TECHNICAL UNIVERSITY OF CLUJ-NAPOCA
Series: Applied Mathematics and Mechanics
Vol. 55, Issue II, 2012

THE RELIABILITY OF DIESEL MOTORS STUDIED THROUGH AHP OF FUZZY-WEIBULL TYPE

Paul MUREŞAN, Marcel POPA

Abstract: *Reliability and life expectancy represent the capacity of a motor to function without drops, respectively, the functioning period until damaged. This study is done for the following purposes: - adopt a more rational choice of construction material destined for the fabrication of the component pieces of the motor, meant to satisfy a certain type of given conditions, - improvement of technologies in order to produce the component pieces; - establishing a link between the characteristics of life duration of the different component parts, in order to make possible the use of preventive repairs of the whole ensemble; - highlighting and an accurate specification of the resistance calculation; - the rationalization and modernization of the experimental techniques on the validation of the proposed mathematical model. The used mathematical model is formed out of the normal and Weibull distributions and fuzzy triangular numbers, in order to build the AHP algorithm, which leads to the determination of local and global weights. Finally, a study of behavior of the fiability function is made, for a couple of particular chases of the Weibull parameters.*
Keywords*: reliability, probability density, repartition function, fuzzy numbers*

I. RANDOM VARIABLE AND PROBABILITY DISTRIBUTIONS ACCORDING TO THE TYPE OF FALLS AND DAMAGE

The life duration of a certain type makes no sense unless for a population of parts manufactured under similar manufacturing conditions and subjected to approximately the same functioning conditions.
If we take a look at the population of the parts of a diesel motor, from a mathematical point of view, this population is a random variable of discrete or continuous type.
In chase of the discrete type, we note with ξ the random variable, which has the distribution:

$$\xi: \begin{pmatrix} \xi_1 & \xi_2 \dots & \xi_n \\ p_1 & p_2 \dots & p_n \end{pmatrix} \quad (1)$$

where $\xi_k, k = 1 + n$ forms the values of the experimental measurments, respectively, p_k, the probabilities with which p_k, the probabilities with which ξ takes those values, meaning

$$p_k = P(\xi = \xi_k), \sum_{k=1}^{n} p_k = 1 \quad (2)$$

We mention that the random variable can be in a permanant or tranzitory state. In the chase ξ is continuous, so tranzitory,

$$\xi = \xi(x), x \in \mathbb{R} \quad (3)$$

the function defined in (3) becomes a random variable, if it's integrable and allows a primitive $F(x), x \in \mathbb{R}$, $F(x) \in [0,1]$ furthemore

$$F(x) = P(\xi \leq x) \quad (4)$$

with the propreties:

$$\lim_{x\to\infty} F(x) = 1 \text{ and } F(0) = 0 \quad (5)$$

In the considered aplications, we will word with random contious variables, defined for x≥0 and prolonged by 0 for x<0.
This mathematical model of random variables is gifted with four functions, which will have physical interpretations specific to the reliability process and life duration.
These functions are:

$f(x)$- probability density or the relative frecquency of breakdown occurrence

$F(x)$ - repartition function

$R(x) = 1 - F(x)$- reliability function

$\mu(x) = \frac{f(x)}{1-F(x)}$ -damage function

Between these functions there are dependence relationships, so as, if one of them is known, the other three can be determined and furthermore they can get the before mentioned interpretations. Before seting the additional binding formulas between the four functions, we mention that the probability density $f(x)$ must have the following qualities:

$$\int_{-\infty}^{+\infty} f(x)dx = 1 \qquad (6)$$

$$f(x) = F'(x) = \frac{dF(x)}{dx} \qquad (7)$$

$$F(x) = \int_{-\infty}^{x} f(t)dt = \int_{0}^{x} f(t)dt \qquad (8)$$

$$R(x) = P(\xi > x) \qquad (9)$$

$$F(x) + R(x) = 1 \qquad (10)$$

With this being said we can show that for μ(x) there is a conditioned probability which is, the conditioned probability that indicates the speed of degradation (of aging) in the x point.
If the definition of the four before mentioned functions and the fact that μ(x) can be written in the form of:

$$\mu(x) = -\frac{1}{R(x)} \cdot \frac{dR(x)}{dx} \qquad (11)$$

are taken into consideration, we have:

$$\mu(x)dx = \frac{P(x<\xi\leq x+dx)}{P(\xi>x)} \qquad (12).$$

It can be noticed that μ(x) is a function which leads to the assessment of the degradation speed, and shows that the degradation process is of Markov type. Between the four functions the following relations can we written: $R(x) = 1 - F(x) = e^{-\int_0^x \mu(t)dt}$
and

$$F'(x) = -R'(x) \qquad (13)$$

In this paper, the density of probability will be taken the bi- or triparametric Weibull function. By the manifestation model of the wear of the diesel motors parts, the densities of probability are associated as follows:
The component parts are not subject of an ageing process, excluding the grinding and wear periods. The fails in this case occur sudden, not being preceded by symptoms, partial fails or secondary damage.
Such a behavior is being mathematical shaped with a density of probability of Poisson type, that is with a discrete random variable.
The component parts subject to empirical observation, suffers in time an aging process, the rhythm of fails accelerating as time goes by, the fails being more dependent one of the other.
The mathematical shaping of these phenomenon's is realized through a normal law (Gauss Poisson), or through a lognormal law.
The case when parts present a complex structure and composition(dependent one of the other), having the component parts mechanical, as well as electrical or electronically, the reliability and life duration model through the density of probability Weibull, respectively, density of probability Weibull – of mixing.
This last type of processes, regarding the reliability and life duration of the diesel motor, makes the object of the present paper (the use of the Weibull model).

The biparametric Weibull model, used as relative frequency of fails (density of probability), has the form:

$$f_\xi(x,\beta,\lambda) = \begin{cases} 0, x < 0 \\ \beta\lambda x^{\beta-1} e^{-\lambda x^\beta} \end{cases} \qquad (14)$$

It can be observed that the density of probability f_ξ attached to the random variable $\xi = \xi(x)$ is a continuous function, because $f_\xi(0-0,\beta,\lambda) = f_\xi(0+0,\beta,\lambda) = f_\xi(0,\beta,\lambda) = 0$

.

The repartition function or the probability of appearance of fails can be calculated through:

$$F_\xi(x,\beta,\lambda) = \int_0^x f_\xi(t,\beta,\lambda)dt = \begin{cases} 0, x < 0 \\ 1 - e^{-\lambda x^\beta}, x \geq 0 \end{cases} \qquad (15)$$

It can be observed that   is a derivable function in relation with   , because $F_\xi'(0-0,\beta,\lambda) = F_\xi'(0+0,\beta,\lambda) = f_\xi(0,\beta,\lambda)$ furthermore:

$$\frac{dF_\xi(x,\beta,\lambda)}{dx} = f_\xi(x,\beta,\lambda), \forall x \in \mathbb{R} \qquad (16)$$

Similarly we study the reliability function $R_\xi(x) = P(\xi > x)$.

If $F_\xi(x) = P(\xi \leq x)$ shows us that in this chase the fails appear with this probability, the function $R_\xi(x)$ is the probability complementary to the event which indicates these fails, so $R_\xi(x)$ shows us the probability with which the fails do not happen, that means that the component part of the motor is fiable with this probability.

For this reason $R_\xi(x)$ caries the name of <u>reliability function</u>.

Therefor, the reliability function of Weibull type, using (10), has the form:

$$R_\xi(x) = e^{-\lambda x^\beta} \qquad (17)$$

Finally, $\mu_\xi(x)$, as defined in (11), leads to the conditioned probability: $\mu_\xi(x)dx = \frac{P(x<\xi\leq x+dx)}{P(\xi>x)}$ which is justified as follows:

From the propreties of the repartition function 8 F(x), we have: $P(a < X \leq b) = F(b) - F(a)$

According to the definition of the differential of a function g(x), we have :

$dg(x) = g(x+dx) - g(x)$

Using (7) we can write:

$$f_\xi(x)dx = dF_\xi(x) = F_\xi(x+dx) - F_\xi(x) = P(x < \xi \leq x+dx) \qquad (18)$$

Considering the relation:

$$\mu_\xi(x) = \frac{f_\xi(x)}{R_\xi(x)} \qquad (19)$$

results that $\mu_\xi(x)dx = \frac{f_\xi(x)dx}{R_\xi(x)} = \frac{P(x<\xi\leq x+dx)}{P(\xi>x)}$, as said in (12).

This formula has a special meaning regarding the physical sense given to the $\mu_\xi(x)$ function. This physical sense is justified by the notions of reliability and life duration of a mechanical part of a diesel motor.

Through the damage (death) of a part we understand the state of impossibility of further use in functioning of the motor, even if previously the part has been subject to a maintenance process, without her being replaced.

The fail of a part can mean damage, but even without it being repaired of maintained. Yet, the fail is a state of the part in which it doesn't function normally, at projected parameters.

So, we can observe that damage, in the proper sense of the word, is preceded (conditioned) by a fail.

Consequently, the probability of the appearance of damage is a probability conditioned by the appearance of fail.

From a mathematical point of view, the formula (12) expresses exactly that, so it is a probability of damage occurrence, conditioned by the appearance of fail.

This is the reason for which the $\mu_\xi(x)$ function carries the name of damage function, furthermore, it expresses the relative speed of degradation of the part. This statement is justified by the formula (11).

This is the physical sense given by the four functions underlying the mathematical modelation of the reliability phenomenon and the life duration of a part, or of the diesel motor which has it as a component.

II. THE GRAFIC ANALYSIS OF THE DAMAGE AND RELIABILITY FUNCTIONS FOR THE DIESEL MOTOR IN THE WEIBULL CHASE

In Figure 1.a) b) c) are presented the geometrical images of behavior (variation) of the monotony and dash, respectively of the asymptotic states for different particular cases of the damage function. To this purpose, we give a frequent used form, which leads to the definition of some physical signification for the parameters which are involved in the definition of the Weibull function $f_\xi(x,\beta,\lambda)$.

If the substitution is made:

$$\lambda = \frac{1}{\delta^{\beta}} \ or \ \delta = \lambda^{-\frac{1}{\beta}} \qquad (20)$$

then: $f_\xi(x,\beta,\delta) = \frac{\beta x^{\beta-1}}{\delta^{\beta}} e^{-\left(\frac{x}{\delta}\right)^{\beta}}$ (21)

For the form (21), we study the particular chases:

a) $\beta = \frac{1}{2}, \delta = \frac{1}{4}$

b) $\beta = 3, \delta = 2$

c) $\beta = 2, \delta = \sqrt{2}$

These particular chases were frequently used in application 12:

a) In this chase:

$$\mu_\xi\left(x, \frac{1}{2}, \frac{1}{4}\right) = \frac{1}{\sqrt{x}} e^{-2\sqrt{x}}, x > 0$$

The graphic of this function allows an vertical asymptote, for value of x=0 and a horizontal one, for value of y=0. We can see that the deviation from the asymptote is larger for the horizontal asymptote.

Therefore, if we consider that

$$\mu_\xi(x,\beta,\lambda) = -\frac{1}{R_\xi(x)} R'(x) \ and$$

$R_\xi(x) = e^{-\lambda x^{\beta}}$,

$R'(x) = -\lambda\beta x^{\beta-1} R_\xi(x)$

we have:

$$\mu_\xi(x,\beta,\lambda) = \lambda\beta x^{\beta-1} \qquad (22)$$

respectively

$$\mu_\xi(x,\beta,\lambda) = \frac{\beta}{\delta^{\beta}} x^{\beta-1} \qquad (23)$$

We use these three general formulas for three particular chases:

a) $\mu_\xi\left(x, \frac{1}{2}, \frac{1}{4}\right) = \frac{1}{\sqrt{x}}$

The graphic of this function is given by Fig 1.a

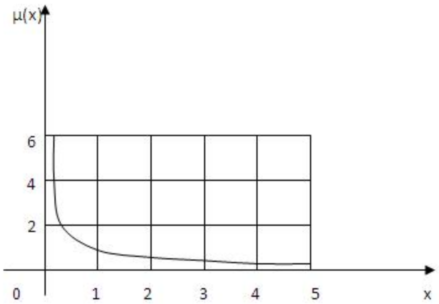

Fig. 1.a. Allure of the damage function

b) For $\beta = 3, \delta = 2$ we get:

$$f_\xi(x,3,2) = \frac{3x^2}{8} e^{-\frac{1}{8}x^3}$$

respectively $\mu_\xi(x,3,2) = \frac{3}{8}x^2$

In this chase the graphic for μ_ξ in the one in Figure 1.b

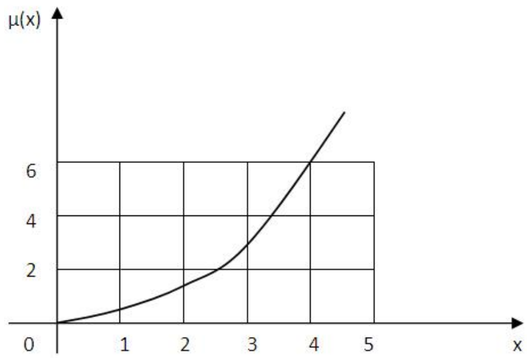

Fig. 1.b Allure of the damage function

c) For $\beta = 2, \delta = \sqrt{2}$ we get for the density of probability a particular chase, which is the Gauss-Laplace chase of the density of probability, so:

$$f_\xi(x, 2, \sqrt{2}) = xe^{-\frac{x^2}{2}}$$

respectively $\mu_\xi(x, 3, 2) = x$

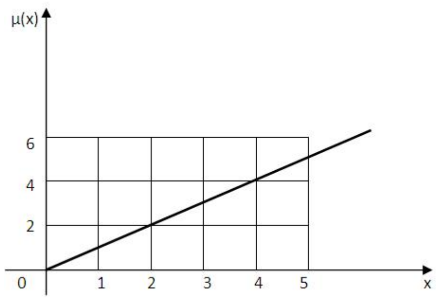

Fig. 1.c Allure of the damage function

These three remarkable examples show us as suggestively as possible that the problem of the parameter estimation from the Weibull model is very important, because at a grossly approximation of them, we can be driven to theoretical results, which don't match the studied phenomenon.

With the same values of the Weibull parameters, the reliability function is:

a) $R_\xi\left(x, \frac{1}{2}, \frac{1}{4}\right) = e^{-2\sqrt{x}}$

b) $R_\xi(x, 3, 2) = e^{-\frac{1}{8}x^3}$

c) $R_\xi(x, 2, \sqrt{2}) = e^{-\frac{x^2}{2}}$

The graphs for the three functions are given in Fig. 2 a,b,c.

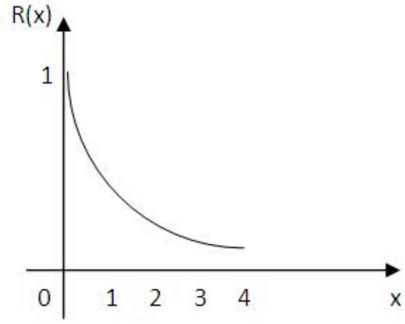

Fig. 2.a Allure of the reliability function

It is noted that compared to the graphs of the damage functions, the graphics of the functions of reliability are much more stable, so they show the behavior of the car body, but at different values given to the Weibull parameters, the probabilities R (x) don't differ so much. This shows that the damage function is more sensitive in relation to the behavior of the diesel motor part.

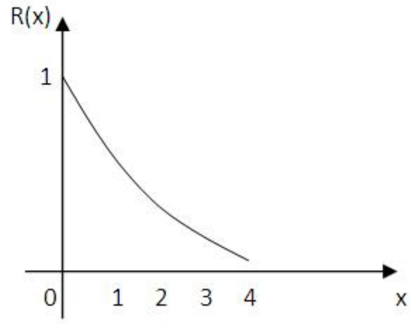

Fig. 2.b Allure of the reliability function

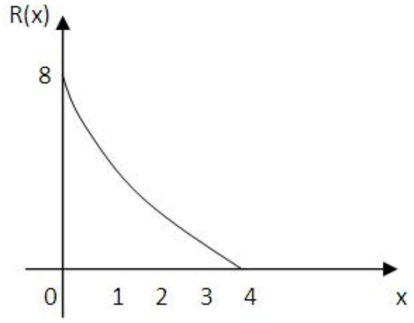

Fig.2.c Allure of the reliability function

Another important observation is related to lifetime (down time) which as we saw more accurately indicates the motors behavior from a mechanical point of view. If carried out an empirical study of a particular type of parts, components of diesel engine in two time periods, the first, before the first revision and the second one after the first revision, the relative frequency curve obtained by interpolation or a regression spline, it is possible (often) to come close to the normal distribution (Gauss-Laplace). This shows that, with all the Weibull distribution advantages, it is not a density of probability that solves in a good manner any problem that comes up in the diesel motors functioning process. In this sense, if wear is caused by the friction (lack of lubrication, lack of sufficient or too much cooling, etc.) empirical distribution shows a curve that is close to the bell of Gauss. For automotive parts two laws of probability can be noted.

The first relates to life duration in km (number of kilometers driven since the implementation of the automobile until the damage of the part taken into account).

The second law of probability of life duration refers to the physical time since the engine has been put into service, after sale, until the damage of the specific part.

Basically the two aspects show that the independent variable of the law of probability can be taken as a measure in kilometers or in time.

However the link between the two independent variables, does not refer to car speed, meaning the ratio km / hour. This relationship between km and physical time will be considered as caused by working conditions (exploitation), for example, will the report called monthly mileage, or mileage per year.

Empirical curves of life duration in km are obtained from maintenance workshops through their output data (by measurements referring to friction degradation or fatigue, etc).

These experimental data lead us to a random phenomenon, to be approximated by a law of probability (a relative frequency component of random variable that characterizes life duration). This law is suggested (in general) by the form of the empirical curve. Theoretical combined with experimental research for the feature called life duration, is generally realized, as already mentioned in the second time period. The first time period is the one between commissioning of the engine and the first overhaul, while the second is one that begins after the first overhaul and ends depending on the length of life duration. Empirical distributions of life duration on the two time periods or portions of these two time periods are generally different. It is therefore possible that these empirical distributions lead us to different theoretical probability laws, which are used for theoretical study of life duration of the same part. Of laborious experimental investigations it was found that the life duration due to mechanical abrasion on the first time period (until the first overhaul) of the piece under study is best approximated by the normal distribution (Gauss-Laplace).

In the second time period (after the first overhaul) when the engine is operated under normal conditions and the parts don’t have a too loose compliance (projected tolerance is not exceeded significantly) it is suggested as theoretical distribution, the gamma distribution (Γ). But if after the first overhaul the engine happens to be excesively exploited by large or very large dynamic loads, mechanical friction can lead to great compliance, it is recommended as theoretical distribution for the life duration of the part under study, the Weibull distribution.

III. THE AHP ALGORYTHM OF WEIBULL TYPE APLIED TO THE ESTIMATION OF THE RELIABILITY AND DAMAGE FUNCTIONS

The determination of the parameters which intervenes in the biparametric Weibull distribution of probability is done by applying the smallest square method, after a double logarithm in the reliability function has been made, which leads to the regression line 8.

Here we make reference to two types of parts A and B.
For this purpose we have:

$$\lambda = e^{a} \quad and \qquad (24)$$

$$\beta = \frac{C(T_n, Y_n)}{D^2(T_n)} \qquad (25)$$

$$a = M(Y_n) - \frac{M(T_n)C(T_n, Y_n)}{D^2(T_n)} \qquad (26)$$

where we used the notation from the average value, dispersion and correlation of the random variables.

In this case, the random variables are:

$$Y_n = \begin{pmatrix} y_1 & y_2 & \cdots & y_n \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \end{pmatrix} \qquad (27)$$

where is the weighted average value of the tolerance measurments from the „i_A” group of measured parts,

$$y_t = \sum_{k=1}^{t} y_{k,t} w_{k,t}^{A} \qquad (28)$$

$y_{k,t}$ represents the values of the tolerance measurments from the „i_A” group, and $w_{k,t}^{A}$ the local weights of the „i_A” group of measured parts, obtained through the FAHP algorythm.

The random variable T_n,

$$T_n = \begin{pmatrix} t_1 & t_2 & \cdots & t_n \\ \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \end{pmatrix} \qquad (29)$$

where $t_i, i = 1+n, t_i = \ln x_i, x_i > 0$ and x_i represents the average value of the tolerance measurments from the „i_A” group of measured parts. To simplify the exposure of the averages of the two types of parts, tehy will be calculated with the same type of fuzzy numbers, the truncated triangular ones, given through:

$$\begin{cases} \tilde{1} = [1, 3 - 2\alpha] \\ \tilde{3} = [1 + 2\alpha, 5 - 2\alpha] \\ \tilde{5} = [3 + 2\alpha, 7 - 2\alpha] \\ \tilde{7} = [5 + 2\alpha, 9 - 2\alpha] \\ \tilde{9} = [7 + 2\alpha, 11 - 2\alpha] \end{cases} \qquad (30)$$

and their inverses are given by:

$$\begin{cases} \tilde{1}^{-1} = \left[\frac{1}{3-2\alpha}, 1\right] \\ \tilde{3}^{-1} = \left[\frac{1}{5-2\alpha}, \frac{1}{1+2\alpha}\right] \\ \tilde{5}^{-1} = \left[\frac{1}{7-2\alpha}, \frac{1}{5+2\alpha}\right] \\ \tilde{7}^{-1} = \left[\frac{1}{9-2\alpha}, \frac{1}{7+2\alpha}\right] \\ \tilde{9}^{-1} = \left[\frac{1}{11-2\alpha}, \frac{1}{9+2\alpha}\right] \end{cases} \qquad (31)$$

The geometric image of the (α) cut fuzzy numbers, $\alpha \in (0,1), \tilde{1}, \tilde{3}, \tilde{5}, \tilde{7}, \tilde{9}$ is given in Figure 3.

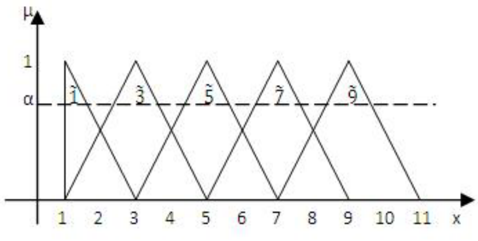

Fig.3. Triangular cut fuzzy numbers

Following this particular way of choosing the fuzzy numbers we can notice that, the number of the „i_A” or „i_B” group of measured parts can't be larger than five, but a numeric analysis of the obtained weights with these fuzzy numbers, can be made using the parameter. It is known that in the order matrix of the fuzzy numbers, at the occurrance of $\tilde{i}$, appears also his invert $\tilde{i}^{-1}$,

$i = 1,3,5,7,9$. We also make the statement that the average value obtained through this procedure, is of the best estimation. This way, we estimate that this procedure is of novelty regarding the study of reliability and life duration for diesel motors.

IV. CONCLUSIONS

This research project regarding reliability and life duration for diesel motors is general enough in nature, although the determination of the parameters which intervene in the reliability function, respectively the damage function, λ and μ is subject to the use of couple of two types of component parts A and B.

Generally, the forming of the A and B type parts couple is natural, because we make reference to the wear by friction, so, the friction must take place between at least two parts. In many papers, the reliability function respectively the damage function, are determined by other principles, not strongly binded with the experimental measurements. That is why, the suggested approximation is of superior nature. The results of the study are considerably better if the part has entered in the phase where the compliances become increasingly larger (the projected tolerances are significantly exceeded). We mention the fact that this numerical approximation procedure can be applied successfully even when the density of probability is one of Gauss-Laplace type, respectively Γ.

REFERENCES

[1] J.J.Buckley, T.Feuring and Y.Hayashi,*Fuzzy hierarchical abalysis revised*, European Journal of Operation Research 129, 2001

[2] D.Dubois and H.Prade, *The mean value of a fuzzy number, Fuzzy Sets and Systems*, vol 24, 1987

[3] A.Kaufmann and M.M.Gupta, *Introduction to Fuzzy Arithmetic Theory and Application,* Van Nostrand Reinhold, New York, 1991

[4] P.W.Leth, *Defence engineering plans military vehicle reliability*, Autom.Ind, vol 121, No12,1959

[5] H-X.Li and V.C.Yen, *Fuzzy sets and fuzzy decision making,* USA:CRC Press,New York, 1995

[6] J.H.K. Kao, *The Weibull distribution in the life testing of election tubes*, unpubl. paper pres.at meeting of Amer.Stat.Ass.,1955

[7] C.Kendreck, *Vehicle component replacement policy,* Rep. No5960-2, Dept.of.Eng.Proc.Univ. of Birmingham, 1961

[8] T.G.Potra, *Probabilităţi şi statistică matematică, Procese stochastice*, Ed.Transilvania Press,2005

[9] M.Vigier, *L'interprétation des essais d'endurance grâce aux technique de la fiabilite*, Bull.AFCIQ Mars 1965

[10] W.Weibull, *The phenomenon of rupture in solids*, Ingen.Vetesk,Akad.Handl.1953

[11] L.Zadeh, *The linguistic approach and its application to decision analysis, Proceedings of conference on directions in decentralized control*, Plenum Press, 1976

[12] A.H.Zaludova. *Problèmes de durée de vie: applications à l'industrie automobile, Revue de statistiqu appliquée*, tome 13, no.4(1965), p.75-98

Fiabilitatea motoarelor DIESEL studiată prin algoritmul AHP de tip Fuzzy-Weibull

Rezumat: Fiabilitatea şi durata de viaţă reprezintă capacitatea motorului de a funcţiona fără căderi, respectiv, perioada de funcţionare până la avarie. Modelul matematic folosit este format din distribuţiile normală şi Weibull, precum şi numere fuzzy triunghiulare, în vederea construirii algoritmului AHP, care conduce la determinarea ponderilor locale şi globale. Se realizează în final un studiu de comportament al funcţiei de fiabilitate pentru câteva cazuri particulare ale parametrilor lui Weibull.

Paul MURESAN, PhD Student, Ing., Technical University of Cluj-Napoca, Faculty of Machine Building,paul.muresan@evwholding.ro, +40720770078, str. Lunetei, no. 5, 400504 Cluj-Napoca.

Sabin Marcel POPA, Prof. Dr. Ing., Technical University of Cluj-Napoca, Faculty of Machine Building,marcel.popa@tcm.utcluj.ro , +40722365475, str. Baia Mare, no. 25, 400171 Cluj-Napoca.