by . $c_2\omega_1^2 = 1$

Also it is noted that at start the secondary torque M_l, has the mass value (for $\omega_1 = 0$). If we express the torque at the beginning with M_s and we take as a measurement unit the secondary shaft torque ω_1 , this maximum torque ω_s , corresponding to zero load conditions, and represent through ω_1 the number that measures ω_1 with such a unit, the formula (29) takes the following form:

$$\frac{M_1}{\omega_1} = 0 = M_s$$

Or out of

$$\frac{M_1}{\omega_1} \neq 0 = M_s \sqrt{1 - c_2 \omega_1{}^2}$$

We note

$$a_1 = \frac{\omega_1}{\omega_{\max}}$$

Thus

$$\frac{M_1}{\omega_1} \neq 0 = M_s \sqrt{1 - c_2 a_1{}^2 \omega_{\max}{}^2} = M_s \sqrt{1 - c_4 a_1{}^2}$$

so

$$M_1 = M_s \sqrt{1 - c_4 a_1{}^2} \quad (31)$$

Formula (30) shows that the primary engine torque M, is zero at the start and increased to a maximum value M_{max}, which is reached when the derivatives cancel each other:

If we note

$$a_1 = \frac{\omega_1}{\omega_{\max}}$$

We obtain

$$\omega_1 = a_1 \omega_{\max}$$

Hence relationship *(30)* becomes:

$$M = c_1 a_1 \omega_{\max} \sqrt{1 - c_2 a_1{}^2 \omega_{\max}{}^2} = c_4 a_1 \sqrt{1 - c_5 a_1{}^2}$$

We note

$$c_4 = c_1 \omega_{\max}$$

And raise to the square root

$$M^2 = c_4{}^2 a_1{}^2 - c_4{}^2 a_1{}^2 c_5 a_1{}^2 = c_4{}^2 a_1{}^2 - c_4{}^2 c_5 a_1{}^4$$

Replacing and canceling the derivative results::

$$2 c_4{}^2 a_1 - 4 c_4{}^2 c_5 a_1{}^2 = 0$$

Or

$$1 - 2 c_5 a_1{}^2$$

So

$$a_1 = \frac{1}{\sqrt{2 c_5}} = \frac{\sqrt{2}}{2} \frac{1}{\sqrt{c_5}} = 0{,}71\, c_6$$

Where

$$c_6 = \frac{1}{\sqrt{c_5}}$$

Having reached this point, the primary engine torque decreases again and is cancelled for

$a_1 = 1$.

The transmission ratio between the secondary torque ω_1 and the primary ω is maximum at this point. If we note this maximum ratio with $i_{t\,max}$ one can write:

$$i_{t\,\max} = \frac{\omega_1}{\omega} = \frac{M}{M_1}$$

Or

$$i_{t\,\max} = \frac{a_1 \omega_{\max}}{a\, \omega_{\max}} = \frac{M}{M_1}$$

We obtain

$$i_{t\,\max} = \frac{a_1}{q_{\,a_1 = 1}} = \frac{M}{M_{\,a_1 = 1}\, 1} \quad (32)$$

If we note with M_{max} the maximum value of the primarys transmission timing, the equation *(30)* becomes:

$$M = 2 M_{\max}\, a_1 \sqrt{1 - a_1{}^2} \quad (33)$$

And from *(32)*, taking into account *(31)* and *(36)* results:

$$M_{\max} = \frac{1}{2} i_{t\,\max} M_s \quad (34)$$

7. CONCLUSION

1. The first conclusion is that: determining the dynamic behavior of the mechanism can be done experimentally, without the need for analytical determination of dependency relationship between the angle of the input shaft and the swinging element. This relationship can be approximated using stochastic algorithms without coupling valve dynamics study.
2. The second conclusion is that: the analytical form of the dependence between the angles , can be used for a complete dynamic study of the mechanism. Coefficients, which enter into this relationship can be determined by imposing dynamic conditions

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Calculul unghiului de iesire fata de cel de intrarea la un convertor

Rezumat: Convertor este un mecanism proiectate astfel încât toate părţile funcționale, în afară de arbori primare şi secundare oscila în jurul anumite poziții medie cu aceeaşi frecvență armonice mişcare.Toate piesele de acest tip sunt supuse forțe interne care sunt, de asemenea, armonice şi au aceeaşi frecvență.

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