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MATHEMATICAL MODEL & ROBOT DESIGN OF AUTONOMOUS UNDERWATER VEHICLES (AUV) AND REMOTELY OPERATED VEHICLES (ROV)

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Abstract: *Autonomous underwater vehicles are currently being utilized for a large scale of applications like: military application, research application, commercial or rescue application. These vehicles require autonomous guidance and control systems in order to perform underwater tasks. Modeling and control of these vehicles are still major active area of research and development, involving many of the research centers around the world. Modeling of these robots, involve the application of the dynamic model of an underwater vehicle and the identification of the relevant parameters that affect the movement of the robot. Initially these parameters have to approximate, and improve after the model of robot.*
Key words: *AUV, Autonomous underwater vehicle, mathematical model, robot design, ROV, remote operate vehicle.*

1. INTRODUCTION

The last years revealed a tendency for use the Autonomous Underwater (AUV) and also Remotely Operated Vehicles (ROV) for many of the tasks that involve a deep diving, a dangerous environment or any other reason that make inaccessible or difficult for the mission of the human operator. There are a plenty robots developed for certain specified task: like ship hull cleaning [3], underwater welding [4], underwater rescue missions[5], research missions, underwater surveillance of borders, underwater cartography[6]; robot platforms that can be customized and equipped by client to meet specified tasks[4].

The robot proposed to be analyzed below is intended to be a platform, easy to be customized, to add software and hardware, and it intended to have also the capability to operate in remote mode and in autonomous mode. This will be a facility that removes the needs for two robots, and it will add a large field of research and expansions on the same platform. Also the low cost of the robot is another standpoint.

This field of underwater robots involves a multidisciplinary knowledge, starting from the data acquisition of the robot to the mechanical build, control systems, data acquisition systems, mathematical modeling, fluid dynamics, and a lot of other field. This paper focused on the mathematical modeling of an underwater robot proposed with four degrees of freedom.

2. MECHANICAL SYSTEM DESCRIPTION

The robot has two major parts for move itself in aquatic environment:

- the mathematical model based control
- correction from sensors acquisitions data

The mathematical based model is based on a system predefined by mathematical laws that describe the movement trough the surrounding environment of the robot. This analyzes the forces involved in movement, in correlation with degrees of freedom and the configuration of the vehicle.

The robot has a torpedo shape with two side thrusters (fig. 1). It has also neutral buoyancy. Inside of the robot is placed a two degree of freedom platform on which is placed the battery pack which has the biggest mass from the components of the robot. The position of the robot is determined by the position of the center of volume that is positioned above the center of weight (fig.2). Moving the battery pack on the X axis will displace the center of weight forward or backward. This will cause the reorientation of the robot respecting the above law of two centers: the center of weight and the center of volume. Also displacing the battery pack left or right will reorient the robot and force to roll with a specified angle.

This kind of approach is designed for a low speed robot, and is a battery saver method because on other robots to maintain robots at a specified angle involve a continuous movement of the thrusters for that direction of movement. Also this approach, reduce the cost using only two thrusters for displacing the robot.

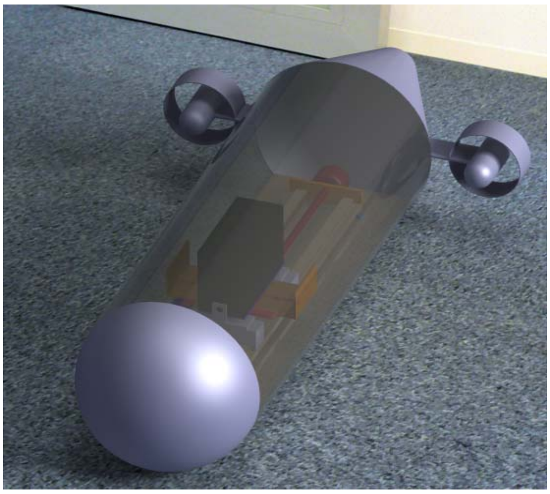

Fig. 1. Robot configuration with two thrusters

3. GEOMETRICAL MODELING THE UNDERWATER ROBOT

The equation from the direct geometrical model, give us the position and the orientation of the robot structure according to the fixed reference system of coordinates. The mobile underwater platform, execute a general movement, so the law of movement according to a reference system of coordinates fixed 0 is express by relation (1.1):

The above expression contains all the six independent parameters, the first three of them are related to position and the other three are related to orientation for robot. Therefore the robot, theoretically possess all the six degrees of freedom.

$$\underset{(3\times1)}{{}^{0}\overline{X}} = \begin{pmatrix}\overline{r}_P \\ \overline{\Omega}\end{pmatrix} = \left[\begin{array}{ccc} \big(x_p(t)=q_1(t) & y_p(t)=q_2(t) & z_p(t)=q_3(t)\big)^T \\ \hline \big(\psi(t)=q_4(t) & \theta(t)=q_5(t) & \phi(t)=q_6(t)\big)^T \end{array}\right] \quad (1)$$

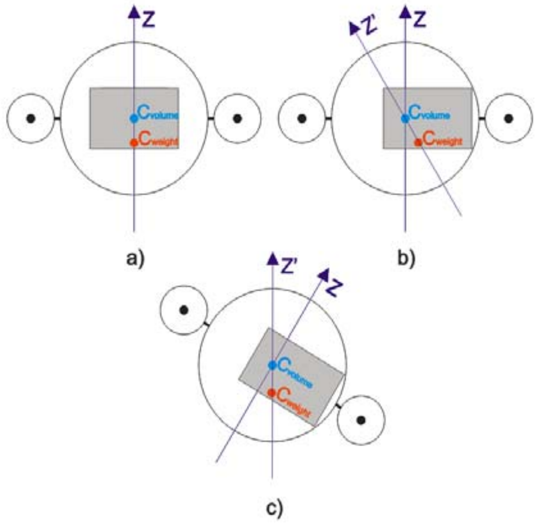

Fig. 2. Robot configuration with two thrusters Position for center of volumes according to center of weight: a) the center of volumes is above the center of weight; b) displacing the center of masses by moving the battery pack will reposition the center of weight; c) the whole system will be reposition itself according to the new inputs.

The orientation of the platform are realized with three independent rotations on the axis z, x_1 and z_2 defined with the following matrix of rotation:

$$R(\overline{z};\, q_4) = \begin{bmatrix} \cos q_4 & -\sin q_4 & 0 \\ \sin q_4 & \cos q_4 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$R(\overline{x}_1;\, q_5) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos q_5 & -\sin q_5 \\ 0 & \sin q_5 & \cos q_5 \end{bmatrix} \quad (2)$$

$$R(\overline{z}_2;\, q_6) = \begin{bmatrix} \cos q_6 & -\sin q_6 & 0 \\ \sin q_6 & \cos q_6 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

The resulting matrix of rotation that characterize the orientation of system R with origin in P attach to the mobile structure of the robot according to reference system 0 and the inverse of matrix that represent the orientation of the fixed system reported to the mobile system is described in following equations [1]:

$$ {}^{0}_{R}[R] = \begin{bmatrix} -sq_1 \cdot cq_2 \cdot sq_3 + cq_1 \cdot cq_3 & -sq_1 \cdot cq_2 \cdot cq_3 - cq_1 \cdot sq_3 & sq_1 \cdot sq_2 \\ cq_1 \cdot cq_2 \cdot sq_3 + sq_1 \cdot cq_3 & cq_1 \cdot cq_2 \cdot cq_3 - sq_1 \cdot sq_3 & -cq_1 \cdot sq_2 \\ sq_2 \cdot sq_3 & sq_2 \cdot cq_3 & cq_2 \end{bmatrix} $$

$$ {}^{R}_{0}[R] = {}^{0}_{R}[R]^{-1} = \begin{bmatrix} -sq_1 \cdot cq_2 \cdot sq_3 + cq_1 \cdot cq_3 & cq_1 \cdot cq_2 \cdot sq_3 + sq_1 \cdot cq_3 & sq_2 \cdot sq_3 \\ -sq_1 \cdot cq_2 \cdot cq_3 - cq_1 \cdot sq_3 & cq_1 \cdot cq_2 \cdot cq_3 - sq_1 \cdot sq_3 & sq_2 \cdot cq_3 \\ sq_1 \cdot sq_2 & -cq_1 \cdot sq_2 & cq_2 \end{bmatrix} \tag{3} $$

From now the notation for trigonometrically equations will be used as follow: $\cos x = cx$ and $\sin x = sx$ in order to simplify the equations. The vector of speeds that describe the absolute movement of the robot has the following form:

$$ {}^{0}\dot{\bar{X}} = \begin{pmatrix} \dot{\bar{r}}_p \\ \dot{\bar{\Omega}} \end{pmatrix} = \begin{bmatrix} \left(\dot{x}_p(t) = \dot{q}_1(t) \quad \dot{y}_p(t) = \dot{q}_2(t) \quad \dot{z}_p(t) = \dot{q}_3(t) \right)^T \\ \left(\dot{\psi}(t) = \dot{q}_4(t) \quad \dot{\theta}(t) = \dot{q}_5(t) \quad \dot{\phi}(t) = \dot{q}_6(t) \right)^T \end{bmatrix} \tag{4} $$

Transfer of the vectors of the linear speed from the fix system of coordinates 0 in the system attached to the robot R is described by the matrix equation with has the following form:

$$ {}^{R}\dot{\bar{r}}_p = \begin{bmatrix} {}^{R}\dot{x}_p \\ {}^{R}\dot{y}_p \\ {}^{R}\dot{z}_p \end{bmatrix} = {}^{0}_{R}[R]^{-1} \cdot \dot{\bar{r}}_p = \begin{bmatrix} \dot{q}_1(cq_1 \cdot cq_3 - sq_1 \cdot cq_2 \cdot sq_3) + \dot{q}_2(cq_1 \cdot cq_2 \cdot sq_3 + sq_1 \cdot cq_3) + \dot{q}_3(sq_2 \cdot sq_3) \\ \dot{q}_1(-sq_1 \cdot cq_2 \cdot cq_3 - cq_1 \cdot sq_3) + \dot{q}_2(cq_1 \cdot cq_2 \cdot cq_3 - sq_1 \cdot sq_3) + \dot{q}_3(sq_2 \cdot cq_3) \\ \dot{q}_1(sq_1 \cdot sq_2) + \dot{q}_2(-cq_1 \cdot sq_2) + \dot{q}_3(cq_2) \end{bmatrix}; \tag{5} $$

If the movement of the robot is realized only after x_R axis, what denote that de slip on the axis y_R and z_R is impossible in infinitesimals movement, then the vector of speed has to have the following form:

$$ {}^{R}\dot{\bar{r}}_P = \left[{}^{R}\dot{x}_p \quad 0 \quad 0 \right]^T \tag{6} $$

and as result a constrain appears for axis of the mobile system for y_R and z_R:

$$ \begin{cases} \dot{q}_1(-sq_1 \cdot cq_2 \cdot cq_3 - cq_1 \cdot sq_3) + \dot{q}_2(cq_1 \cdot cq_2 \cdot cq_3 - sq_1 \cdot sq_3) + \dot{q}_3(sq_2 \cdot cq_3) = 0 \\ \dot{q}_1(sq_1 \cdot sq_2) + \dot{q}_2(-cq_1 \cdot sq_2) + \dot{q}_3(cq_2) = 0 \end{cases} \tag{7} $$

This equation can be rewritten by its differential form like:

$$ \begin{cases} dq_1(-sq_1 \cdot cq_2 \cdot cq_3 - cq_1 \cdot sq_3) + dq_2(cq_1 \cdot cq_2 \cdot cq_3 - sq_1 \cdot sq_3) + dq_3(sq_2 \cdot cq_3) + 0 \cdot \sum_{i=4}^{6} dq_i = 0 \\ dq_1(sq_1 \cdot sq_2) + dq_2(-cq_1 \cdot sq_2) + dq_3(cq_2) + 0 \cdot \sum_{i=4}^{6} dq_i = 0 \end{cases} \tag{8} $$

The above equation did not satisfy the conditions of Cauchy so as a result the robot has only 4 degrees of freedom.

4. CONSIDERATION ABOUT THE TRUSTER AND PROPULSION EQUATIONS

There are two ways most used for underwater propulsion:

- Jet propulsion
- Propeller propulsion (most used)

I will consider the best solution for this robot the propeller propulsion solution because it is easy to use no needs for special body design and is also low cost.

The propeller propulsion is based on a propeller blade controlled by an engine (electrical, combustion, etc.).

Definition of power:

$$ W = F * \Delta d \tag{9} $$

where: W – is the work done on an object
F – is force
Δd - are the displacement of the object

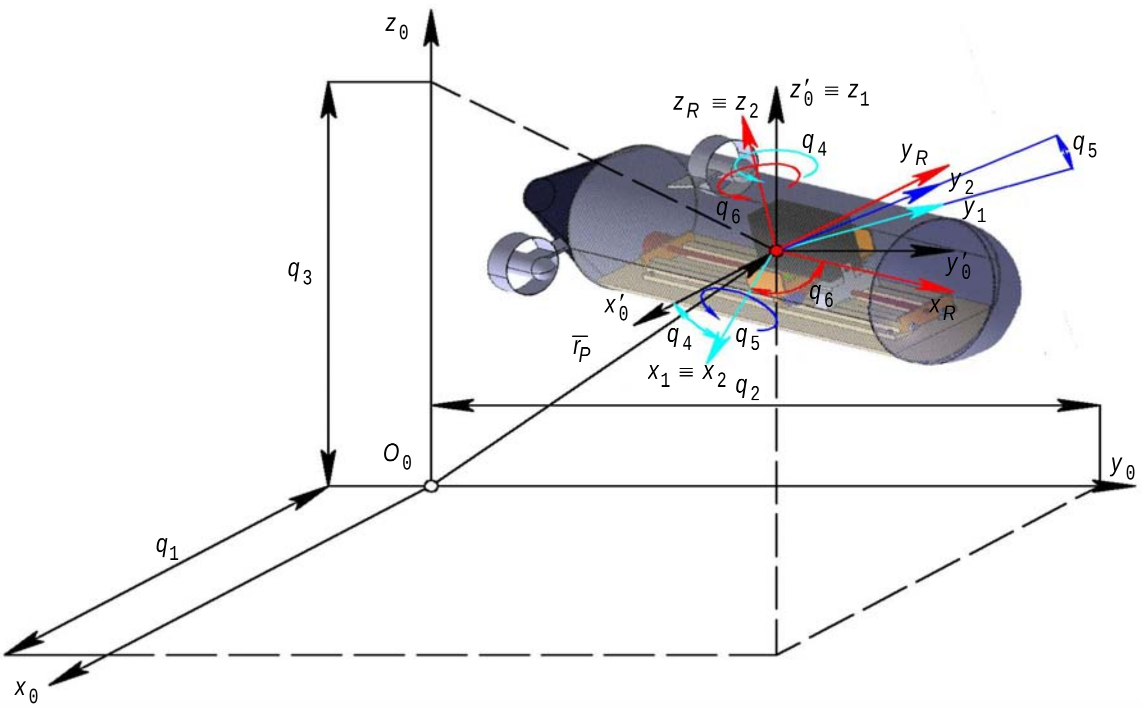

Fig. 3. The six degree of freedom related to a fixed system of coordinates

The shape of propeller and its diameter are the major determinants factor for the propulsion efficiency. The surface of the blade is a helicoidally portion. The blade act like a screw, in water a full rotation are theoretically equivalent to an axial movement at a certain distance determined by the helicoidally parameters of the blade.

If a point on blade is considered, at a full rotation the blade movement of the point on the surface swept out a helicoidally surface.

Angle of attack is the angle between the line of reference (on a body) and the flow lines oncoming on the body.

The second important parameter of the blade is the length of the blade. The length of the blade is determined by the difference between diameter of the propeller and the boss diameter. The boss diameter is the diameter on which the blades are attached to propeller shaft.

The major forces (F) acting on any point of the blades are determined by the surface of blade (area of blade), the angle of attack (α) and velocity (V).

The force has two components:

- the force, normal to the direction of flow give the lift/displacement component (L);
- the drag force – this have the same direction with the direction of flow (D);

Reynolds number is a dimensionless number used in fluid mechanics that in a given flow conditions give a measure for the ratio of inertial forces to viscous forces [2].

$$R_e = \frac{\rho V L}{\mu} = \frac{VL}{\nu}\ ; \qquad \nu = \frac{\mu}{\rho}\ ; \qquad (10)$$

where:

V – Velocity of the object relative to fluid (m/s)

L – Characteristic linear dimension (m)

μ – Dynamic viscosity of the fluid (m*s)

ν – Kinematic viscosity (m^2/s)

ρ – Density of the fluid (kg/m^3)

Conform to this we obtain from the relation:

$$\frac{F}{\rho A V^2} = f(R_n, \alpha)\ ; \qquad (11)$$

this:

$$C_L = \frac{L}{\frac{1}{2}\rho AV^2};\quad (12)$$

$$C_D = \frac{D}{\frac{1}{2}\rho AV^2};\quad (13)$$

Each coefficient: lift coefficient (C_L) and coefficient of direction of flow (C_D) are function of Reynolds' number and angle of attack.

Propeller thrust

This is calculated for a single blade, taking an arbitrary radial section of a blade at r; the number of revolutions is N. Consider this the velocity of rotation is: $2 \cdot \Pi \cdot N \cdot r$. Note P the pitch of the blade. Theoretically, in a solid at a complete rotation the blade, advance at rate: NP. But if the blade is submerged in water, the advance rate is lower than a solid. Let's note V_a the water advance. So the difference is the "slip" ratio. Refer to these notations the slip ratio is:

$$Slip\_ratio = \frac{NP - V_a}{NP} = 1 - \frac{J}{p}\quad (14)$$

Where:

$$J = \frac{V_a}{ND} \text{ the advance coeficient}\quad (15)$$

$$p = \frac{P}{D} \text{ the pitch ratio}\quad (16)$$

Consider the dA force of drag on blade and dL the force normal to the surface:

$$dL = \frac{1}{2}\rho C_L [V_a^2(1+a)^2 + 4\pi^2 r^2 (1-a')^2] bdr \quad ..\quad (17)$$

$$dD = \frac{1}{2}\rho C_D [V_a^2(1+a)^2 + 4\pi^2 r^2 (1-a')^2] bdr\quad (18)$$

The thrust force T on blade:

$$dT = dL\cos\varphi - dD\sin\varphi\quad (19)$$

where note with :

$$\tan\beta = \frac{dD}{dL} = \frac{C_D}{C_L}\quad (20)$$

$$V_1 = \frac{V_a(1+a)}{\sin\varphi}\quad (21)$$

$$dT = \frac{1}{2}\rho C_L \frac{\cos(\varphi+\beta)}{\sin^2\varphi\cos\beta} bdr\quad (22)$$

From above we obtain the thrust and transverse forces by integrating expression along the blade:

$$dM = \frac{1}{2}\rho V_1^2 C_L \frac{\sin(\varphi+\beta)}{\cos\varphi} brdr\quad (23)$$

The torque is obtained by substitute V_1 and multiply by r :

$$dQ = rdM = \frac{1}{2}\rho C_L \frac{V_a^2(1+a)^2\sin(\beta+\varphi)}{\cos\beta\sin^2\varphi} brdr\quad (24)$$

The total power of thrust is proportional to TV_a and the shaft power to $2\pi NQ$.

From this consideration the efficiency is:

$$\frac{TV_a}{2\pi NQ}\quad (25)$$

and the blade element efficiency is:

$$B_{el.ef} = \frac{V_a}{2\pi Nr} \cdot \frac{1}{\tan(\varphi+\beta)}\quad (26)$$

So, the thrust (T) and torque (Q) depend on propeller diameter (D) and the rate of advance (V_a) number of revolution (N).

5. THE TOTAL PERFORMANCE

The total performance is a function of performance of the hull (here come the shape analysis), propeller efficiency, relative to the rotative efficiency divided by the appendage coefficient and all multiply by transmission efficiency.

$$TP = \frac{H_e \cdot P_e \cdot R_{re}}{A_c} \cdot T_e \qquad (27)$$

where:

TP-total performance

H_e -hull efficiency;

P_e -propeller efficiency;

A_c -appendage coefficient;

T_e -transmission efficiency.

6. CONCLUSIONS

The evaluation of parameters offer a complex image that can help us in build physical the robot, but also can help us to start programming the robot, evaluate the forces obtained in real environment, or to build and program the simulators.

The model, provide only an approach to the real environment because this model did not contain the turbulences from areal environment and other factors that can affect the performance and stability.

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REFERENCES

[1] I. Negrean “Robotics:kinematic and dynamic modeling”, Bucuresti:EDP, 1998

[2] D. Opruta ,”Dinamica fluidelor” -[curs], Cluj-Napoca: Universitatea Tehnica din Cluj-Napoca, 1998

[3] Embient cooperates - ROVING BAT data sheet - http://embient.de/products_ROV.html

[4] http://www.seaeye.com/products.html

[5] WHOI-operated robotic vehicle *Jason* - www.**whoi**.edu

[6] Sonobot Advanced Autonomous Cartography - http://www.evologics.de/en/products/sonobot/index.html

Modelarea matematică & proiectarea robotului AUV şi operarea de la distanţă a acestuia

Rezumat: Vehicolele autonome submersibile sunt utilizate pe o scala larga de aplicatii cum ar fi apliccatii militare, aplicatii de cercetare sau comerciale, etc. Aceste AUV-uri ecesita sisteme de cotrol si comanda pentru a realiza anumite sarcini. Modelarea si cotrolul lor ridica si in ziua de astazi probleme deoarece sistemele actuale sunt continuu imbunatatite si perfectionate, insa ramanand lipsuri contiuue lasand loc cercetarii si dezvoltarii.

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