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COMPARATIVE METHODS FOR DETERMINING THE LINE OF INTERSECTION BETWEEN TWO PLANES

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Abstract: *In geometrical configuration of the parts of machine building, the edge parts can be treated as a line segment, which result from the intersection of two planes. The objective of this paper is to present two comparative methods for determine the intersection line between two planes. In the first part of this paper is determined the line of intersection between two planes with mathematical approach, and in the second part this problem is solved using descriptive geometry methods. In the mathematical model, each plane is defined by three points with respect to the reference system OXYZ. Using the cross product is determined the general equation of these two planes and the parametric form of the intersection line. In the graphical method, each planes are defined by the projections of three points in front view and top view, also is presented the axonometric representation for a better view of the intersection studied.*

Key words: *descriptive geometry, cross product, planes intersection.*

1. INTRODUCTION

The paper presents two comparative methods for determine the line of intersection between two planes. In the first part, applying the vector calculation is determined the general equation of these two planes and the parametric equation of the line of intersection between two planes [1], [2]. Each plane is determined by the three points with respect to the reference system *OXYZ*. The end of mathematical approach presents a tridimensional plot of these two planes and the intersection line generated in the Matlab programming environment, for a better visualization in tridimensional view [3].

The second part present the cutting plane method for determinate the line of intersection between two planes, using descriptive geometry methods [4]. The line of intersection is determined by locating the piercing points of lines from one plane with the other plane and drawing a line between the points.

2. MATHEMATICAL APPROACH

Applying the vector calculation is presented the mathematical approach for determining the line of intersection between two planes. The planes *[ABC]* and *[MNK]* are defined by the points: *A (65, 20, 40)*, *B (7, 50, 5)*, *C (3, 5, 30)* and *M (55, 12, 20)*, *N (28, 48, 40)*, *K (12, 10, 3)*. All dimensions from this paper are given in millimeters. The start point of this calculus is to find the normal vector for the plane *[ABC]*. The equation of the normal vector of the plane *[ABC]* is given by the next relation:

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} \qquad (1)$$

For determining the normal vector it must to find two directions in the plane. These two directions are given by the difference between the point *A*, *B* and *C* as follows:

$$\overrightarrow{AB} = (-58,\ 30,\ -35) \qquad (2)$$

$$\overrightarrow{AC} = (-62,\ -15,\ -10) \qquad (3)$$

The cross product $\overrightarrow{AB} \times \overrightarrow{AC}$ is a perpendicular vector to both, $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Next relation shows the cross product between these two directions from the plane [ABC]:

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} i & j & k \\ -58 & 30 & -35 \\ -62 & -15 & -10 \end{vmatrix} \quad (4)$$
$$=-825i+1590j+2730k$$

From the vector obtained derives the equation of the plane. The general equation of a plane [3] is written in next relation:

$$ax+by+cz+d=0 \quad (5)$$

To verify the results it used *B (7, 50, 5)* point as follows:

$$-825\cdot(x-7)+1590\cdot(y-50)+2730\cdot$$
$$\cdot(z-5)+d=0 \quad (6)$$
$$d=-87375$$

The general equation of plane *[ABC]* can be written as follows:

$$-825x+1590y+2730z=87375 \quad (7)$$

For second plane *[MNK]* the calculations keeps the same path as the first plane. The directions are given by the next two relations:

$$\overrightarrow{MN}=(-27,\ 36,\ 20) \quad (8)$$

$$\overrightarrow{MK}=(-43,\ -2,\ -17) \quad (9)$$

The cross product for the second plane is written as follows:

$$\overrightarrow{MN}\times\overrightarrow{MK}=\begin{vmatrix} i & j & k \\ -27 & 36 & 20 \\ -43 & -2 & -17 \end{vmatrix} \quad (10)$$
$$=-527i-1319j+1602k$$

The result is checked for *N (28, 48, 40)*, as follows:

$$-527\cdot(x-28)-1319\cdot(y-48)+1602\cdot$$
$$\cdot(z-40)+d=0 \quad (11)$$
$$d=15248$$

The general equation of plane *[MNK]* can be written as follows:

$$-572x+1319y+1602z=-15248 \quad (12)$$

Next step in this calculus is to find equation of the intersection line between planes *[ABC]* and *[MNK]*. To find an equation of this line it must to have a point and a direction. The planes equations are given by the next relation:

$$\begin{cases} -825x+1590y+2730z=87375 \\ -572x+1319y+1602z=-15248 \end{cases} \quad (13)$$

for determination of the point, it considers $y=0$:

$$\begin{cases} -825\cdot x+2730\cdot z=87375 \\ -572\cdot x+1602\cdot z=-15248 \end{cases} \quad (14)$$

results:

$$x=756,9579842$$
$$z=260,7565337 \quad (15)$$

results values are checked in the next two relations:

$$87375,00004=87375 \quad (16)$$

$$-15247,99997=-15248 \quad (17)$$

the cross product of these two planes is:

$$\overrightarrow{n_1}\times\overrightarrow{n_2}=\begin{vmatrix} i & j & k \\ -825 & 1590 & 2730 \\ -527 & -1319 & -1602 \end{vmatrix} \quad (18)$$
$$=-6148050i-117060j+1926105k$$

results of the cross product are verified by the next two relations:

$$n_1\cdot(\overrightarrow{n_1}\times\overrightarrow{n_2})=0 \quad (19)$$

$$n_2\cdot(\overrightarrow{n_1}\times\overrightarrow{n_2})=0 \quad (20)$$

Parametric form of the intersection line can write as follow:

$$\begin{cases} x(t) = 6148050 \cdot t + 756{,}9579842 \\ y(t) = -117060 \cdot t \\ z(t) = 1926105 \cdot t + 260{,}7567337 \end{cases} \quad (21)$$

Symmetric form of this line can write:

$$\frac{x - 756{,}9579842}{6148050} = \frac{y}{-117060} = \frac{z - 260{,}7567337}{1926105} \quad (22)$$

Next two relations show the results verification using parametric form of the line.

$$87375.5460 = 87375 \quad (23)$$

$$-15247.67958 = -15248 \quad (24)$$

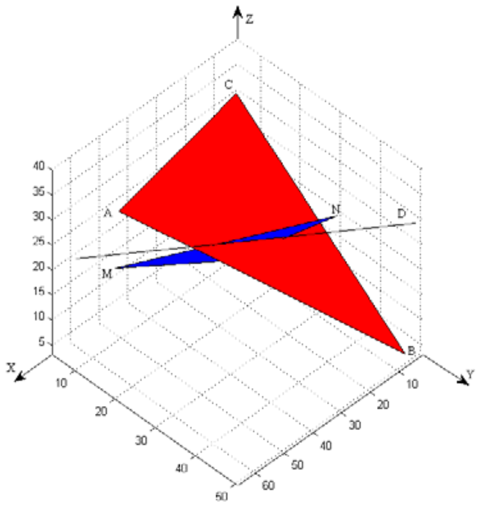

Fig.1. Matlab representation of the plane and the intersection line.

Mathematical solution of this problem has been solved in MATLAB programming environment. For a better view, has been developed a program that generates the planes and the intersection line between them. The tridimensional plot of line of intersection between two is presented in figure 1.

3. GRAPHICAL APPROACH

In this section is presented the necessary steps to determine the intersection line between two planes using cutting plane method. For graphic determination of this problem were study more paper from this field [5], [6], [7]. In figure 2 are represented the vertical and horizontal projection of the planes *ABC (abc, a`,b`,c`)* and *MNK(mnk, m`n`k`)*.

The start point of this method consist in creation of a cutting plane *[Q₁]* through line *m`n`* in the front view. This cutting plane intersects the plane *[a`b`c`]* at the points *1`* and *2`*. Project the points *1`* and *2`* into the top view and mark these points on lines *a`b`* and *a`c`*.

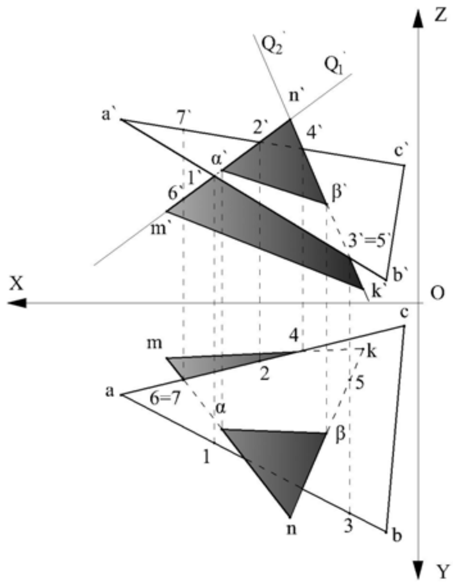

Fig.2. Intersection line between two planes using graphical method.

Draw the trace line given by the point *1* and *2* in the top view, and mark where this trace line crosses the line *mn*. The first piercing point is labeled *α*. Project *α* back into the front view, and mark the intersection point with *α`*.

In the second step of this method is created the second plane *[Q₂]* through the line *n`k`* in the front view. The plane *[Q2]* intersects the plane *[a`b`c`]* at the points *3* and *4*. Project the points *3`* and *4`* into the top plane on lines *ac* and *ab*. Draw a line between this points, and mark where the trace line crosses line *nk*, with *β*. Project *β* back into the front view and mark that point with *β`*. Draw a line in the both view between piercing point *α* and *β*. This is an intersection line between these two planes.

For visibility study in the horizontal projection is considered the points *6≡7* apparently overlapped in top plane and for vertical projection the points *3`≡5`*. In vertical projection is visible the point *3`*, respectively the side *ab*, and in top plane is visible the point *7*, respectively the side *ac*. Visibility and invisibility of the other segments result from the vertical and horizontal projection depending on the intersection line between these two planes. In figure 3 is presented the axonometric view of this plates solved using cutting plane method.

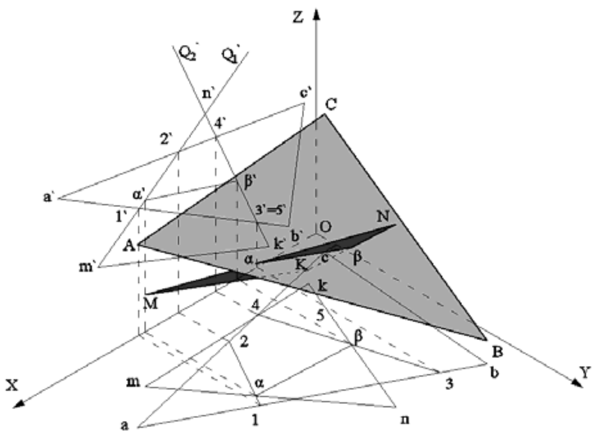

Fig.3. Axonometric view of the intersection between two planes.

4. CONCLUSIONS

Using the cross product it obtains the general equations of the planes. The intersection line between these two planes gives the parametric equation of this line. Plotting the planes and the line in Matlab [2] it studies the plane's orientation and visibility in tridimensional view. Solving this problem through graphical approach, we find an exact and easy way solution in a short time. In conclusion the mathematical method is a more difficult way to find the intersection line between these two planes, because the calculations can be easily mistaken.

5. REFERENCES

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METODE COMPARATIVE PENTRU DETERMINAREA LINIEI DE INTERSECȚIE DINTRE DOUĂ PLANE

În configurația geometrică a pieselor din construcția de maşini, muchiile acestora, asimilate cu segmente dreaptă, rezultă la intersecția a două plane. În această lucrare se prezintă două metode comparative pentru determinare a liniei de intersecție dintre două plăci. În prima parte a lucrării este determinată linia de intersecție dintre plăci prin abordare matematică, iar in a doua parte a lucrării această problemă este rezolvată folosind geometria descriptivă. În modelul matematic, fiecare placă este definită de trei puncte raportate unui sistem de cordonate OXYZ. Folosind produsul vectorial se determină ecuațiile generale ale acestor două plăci şi ecuația parametrică a liniei de intersecție dintre ele. În metoda grafică plăciile sunt definite de către proiecțiile punctelor in plan frontal şi orizontal, de asemenea se prezintă şi reprezentarea axonometrică a problemei studiate pentru o mai bună vizualizare.

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