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CONTRIBUTION TO THE OPTIMIZATION OF THE RELATIVE MOTION ON A VIBRATING CONVEYOR

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Abstract: *In this article the problem of optimizing the relative motion on a vibratory conveyor surface is presented. The relative motion characteristics were determined using a C program developed by the authors. Based on the Grid method it was determined the maximum relative motion speed considering the influence of various parameters.*
Key words: *lamellar spring, linear vibratory conveyor, vibration, C program, optimization, Grid method.*

1. INTRODUCTION

The studies concerning the vibratory conveyors have a great theoretical and practical interest. The vibratory conveyors are used to transport different types of materials with established speeds [2], [3], [6].

Many specialized books and articles studied these conveyors. These equipments are actuated with frequencies close to the resonance. Conveyors are capable to transport a wide variety of materials on long and short distances. The main advantages of such conveyors are their simple construction, their suitability for handling hot or abrasive materials and their applicability as dosage equipments.

The main components of these conveyors are the trough 1, the elastic elements 2, the actuator, the inertial generator 3 and the basement 4 (Figure 1).

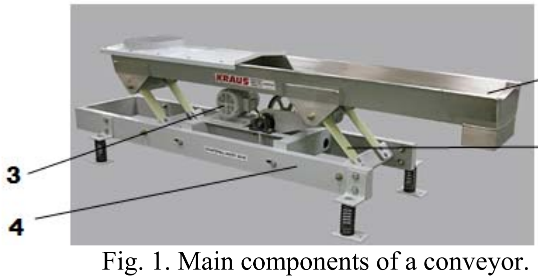

Fig. 1. Main components of a conveyor.

The purpose of our paper is to determine the optimal constructive and kinematics characteristics of the planar conveyor that allow the maximal relative speed of the transported objects.

The optimization techniques are based on different algorithms [1], [7].

In this case the objective function is considered the relative speed of objects on the vibrating surface. The GRID method is used to solve this problem.

The dynamics of different equipments are frequently studied in the literature, using various optimization methods. The vibratory conveyors were optimized with respect to the transmission of the dynamic force considering various design parameters. The positions at which the elastic elements are attached to the plate were considered as design variables. The optimization of these parameters is performed using the mathematical modeling of the system dynamic behavior [4], [8].

2. THEORETICAL DEVELOPMENTS

The objective function is the relative motion speed, and our purpose is to maximize it.

In the figure 2 is presented a vibratory plate that belongs to the conveyor that performs a harmonic motion along to the direction Δ, with the harmonic law

$$S = A \cdot \cos(\omega t) \qquad (1)$$

Due to this motion a mass **m** will move on the vibratory plate.

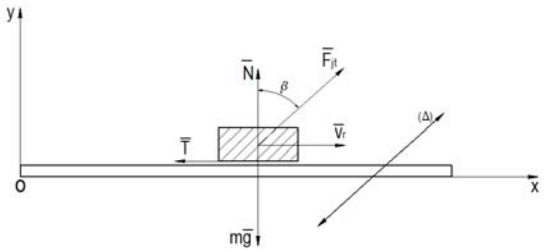

Fig. 2. Schematic representation of a vibratory conveyor

The differential equations of the mass **m** relative motion are determined referring to the Oxy Cartesian frame. The following differential simultaneous equations model the relative motion of the object on the vibrating plate:

$$\begin{cases} m\ddot{x} = F_{jt}\sin(\beta) - T \\ m\ddot{y} = -mg + F_{jt}\cos(\beta) + N \end{cases} \qquad (2)$$

Were:

$$F_{jt} = m \cdot a_t = m\omega^2 A\cos(\omega t) \qquad (3)$$

$$\overline{T} = -\mu N \frac{\overline{v}_r}{|\overline{v}_r|} \qquad (4)$$

μ being the friction coefficient.

Considering that the mass m will never leave the vibratory conveyor plate result the conditions: $y = 0$ and $N \geq 0$.

As a consequence, the normal force N is as follows:

$$N = mg - F_{jt}\cos(\beta) \geq 0 \qquad (5)$$

The resulting differential equation of relative movement is the following:

$$\ddot{x} = \omega^2 A\cos(\omega t) \cdot \sin(\beta) - \\ - \mu \cdot \mathrm{sign} \cdot \left(g - \omega^2 A\cos(\omega t) \cdot \cos(\beta)\right) \qquad (6)$$

The second order differential equation (6) may be writing as a system of two first order differential equations,

$$\begin{cases} \dot{Z}_1 = Z_2 \\ \dot{Z}_2 = \omega^2 A\cos(\omega t) \cdot \sin(\beta) - \mu \cdot \mathrm{sign} \cdot \left(g - \omega^2 A\cos(\omega t) \cdot \cos(\beta)\right) \end{cases} \qquad (7)$$

with the change of variables

$$Z_1 = x, \quad Z_2 = \dot{x}$$

The displacement and speed of the object during the relative motion were determined using a C program, written by the authors, the system of nonlinear equations (7) being solved with fourth degree of accuracy Runge - Kutta method [5].

Also, using the least square method the slope and intersect of the line that fits the obtained displacement was determined, $c_1 t + c_2$.

Being determined the displacement values $x_1, x_2, \ldots, x_n$ at each iteration step performed at times $t_1, t_2, \ldots, t_n$, after the numerical solving of the system (7) we may compute the slope c_1 and intersect c_2 of the line that fits the displacement diagram, that results after solving the system:

$$\begin{cases} \left(\sum_{i=1}^{n} t_i^2\right)c_1 + \left(\sum_{i=1}^{n} t_i\right)c_2 = \sum_{i=1}^{n} x_i \cdot t_i \\ \left(\sum_{i=1}^{n} t_i\right)c_1 + n \cdot c_2 = \sum_{i=1}^{n} x_i \end{cases} \qquad (8)$$

These coefficient values are

$$c_1 = \frac{\left(\sum_{i=1}^{n} x_i \cdot t_i\right) - \left(\sum_{i=1}^{n} t_i\right) \cdot \left(\sum_{i=1}^{n} x_i\right)}{\Delta} \qquad (9)$$

$$c_2 = \frac{\left(\sum_{i=1}^{n} t_i^2\right) \cdot \left(\sum_{i=1}^{n} x_i\right) - \left(\sum_{i=1}^{n} t_i\right) \cdot \left(\sum_{i=1}^{n} x_i \cdot t_i\right)}{\Delta} \qquad (10)$$

were

$$\Delta = n \cdot \left(\sum_{i=1}^{n} t_i^2 \right) - \left(\sum_{i=1}^{n} t_i \right)^2 \qquad (11)$$

3. NUMERICAL RESULTS

Authors have considered different cases for the determination of the relative motion:

1. The coefficient of friction μ and the angle β were considered as variable parameters and the amplitude and frequency were considered as constant parameters.

2. The coefficient of friction μ and the angle β were considered as constant and the amplitude and frequency were considered variables.

3. The coefficient of friction μ and the amplitude were considered as constant and the angle β and the frequency were considered variables.

4. The angle β and the frequency were considered as constants and the coefficient of friction μ and the amplitude were considered variable.

5. The coefficient of friction μ and the frequency were considered variable and the angle β and the amplitude were as constant.

The speed of the relative motion is considered as objective function in any of the previous cases.

Using the optimization Grid method the parameter values that determine the maximal speed in the relative motion are established.

The C program performs the following tasks:

- the diagrams for the relative displacement on the conveyors plate were determined after the numerical solving of the system (7) has been performed.
- after solving the system (7) the coefficients of the line that fits the displacement diagram were obtained.

The slope of the line is corresponding to the relative motion speed.

Completing all required steps and based on Grid method results a matrix which contains the values of the objective function.

In the figures 3, 4 and 5 are presented the diagram of the relative displacement considering some studied cases.

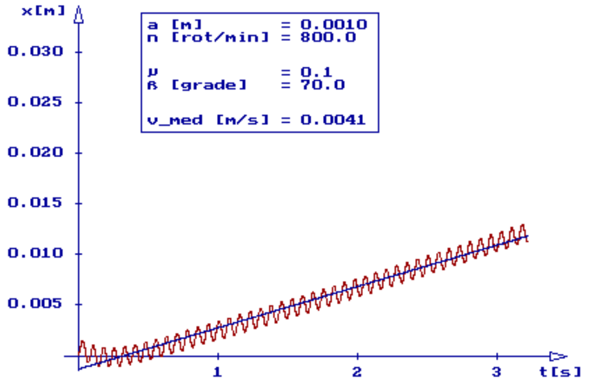

Fig.3. Graphical representation for the relative displacement (variables: μ, β, v_med)

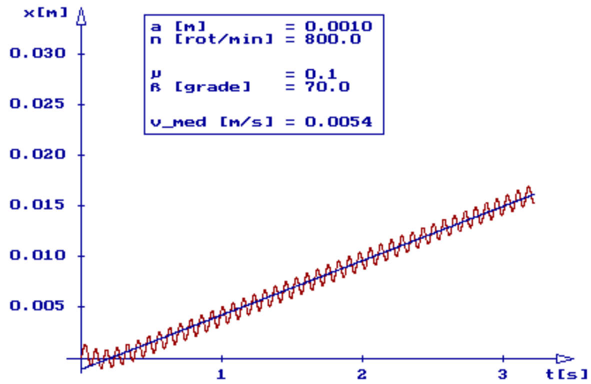

Fig.4. Graphical representation for the relative displacement (variables: μ, β, v_med)

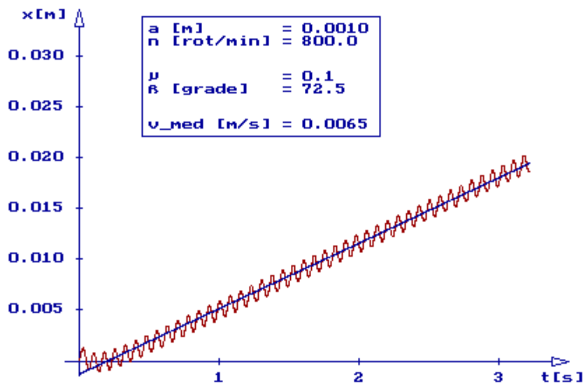

Fig.5. Graphical representation for the relative displacement (variables: μ, β, v_med)

Observing the obtained numerical results it was considered that the first case is the most conclusive regarding the determination of the relative motion speed. A particular interest presents the situation in which the coefficient of friction μ and the angle β were considered variable parameters. The initial data were as follows: amplitude = 0.0010 [m], rotational speed = 800.00 [rpm], number of iterations = 6450, iteration step = 0.00500 [s], μ between 0.0750 and 0.250, β between 60° and 77.5°.

We noticed that the maximum speed of the relative motion is obtained for μ = 0.250 and β = 60°. Using other combinations of the parameters we can perform numerical studies to determine the optimal value.

4. CONCLUSIONS

The system of differential equations modeling the movement of the material particle on the vibrating plate belonging to the conveyor was solved using Runge – Kutta method. Our goal was to find the optimal values of some parameters that allow the obtaining of maximum speed of a vibratory movement on the plate. The Grid optimization method was used. In the future will be also studied the situation in which the material jumps on the vibratory conveyor plate.

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REFERENCES

[1]. Ancău, M., Numerical optimization. Algorithm and programs in C (in Romanian), Publisher Casa Cărții de Știință, Cluj-Napoca, 2005, 202 pp.

[2]. Awrejcewicz, J., Olejnik, P., Analysis of dynamics systems with various friction law, Applied Mechanics Reviews, Vol. 58, issue 6, 2005, pp. 389 – 411.

[3]. Потураев, В. Н. и др., Динамика и прочность транспортно технологических машин, Ленинград, Машиностроение, 1989, 112 стр.

[4]. Reznik, D., Canny, J., Design verification and optimization of novel parts feeders with impulse dynamic simulator, EECS Dept. UC-Berkeley, Berkeley, CA 94720 – 1776 USA, http://www.cs.berkeley.edu/~jfc/dreznik/Pubs/iros97-ws.pdf, 2007, 11 pp.

[5]. Ursu-Fischer, N., Ursu, M., Numerical Methods in Engineering and Programming in C/C++ (in Romanian), Vol. II, Publisher Casa Cărții de Știință, Cluj-Napoca, 2003.

[6]. N. Ursu-Fischer, Mechanical System Vibration. Theory and Applications, Vibrațiile Sistemelor Mecanice. Teorie şi Aplicații, Publisher Casa Cărții de Știință, Cluj-Napoca, 1998, 452 pp.

[7]. G. N. Vanderplaats, Numerical Optimization for Engineering Design, McGraw Hill, 1989, 333 pp.

[8]. A.J. Wyk, J.A. Snyman, P.S. Heyns, Optimization of a vibratory conveyor for reduced support reaction force, N&O Journal, Vol. 10, No 1, 1994, pp. 12-17.

CONTRIBUȚII PRIVIND OPTIMIZAREA MIŞCĂRII RELATIVE PE UN VIBROTRANSPORTOR PLAN

Rezumat: *Lucrarea prezintă problema optimizării vitezei relative pe suprafața unui vibrotransportor. Viteza relativă a fost determinată folosind un program realizat în limbajul C, dezvoltat de autori. Folosind metoda Grid s-a determinat viteza maximă având în vedere influența diverşilor parametrii.*

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