TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue II, 2012

# DETERMINING THE MILLING TECHNOLOGICAL PARAMETERS USING THE STABILITY DIAGRAMS

**Dumitru POP, Liviu MORAR, Emilia CÂMPEAN, Ionela TĂNASE**

***Abstract:*** *Due to the dynamic behavior of the milling machine, the milling operation can proceed from the stable milling domain in the unstable domain although the depth of milling, the cutting speed and feed rate are rigorously calculated. The stable milling domains can be identified through stability charts which are calculated with specific algorithms that use as input data the modal parameters of the milling machine. We calculated and drew the stability diagrams modeling the cutter vibrations as a one degree of freedom oscillator. For this we implemented the semi-discretization algorithm in software written in Pascal language.*

***Keywords:*** *milling machine, stability diagram, semi-discretization algorithm.*

## I.INTRODUCTION

In the milling process is used a cutter that has one or more teeth. The action of each tooth is intermittent. At each entry of the tooth in the work piece the cutter is subjected to mechanical and thermal stresses.

Mechanical stress occurs in the form of vibrations. There are of two types of vibrations: forced vibration due to periodicity of cutting forces and self-excited vibrations.

We consider that the initial surface of the work piece that will be processed is perfectly flat. After the first pass of the cutter on that surface it will be slightly wavy. These undulations are due to vibrations of the spindle-tool holder-cutter-work piece assembly. Primary oscillations take place on one of the natural frequencies of the milling machine. At the next crossing of the cutter interference occur between the oscillation induced by the wavy surface and the primary current oscillation.

The resulting oscillation is a self-sustaining oscillation whose frequency depends on the rigidity of the spindle-tool holder-cutter-work piece system which is influenced by the milling depth.

The phase shift between primary and induced oscillation depends on the feed rate. If secondary oscillation amplitude increase than the milling process is leading to the unstable domain.

The self-sustaining vibrations produce poor surface finish and reduce the durability of the cutter. Usually, in an attempt to avoid the chatter in milling, it will be reduced the depth of cut, cutting speed and feed rate which lower the productivity and does not always guarantee a stable process.

## II. MODELING THE MILLING CUTTER AS A ONE DEGREE OF FREADOM MECHANICAL OSCILLATOR

Modeling the milling operation is made by taking into account the stiffness of the spindle - tool holder - cutter and work piece assembly. If the stiffness is higher the milling cutter will vibrate only in the feed direction. According to Butcher [13], Stephan [14], when the rigidity is low, the cutter vibrates in two directions: with high amplitude in the feed direction and with low amplitude in a perpendicular direction on the feed direction.

The motion equation of a one-dimensional oscillator, performing damped oscillations maintained by the action of external forces is:

$$m.\frac{d^2x}{dt^2}+c.\frac{dx}{dt}+k.x=F(t) \tag{1}$$

where:
m - masss
c - viscous damping coefficient
k - spring constant
x - deformation
t - time
F - the resultant of external forces

For the case of cutter vibration the external force is influenced by surface ripples processed at an earlier time, (t-τ):

$$m.\frac{d^2x}{dt^2}+c.\frac{dx}{dt}+k.x=F[a_0,x(t),x(t-\tau)] \qquad (2)$$

We introduce the following notations:

$$\frac{k}{m}=\omega_n^2 \qquad (3)$$

$$\frac{c}{m}=2.\zeta.\omega_n \qquad (4)$$

Now we can rewrite the equation of motion in the form:

$$\frac{d^2x}{dt^2}+2.\zeta.\omega_n.\frac{dx}{dt}+\omega_n^2.x=\frac{F[a_0,x(t),x(t-\tau)]}{m} \qquad (5)$$

where:
$\omega_n$ - natural pulsation
$\zeta$ - viscous damping factor
$\tau$ - time lag between successive crossings of the cutter teeth
$a_0$ - initial deformation

Equation (5) is a Mathieu type equation. According to Butcher [13] and Stephan [14] for a cutter with a number of z teeth the cutting force F is expressed as:

$$F=l_a.[x(t)-x(t-\tau)].\sum_{j=1}^{z}h[\alpha_j(t)].\sin[\alpha_j(t)].\{K_t.\cos[\alpha_j(t)]+K_n.\sin[\alpha_j(t)]\} \qquad (6)$$

where:
$l_a$ - removed chip width
$K_t$ - specific tangential cutting force
$K_n$ -specific axial cutting force

The function h [α (t)] shows us when the tooth cutter is in the work piece or not.

Because the cutter is acting intermittently due to its teeth, the law of motion is not continuous in time involving special methods for solving it.

Solving Mathieu periodic differential equations with time lag periodic is made using graphic-analytical methods [3], Chebyshev polynomials method [1] or semi discretization method [8].

## III. THE SEMI-DISCRETIZATION METHOD FOR SOLVING MATHIEU DIFFERENTIAL EQUATION

The equation of motion (5) is linearised and written in the form of Mathieu type equation.

In [8] is described in detail the solving of time lag Mathieu differential equations using the semi-discretization method.

The general form of time lag Mathieu differential equation is:

$$\frac{dx}{dt}=A(t).x(t)+B(t).x(t-\tau) \qquad (7)$$

$$A(t)=A(t+T_e) \qquad (8)$$

$$B(t)=B(t+T_e) \qquad (9)$$

**A, x** and **B** are n-dimensional matrix of type :

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ . \\ x_n(t) \end{bmatrix} \qquad (10)$$

$$A(t)=\begin{bmatrix} a_{11}(t) & . & . & a_{1n}(t) \\ a_{21}(t) & . & . & . \\ . & . & . & . \\ a_{n1}(t) & . & . & a_{nn}(t) \end{bmatrix} \qquad (11)$$

$$B(t)=\begin{bmatrix} b_{11}(t) & . & . & b_{1n}(t) \\ b_{21}(t) & . & . & . \\ . & . & . & . \\ b_{n1}(t) & . & . & b_{nn}(t) \end{bmatrix} \qquad (12)$$

Mathieu equation solution is:

$$X_k=\Phi.X_0 \qquad (13)$$

where: Φ is the transition matrix.
The transition matrix has the form :

$$\Phi=C_{k-1}.C_{k-2}...C_1.C_0 \qquad (14)$$

The stability of the solutions of this type of differential equations involves a rigorous demonstration because we have to treat all cases of indeterminacy.

In [8] is demonstrated that the solutions of the differential equation are stable when the eigenvalues of transition matrix Φ are smaller than unity. When the eigenvalues are complex numbers their module must be calculated.

The eigenvalue problem is represented on the form:

$$\Phi.X = \lambda.X \qquad (15)$$

where:
λ - eigenvalues
X - eigenvectors.

$$(\Phi - \lambda I).X = 0 \qquad (16)$$

In our particular problem, when the differential equation models the vibrations of the milling cutter, we plotted the graph on a Cartesian coordinate system. Depth cut is representing on the ordinate axis and spindle speed is representing on the abscissa axis. The area delimited by the graph and axes represents the stable domain of the milling process.

Matrices A and B will have the particular form:

$$A = \begin{bmatrix} 0 & 1 \\ -(\omega_n^2 + \dfrac{t_d.h_K}{m_m}) & -2.\zeta.\omega_n \end{bmatrix} \qquad (17)$$

$$B = \begin{bmatrix} 0 & 0 \\ \dfrac{t_d.h_K}{m_m} & 0 \end{bmatrix} \qquad (18)$$

where:
$t_d$ - milling depth
$m_m$ - modal mass

Time is divided into discrete intervals of the form: [ti, ti+1 ]:

$$\Delta t = t_{i+1} - t_i, i = 0,1,... \qquad (19)$$

The period of time $T_e$ on which we made calculations is:

$$T_e = n.\Delta t \qquad (20)$$

where:

$$n = \mathrm{int}\left(\frac{\tau + \dfrac{\Delta t}{2}}{\Delta t}\right) \qquad (21)$$

## IV. CALCULUS AND GRAPHICAL REPRESENTATION OF STABILITY DIAGRAMS

For calculus and graphical representation of stability diagrams was written a Pascal program using the Free Pascal Compiler 2.0.2 [7], [9]. In this program is implemented the semi-discretization algorithm used in solving the Mathieu equation. Current calculations have been made with the FMATH library. For complex matrix operations and eigenvalues were used the specific Matrices and Eigen, [4], [6] libraries.

The graphical representation was carried out using the scaling, rotation, translation and cropping routines described in [10].

Modal parameters presented in table 1, were obtained using experimental modal analysis on a FUS22 CNC milling machine at the spindle-tool holder-cutter level, as demonstrated in [11].

*Table1.*

**The modal parameters for cutters**

| Cutter D (mm) | Frequency (Hz) | Damping factor - | Modal mass - |
|---|---|---|---|
| 6 | 318.8 | 4.75 | 7.4234 |
| 6 | 940 | 3.75 | 0.1460 |
| 6 | 1297 | 1.48 | 0.0326 |
| 8 | 315.6 | 6.55 | 3.2244 |
| 8 | 803.1 | 10.4 | 0.2885 |
| 8 | 1509 | 2.56 | 0.0440 |
| 10 | 318.8 | 5.11 | 6.5944 |
| 10 | 834 | 9.8 | 0.4780 |
| 10 | 1609 | 2.84 | 0.0556 |
| 12 | 315.6 | 5.56 | 6.8925 |
| 12 | 809.4 | 8.14 | 0.6359 |
| 12 | 1656 | 1.85 | 0.1449 |
| 14 | 318.9 | 4.71 | 6.8640 |
| 14 | 843.8 | 7.28 | 0.5558 |
| 14 | 1706 | 3.72 | 0.0918 |

Stability diagrams are presented in Figures 1 to 15. In this diagram is plotted depth of cut dependence by spindle speed to the point where the eigenvalues of transition matrix Φ is equal to unity.

Field determined by the graphic and abscises axis represent the stable milling domain.

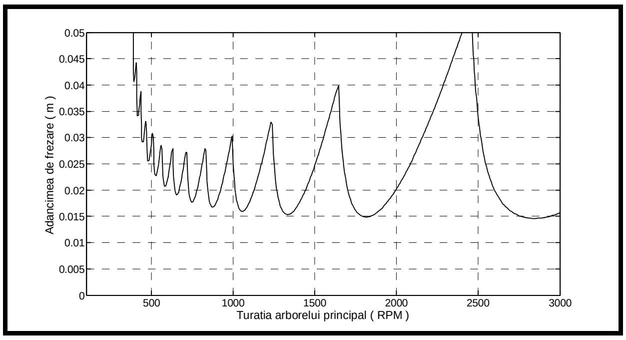

**Fig.1** The stability diagram for a 6 mm diameter Kennametal milling cutter at a frequency of 318 Hz

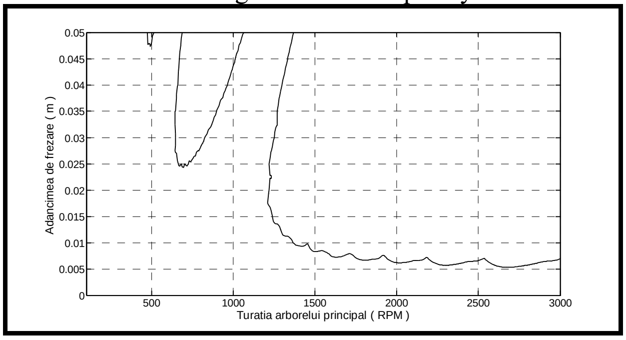

**Fig.2** The stability diagram for a 6 mm diameter Kennametal milling cutter at a frequency of 940 Hz

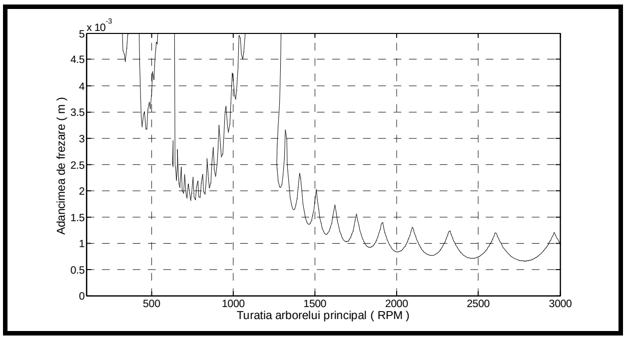

**Fig.3** The stability diagram for a 6 mm diameter Kennametal milling cutter at a frequency of 1297 Hz

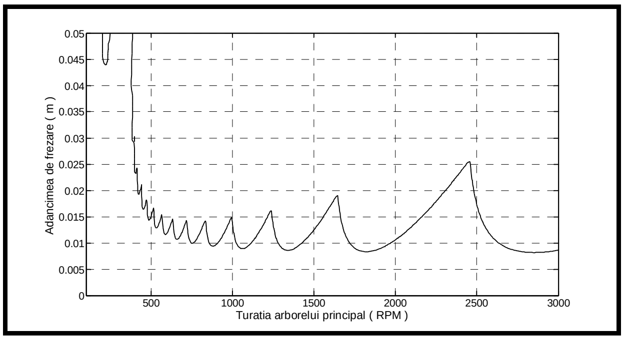

**Fig.4** The stability diagram for a 8 mm diameter Kennametal milling cutter at a frequency of 315 Hz

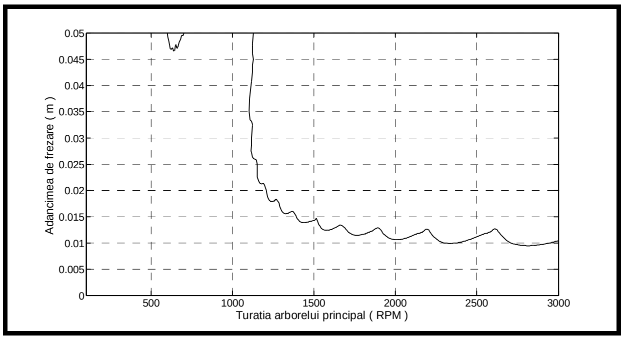

**Fig.5** The stability diagram for a 8 mm diameter Kennametal milling cutter at a frequency of 803 Hz

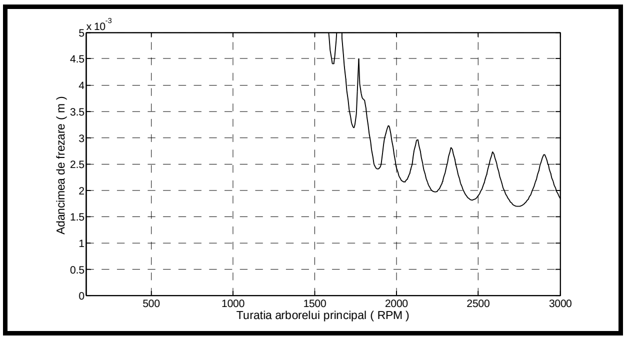

**Fig.6** The stability diagram for a 8 mm diameter Kennametal milling cutter at a frequency of 1509 Hz

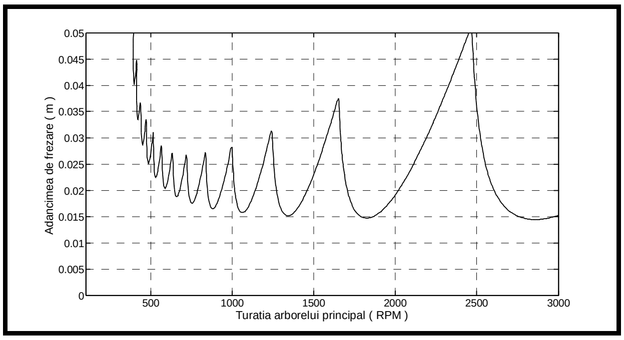

**Fig.7** The stability diagram for a 10 mm diameter Kennametal milling cutter at a frequency of 318 Hz

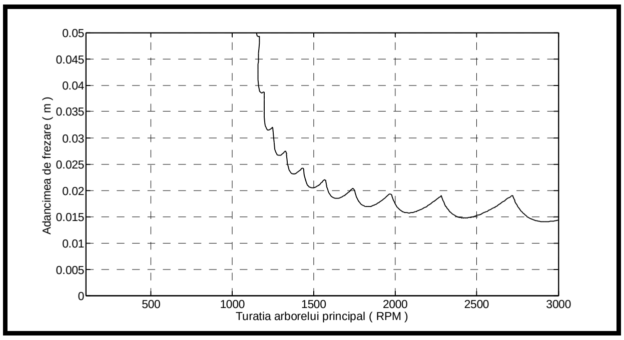

**Fig.8** The stability diagram for a 10 mm diameter Kennametal milling cutter at a frequency of 834 Hz

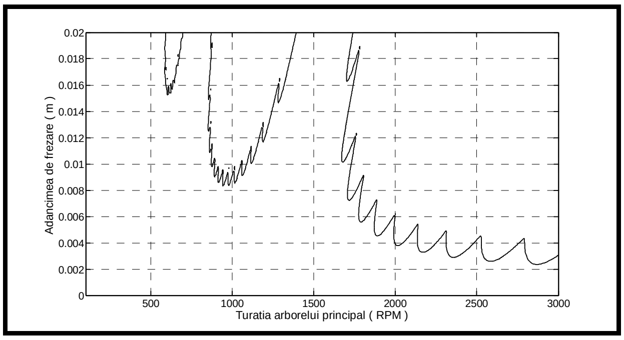

**Fig.9** The stability diagram for a 10 mm diameter Kennametal milling cutter at a frequency of 1609 Hz

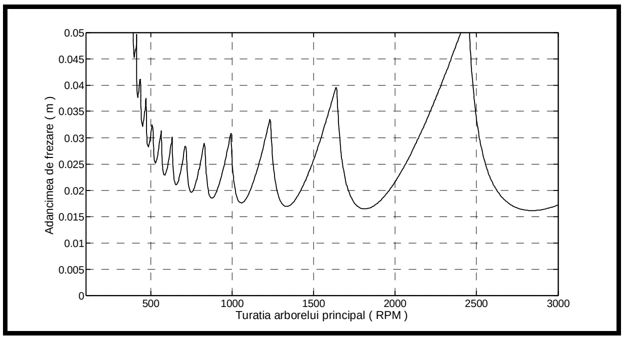

**Fig.10** The stability diagram for a 12 mm diameter Kennametal milling cutter at a frequency of 315 Hz

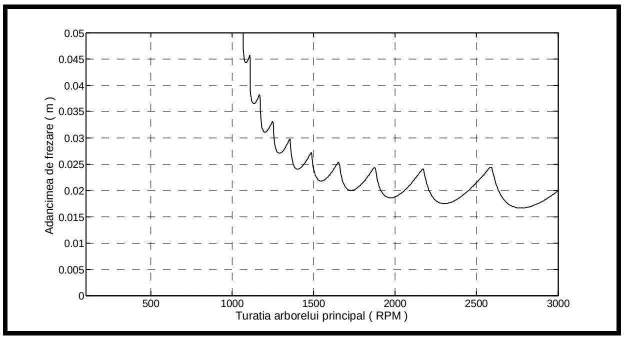

**Fig.11** The stability diagram for a 12 mm diameter Kennametal milling cutter at a frequency of 809 Hz

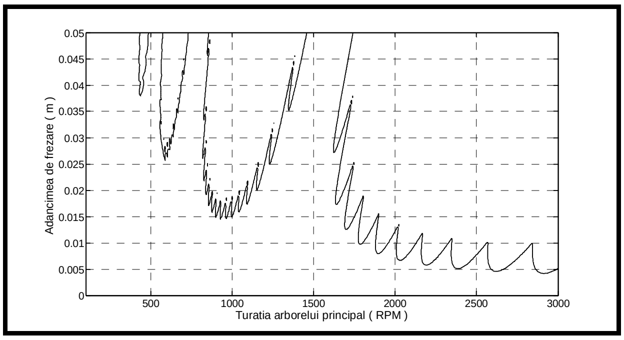

**Fig.12** The stability diagram for a 12 mm diameter Kennametal milling cutter at a frequency of 1656 Hz

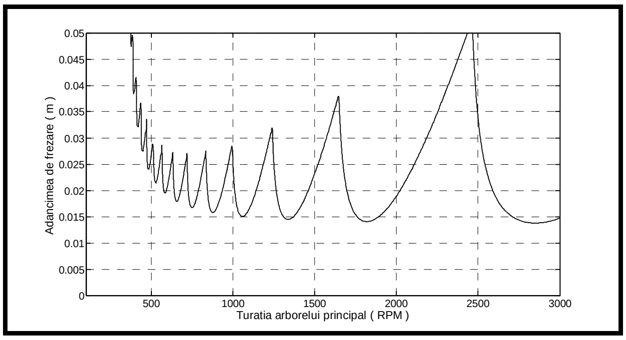

**Fig.13** The stability diagram for a 14 mm diameter Kennametal milling cutter at a frequency of 318 Hz

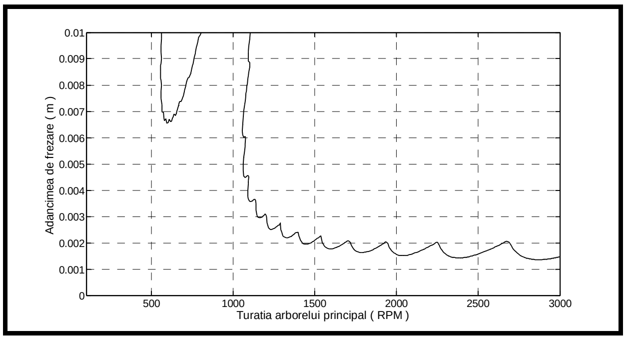

**Fig.14** The stability diagram for a 14 mm diameter Kennametal milling cutter at a frequency of 943 Hz

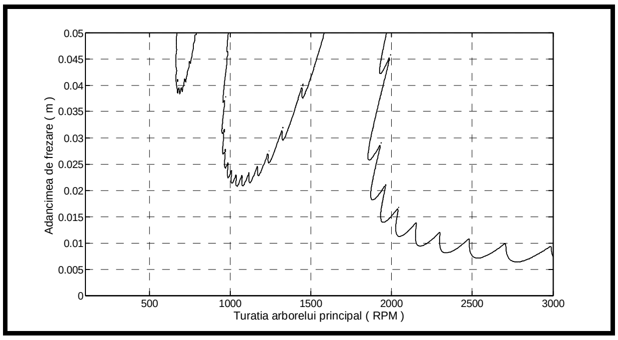

**Fig.15** The stability diagram for a 14 mm diameter Kennametal milling cutter at a frequency of 1706 Hz

## VI. ANALYSIS OF STABILITY CHARTS. CONCLUSIONS

For the cutter with 6 mm diameter at 2400 RPM the milling depth is limited to 5 mm by vibration mode at 940 Hz.

For the cutter with 8 mm diameter and a cutting speed in the 0 - 3000 RPM range, the milling process is stable for a depth of cut up to 1.5 mm. In the 0 - 1700 RPM range, the milling process is stable for a depth of cut up to 5 mm.

For the cutter with 10 mm diameter and a cutting speed in the 0 - 3000 RPM range, the milling process is stable for a depth of cut up to 2.5 mm. In the 0 - 1800 RPM range, the milling process is stable for a depth of cut up to 8 mm.

For the cutter with 12 mm diameter and a cutting speed in the 0 - 3000 RPM range, the milling process is stable for a depth of cut up to 4 mm. In the 0 - 1700 RPM range, the milling process is stable for a depth of cut up to 12 mm.

For the cutter with 16 mm diameter and a cutting speed in the 0 - 3000 RPM range, the milling process is stable for a depth of cut up to 6 mm. In the 0 - 1200 RPM range, the milling process is stable for a depth of cut up to 14 mm.

Restrictions are determined by cutter vibrations at frequencies of 1297 Hz, 1509 Hz, 1609 Hz, 1656 Hz and 1706 Hz. The values of these frequencies are slightly influenced by the tool holder, spindle and by the stiffness of the work piece.When the milling process takes place in the unstable domain, simple lowering of the cutting speed does not necessarily lead to the stable cutting.

We can conclude that the stability diagrams can be use to maintain a high productivity by changing simultaneously cutting speed and feed rate so total amount of metal chipped in a time equal to the durability of the cutter will be maximized [12].

## REFERENCES

[1] Butcher,E.A., H Ma , E., Bueler , Averina V, Szabó Zs., *Stability of time-periodic delay-differential equations via Chebyshev polynomials.* International Journal for Numerical Methods in Engineering ; 59(7):895–922, 2004

[2] Chiriacescu, S.,T., *Sisteme mecanice liniare,* Editura Academiei Române, 2007

[3] Chiriacescu, S.T., D.Mangeron, Crăciuanş A., *Metode grafoanalitice folosite în studiul stabilităţii sistemului dinamic de prelucrare pe maşini-unelte,* Edituara Academiei Republicii Socialiste România, 1986

[4]http://mathworld.wolfram.com/MatrixExponential.html

[5]http://www.mathworks.com/help/techdoc/ref/dde23.html

[6] http://matriisi.ee.tut.fi/~piche/ode/expm2/

[7] http://www.freepascal.org/

[8] Insperger, T., Stépán , G., *Semi-discretization method for delayed systems.* International Journal for Numerical Methods in Engineering 2002; 55(5):503–518

[9] Miller, A., *Pascal Programs for scientists and engineers*, Sybex Inc., 1981

[10] Pop, D., *Subrutine grafice 2D în limbajul BASIC implementate pe calculatorul Sinclair Spectrum*, Simpozionul Condinf, Cluj Napoca, 1988

[11] Pop, D.,Morar,L., Câmpean,E.,Tănase, I., *Experimental modal analysis of a spindle-tool holder-cutter assembly,* Acta Technica Napocensis, Vol. 55, Issue I, 2012, ISSN 1221-5872, pag.227-233

[12] Pop, D.,Morar,L., Câmpean,E.,Tănase, I., *Optimization of the technological parameters in the milling process,* Acta Technica Napocensis, Vol. 55, Issue I, 2012, ISSN 1221-5872, pag.221-227

[13] Seagalman, D.J., Butcher E.A., *Suppression of regenerative chatter via impedance modulation.* Journal of Vibration and Control 2000; 6:243–256

[14] Stépán, G., *Modelling non-linear regenerative effects in metal cutting.* Philosophical Transactions of the Royal Society 2001; 359:739–757

## Determinarea parametrilor tehnologici de frezare cu ajutorul diagramelor de stabilitate

**Rezumat**: *Datorită comportamentului dinamic al maşinii de frezat, operaţiunea de frezare poate trece din domeniul de frezare stabil în domeniul instabil deşi adâncimea de frezare, viteza de aşchiere şi viteza de avans sunt riguros calculate. Domeniile stabile de frezare pot fi identificate prin diagrame de stabilitate calculate cu algoritmi specifici care folosesc ca şi date de intrare parametrii modali ai maşinii de frezat. Am calculat şi reprezentat grafic diagramele de stabilitate asimilând vibraţiile frezei cu vibraţiile unui oscilator cu un singur grad de libertate. Pentru aceasta am implementat algoritmul de semidiscretizare într-un software scris în limbaj Pascal.*

**POP Dumitru,** prof.dr., Grupul Şcolar "V.Gelu" Zalău, S.Bărnuţiu 51, 0260-617697, dumitrupop@yahoo.com
**MORAR Liviu,**prof.univ.dr.ing,Universitatea Tehnică Cluj-Napoca, Department of Machine Tools and Industrial Robots, B-dul Muncii, 103-105, 0264401698, Liviu.Morar@staff.utcluj.ro
**CÂMPEAN Emilia**,drd.ing,Universitatea Tehnică Cluj-Napoca,Department of Machine Tools and Industrial Robots, B-dul Muncii, 103-105, 0264401698, Emilia.Campean@muri.utcluj.ro
**TĂNASE Ionela,** drd.ing**.** Universitatea Tehnică Cluj-Napoca, Department of Machine Tools and Industrial Robots, B-dul Muncii, 103-105, 0264401698, Ionela.Dincuta@muri.utcluj.ro