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INFLUENCE OF THE CUTTING TOOL GEOMETRY ON THE SURFACE GENERATION PROCESS

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Abstract: *The paper analyses the influence of the geometric characteristics of Corner Radius End Mill and Ball Nose End Mill on the surface generation process. A calculation of the volume removed and the volume of material left over by each end mill is done. Equations for the volume of material left over by the tool between two adjacent passages for a given length are also developed.*
Key words: *milling, Corner Radius End Mill, Ball End Mill, cusp, machining*

1. INTRODUCTION

Milling is the most common form of machining. It is a material removal process in which a cutting tool removes unwanted material from a workpiece to produce the desired shape.

Milling tools are highly diverse, as can be expected from such a widespread process.

End mills are the primary cutting tools used in a vertical mill. The basically forms of mills are: Flat End Mills, Corner Radius End Mill and Ball End Mills.

When machining a surface it is desired that the amount of stock material leftover in the finished product is minimal. Flat End Mills will cut flat area without any material leftover. Corner Radius End Mill and Ball End Mills will leave some material between two consecutive passes on flat areas.

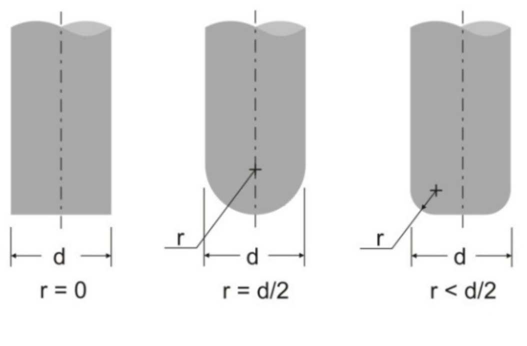

Fig. 1. The most common End Mills [1]

An original mathematical model for calculating the volume removed and the volume left over by each Corner Radius End Mill and Ball End Mill is proposed.

2. POPOSED MATHEMATICAL MODEL

A calculation of the volume removed and the volume of material left over by each end mill is done, without taking into account the tool movement between adjacent passes.

2.1 Corner Radius End Mill Geometry

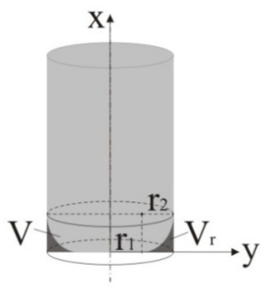

Fig. 2. Corner Radius End Mill material removal volume and its material left volume

Based on the parameters shown in Fig. 2 the volume removal V of the torus end mill is found by:

$$V=\pi\int_0^{r_2} f^2(x)\,dx \qquad (1)$$

where:

$$f^2(x)=y^2=r_2^2+r_2^2-(x-r_2)^2+2r_2\sqrt{r_2^2+(x-r_2)^2} \quad (2)$$

Eq. (2) is replaced in Eq. (1):

$$V=\pi\Big[\int_0^{r_2} 2r_2^2\,dx-\frac{(x-r_2)^3}{3}\Big|_0^{r_2}+ +2r_2\int_0^{r_2}\sqrt{r_2^2+(x-r_2)^2}\,dx\Big] \quad (3)$$

Calculation of the integrals leads to:

$$V=\frac{10\pi+3\pi^2}{6}r_2^3 \quad (4)$$

From the difference of (2) and (6) results the material volume that cannot be removed by the Corner Radius End Mill:

$$V_r=\frac{\pi r_2}{6}\left[6r_1(r_1+r_2)+r_2^2(3\pi-4)\right]\ [mm^3] \quad (5)$$

2.2 Ball End Mill Geometry

The volume of the spherical cap is calculated using the triple integral formula:

$$V_c=\iiint_{V_c} dx\,dy\,dt \quad (6)$$

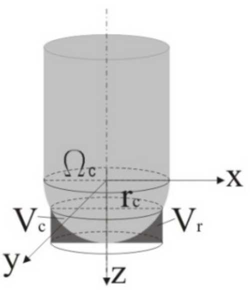

Figure 3. Ball End Mill material removal volume and its material left volume

Based on the parameters shown in Fig. 3 the calculation of triples integral is reduced to a double integral:

$$V_c=\iint_{\Omega_c} dx\,dy\int_{z_1}^{z_2} dz \quad (7)$$

where $z_1=r-h$ and $z_2=\sqrt{r^2-x^2-y^2}$

Considering the sphere eq. (8):

$$S: x^2+y^2+z^2=r^2 \quad (8)$$

Therefore, (8) becomes:

$$V_c=\iint_{\Omega_c}\left(\sqrt{r^2-x^2-y^2}-r+h\right)dx\,dy \quad (9)$$

In (8) and (9) the spherical cap projection occurs in xOy plane, denoted by Ωc. Therefore, Ωc can be defined as:

$$\Omega_c: x^2+y^2\le r_c \quad (10)$$

were the r_c radius is obtained by:

$$r_c^2=r^2-(r-h)^2 \quad (11)$$

Converting to polar coordinates (9) becomes:

$$V_c=\int_0^{r_c}\delta\,d\delta\int_0^{2\pi}\sqrt{r^2-\delta^2}\,d\theta-(r-h)\pi r_c^2 \quad (12)$$

From the integral (13) calculation result:

$$V_c=\frac{\pi h^2(3r-h)}{3} \quad (13)$$

If h=r the spherical cap becomes a semisphere, then:

$$V_c=\frac{2\pi r^3}{3} \quad (14)$$

Knowing the volume of a sphere classic formula:

$$V_s=\frac{4\pi r^3}{3} \quad (15)$$

The left material volume V_r can be written as a difference between the volume of the cylinder of r_c radius and height h:

$$V_r=V_{cil}-V_c \quad (16)$$

Hence,

$$V_f = \frac{\pi}{3}\left[h^3 + 3rh(r-h)\right] \tag{17}$$

If h=r, considering the entire semisphere than, the material left volume is:

$$V_s = \frac{\pi r^3}{3} \tag{18}$$

This result is obtained from (16) writing that:

$$V_{cil} = \pi r^3 \tag{19}$$

and

$$V_c = \frac{1}{2}V_s = \frac{2\pi r^3}{3} \tag{20}$$

2.3 Cusp

Material removal and the amount of left material is a good way to determine which tool is better for a specific application; however, toolpaths require tools to move. For a global perspective a second factor should be used to evaluate the surface quality – cusp.

The cusp corresponds to the material left by the tool between two adjacent passages.

The nomenclature in this field is not unique and differs depending on the authors. In this paper the term cusp is used for the material left over rather than the term scallop.

2.3.1 Corner Radius End Mill

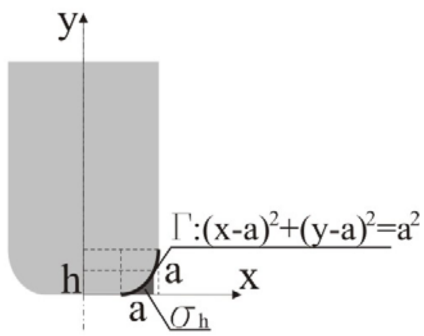

Fig. 4. A half o a cusp area corresponding to the Corner Radius End Mill geometry for a given step over

Based on Corner Radius Mill parameters, shown in Fig. 4, for simplicity only a half of cusp area, σ_h for a given stepover is calculated.

Stepover defines the distance the tool will move horizontally when making the next step. This distance is a percentage of the tool diameter.

$$\sigma_h = \int_h^a \left[a + \sqrt{a^2 - (x-a)^2}\right] dx \tag{21}$$

From the integral (21) calculation, result:

$$\sigma_h = \frac{a(a-h)}{2}\sqrt{1-\left(\frac{a-h}{a}\right)^2} + a(a-h) + \frac{a^2}{2}\arcsin\frac{a-h}{a} \tag{22}$$

If L is the length of the flat surface, the material left volume between two adjacent passages is obtained:

$$V_{cusp} = L\left[a(a-h)\sqrt{1-\left(\frac{a-h}{a}\right)^2} + 2a(a-h) + a^2\arcsin\frac{a-h}{a}\right] \tag{23}$$

2.3.2 Ball End Mill

The σ_h and V_{cusp} are determined for ball end mill.

In zOx plane the σ_h (half cusp area) can be calculated by the double integral:

$$\sigma_h = \iint_{\Omega_h} dz dx \tag{24}$$

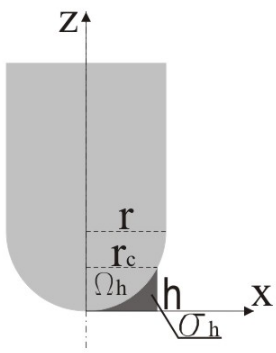

Fig. 5. A half o a cusp area corresponding to the Ball End Mill geometry for a given step over

The integral (24) can be writen as:

$$\sigma_h = h r_c - \iint_{\Omega_c} dy\,dx \tag{25}$$

Ω_c is the complementary of σ_h, see figure 5.

The double integral calculation in (25) is made by:

$$\iint_{\Omega_c} dy\,dx = \int_0^{r_c} dx \int_{r-h}^{\sqrt{r^2-x^2}} dz \tag{26}$$

or

$$\sigma_h = \frac{1}{2}\left(r r_c + h r_c - r^2 \arcsin \frac{r_c}{r} \right) \tag{27}$$

If L is the length of the flat surface, the material left volume between two passes is calculated:

$$\sigma_h = L \left(r r_c + h r_c - r^2 \arcsin \frac{r_c}{r} \right) \tag{28}$$

3. ACKNOWLEDGEMENT

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4. CONCLUSION

Knowledge of the cutting tool, especially their cutting capacity, which represents the essence of their use characteristic, determine in which measure a cutting tool corresponds for a prescribed purpose, allowing the possibility of a quality comparative appreciation of the two cutting tools studied.

5. REFERENCES

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INFLUENȚA GEOMETRIEI SCULEI ASUPRA PROCESULUI DE GENERARE A SUPRAFEȚEI

Rezumat: Lucrarea analizează influența caracteristicilor geometrice ale frezei cilindro-frontale cu rază la vârf şi ale frezei cilindro-frontale sferice în procesul de generare a suprafeței. Pentru fiecare freză în parte se calculează volumul de material îndepărtat respectiv volumul de material rămas. De asemenea, sunt determinate ecuațiile pentru volumul de material rămas între două treceri consecutive pentru o lungime dată.

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