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MATHEMATICAL MODEL OF PARTICLE-PLASMA INTERACTIONS IN THE PLASMA SPRAYING PROCESS

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***Abstract**: This paper presents the momentum transfer between plasma and the particles injected in the plasma jet. It is also presented the mathematical model for the heat transfer between plasma and particles.*
***Key words:** mathematical model, heat transfer, momentum transfer, particle-plasma interaction, plasma spraying process.*

1. INTRODUCTION

Plasma spraying process consists of injecting solid particles into a high temperature and high velocity gas jet, in which the particles are being melted and sprayed on parts surfaces. Plasma spraying can also be described as a connected energy transmission process, starting with the transfer of energy from an electric potential field to the plasma gas, proceeding with the transfer of energy from the plasma to the powder particles, and concluding with the transfer of energy from the particles to the substrate. In the following paper, will be discussed the interactions between particles and plasma jet, considering the speed of the particles and plasma and the heat transfer between plasma-particle. Particles are being accelerated in the plasma jet. The process can be described by a two-phase fluid flow, but is much more complex because of the presence of charged particles, chemical reactions, and large temperature gradients.

The success of plasma spray depositing layers depends on the ability of placing the particles in the plasma jet so that the particles to melt without being vaporized and designed with an optimum speed to impact the substrate.

The temperature of the particles can be controlled by the variation of the flow, the gas flow, the nature of the gas and by the particle size and their physical and chemical propreties.

2. HEAT TRANSFER FROM PLASMA TO PARTICLES

Heat transfer to particles is produced in two successive phases:

- connective transfer from fluid to particle;
- conductive transfer inside the particle.

The heat transmission by conduction and radiation of the plasma is negligible.

Heat flow “q” transmitted by convection is given by Newton’s law:

$$q=\alpha S(T_g - T_p) \tag{2.1}$$

where:

α - thermo convection factor between jet and particle,
S - transfer surface of particles,
T_g - gas temperature,
T_p - particle temperature.

The convective heat transfer takes place through the boundary layer of the particle, shown in figure 1.

Generally, the heat transfer factor will be maximum when the thickness of the boundary layer will be minimum and the turbulence grade of the fluid mass will be high.

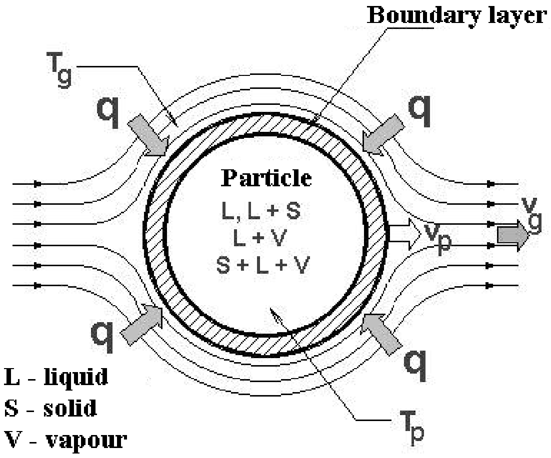

Fig. 1 Heat transfer between plasma and particle

2.1 Superheating temperature of particles T_s

For a pure alloy particle, supposing that the density in solid phase ρ_S is approximately the same as in liquid phase, the superheating temperature T_s is obtained by equating the convection ceded heat amount (2.2) with the received heat of the particle Q_p (2.3), until the superheating temperature T_s in liquid phase is reached.

$$Q_c = \alpha(T_g - T_o)S\tau \qquad (2.2)$$

$$Q_p = m\, c_s\,(T_t - T_o) + m\, c_l\,(T_s - T_t) + mL_t \qquad (2.3)$$

After equating the heat amount, results:

$$T_s = \frac{6\alpha\left(T_g - T_o\right)}{\rho_s \cdot c_l \cdot d_p}\cdot\tau - \frac{T_t\left(c_s - c_l\right) + L_t - c_s \cdot T_o}{c_l} \qquad (2.4)$$

where:

T_s - superheating temperature;
T_o - initial particle temperature;
τ - dwell time
ρ_s - particle density in solid phase
c_l - particle specific heat in liquid phase
d_p - particle diameter
T_t - melting temperature
c_s - specific heat in solid phase
L_t - melting latent heat

We can see from the equation (2.4) that the superheating temperature T_s for a particle depends on α, T_g, τ and d_p.

$$T_s = f\,(\alpha, T_g,\ \tau,\ d_p) \qquad (2.5)$$

2.2 Conductive heat transfer inside the particle

Transient heat flow is given by Fourier's equation:

$$q_t = -\lambda_p \frac{dT}{dx} \qquad (2.6)$$

where:

q_t – heat flow density
λ - termo conductivity factor
T – temperature
x – distance

Solving the transient heat flow problems, involves the determination of temperature variation and the amount of transferred heat depending on time. The equation that describes the thermal field evolution in a globular body with globular coordinates, is given by the following equation:

$$\rho_p c_p \frac{\partial T}{\partial \sigma} = \frac{1}{r_p^2}\cdot\frac{\partial}{\partial r}\left(r_p^2 \cdot \lambda_p \cdot \frac{\partial T}{\partial r_p}\right) \qquad (2.7)$$

where:

τ - time;
r_p - particle rate;
ρ_p - particle density;
c_p - specific heat of particle.

The determination of the temperature field $T(r,\tau)$, is possible when are known the initial and the limit conditions:

$$T(r,o) = T_{ro} \qquad r \le \frac{r_p}{2} \qquad (2.8)$$

$$-\lambda_g \frac{\partial T(r,\tau)}{\partial r} = \alpha\left(T_g - T_{ps}\right) \qquad (2.9)$$

where:

T_{ro} - temperature at distance r and time τ=0

T_{ps} - particle surface temperature.

The heat transfer coefficient can be expressed as:

$$\alpha = \frac{C_g \eta}{\Pr}\frac{Nu}{d_p} \qquad (2.10)$$

where C_g – specific heat of plasma gas,
η – gas viscosity,
d_p – particle diameter,
Nu – Nusslet number,
Pr – Prantl number.

The Nusslet and Prantl numbers are given by the following equations:

$$Nu = \frac{\alpha \cdot dp}{\lambda_g} \quad (2.11)$$

$$\Pr = \frac{c_g \eta_g}{\lambda_g} \quad (2.12)$$

The choice of the numerical value of the Nusselt number is crucial for a realistic estimate of the heat transfer rates from the plasma to the particles in the same way the choice of the drag coefficient is important for the momentum transfer. For the heat transfer between a gas and a spherical particle the expression (2.13) was given by Vardelle.

$$Nu = 2.0 + 0.6 \,\mathrm{Re}^{0.5} \Pr^{0.33} \quad (2.13)$$

where Re is the Reynolds number:

$$R_e = \frac{d_p v_r \rho_g}{\eta_g} \quad (2.14)$$

From the equations (2.13), (2.11), (2.12) and (2.14) we have the heat transfer coefficient:

$$\alpha = \frac{2\lambda_g}{d_p} + 0{,}6\left(\frac{\rho_g^3 \cdot c_g^2 \cdot \lambda_g^4}{\eta_g}\right)^{1/6}\left(\frac{v_r}{d_p}\right)^{1/2} \quad (2.15)$$

The relationship between melting time τ_t and particle dwell time τ_r shows the following :

$\tau_r \gg \tau_t$ - melted particles, partially vaporized,

$\tau_r \geq \tau_t$ - melted particles,

$\tau_r < \tau_t$ - partially melted particles.

3. MOMENTUM TRANSFER. PARTICLE VELOCITY IN PLASMA JET

The speed of the particles at the nozzle exit depends of the procces parameters, the injected material characteristics, the plasma generator configuration and the plasma gas.

The movement of particles in plasma jet is a matter of relative motion. The force acting on the powder grain due to the interaction with the moving gas is given by Newton's law of fluid mechanics:

$$F = m_p \frac{dv_p}{d\tau} = \frac{1}{2} C \cdot \rho_g \cdot S \cdot v_r^2 \quad (3.1)$$

where:

m_p - particle powder mass,
v_p – particle velocity ,
τ - time,
C - drag coefficient of the fluid,
ρ_g - gas density,
S - sectional area of the powder grain,
v_r - relative particle velocity to plasma.

Relative velocity square can be positive if $v_p > v_g$ and negative if $v_p < v_g$, where v_g is the plasma gas velocity.

$$v_r^2 = (v_p - v_g)\,|\, v_p - v_g \,| \quad (3.2)$$

Substituting in relation (3.1), particle mass m_p, the surface S, the particle density ρ_p, d_p the particle diameter and relative velocity, results differential equation of motion of particles in the gas stream:

$$\frac{dv_p}{d\tau} = \frac{3}{4} \cdot C \cdot \frac{\rho_g}{\rho_p \cdot d_p}\left(v_p - v_g\right) \cdot \left|v_p - v_g\right| \quad (3.3)$$

Differential equation (3.3) can be solved in three cases:

- if vp < vg, equation (3.3) becomes:

$$\frac{dv_p}{d\tau} = -\frac{3}{4}\frac{C \cdot \rho_g}{\rho_p \cdot d_p}\left(v_p - v_g\right)^2 \quad (3.4)$$

and particle will be accelerated,

- if vp = vg, equation (3.3) becomes:

$$\frac{dv_p}{d\tau} = 0 \quad (3.5)$$

zero acceleration,

- if vp > vg, equation (3.3) becomes:

$$\frac{dv_p}{d\tau} = \frac{3}{4}\frac{C \cdot \rho_g}{\rho_p \cdot d_p}\left(v_p - v_g\right)^2 \quad (3.6)$$

and particle will be decelerated.

3.1 Estimation of drag coefficient

The particle drag coefficient C, depends on Reynolds number given by the equation:

$$R_e = \frac{\rho_g \left| v_g - v_p \right| d_p}{\eta_g} \qquad (3.7)$$

The particle drag coefficient has different values depending on the following cases:

$$C = \frac{24}{R_e} \quad \text{for} \quad R_e < 1 \qquad (3.8)$$

$$C = \frac{24}{R_e}\left(1 + 0{,}15 R_e^{\,0{,}67}\right) \quad \text{for} \quad 1 < R_e < 10^3 \qquad (3.9)$$

$$C = 0{,}44 \quad \text{for} \quad R_e > 10^3 \qquad (3.10)$$

The velocity of the particles are given by the following equations depending on Reynolds number :

$$v_p = v_g \left[1 - \exp\left(\frac{18\eta_g \cdot \rho_g \cdot \tau}{d_p^{\;2} \cdot \rho_p} \right) \right], \mathrm{Re} > 1 \qquad (3.11)$$

$$v_p = \frac{v_g}{1 + \dfrac{4 d_p \eta_g}{3 C \rho_p \cdot \tau}}, \mathrm{Re} > 2 \qquad (3.12)$$

We can see from this equations, that the velocity of the particles, one of the most important process parameter is influenced by the plasma velocity v_g, time τ, and particle diameter d_p.

4. CONCLUSIONS

The reason why the temperature increases with the radial distance is because the particles that are not in the plasma jet axis have a longer dwell time due to plasma jet radial distribution of plasma flow speed, the speed being smaller at the periphery of the jet.

The acceleration of the particles caused by the viscous drag forces is proportional to the relative velocity of the particles, and inversely proportional to the density and the square of the diameter of the particle.

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Modelul matematic al interacţiunilor particulă – plasmă în procesul de pulverizare cu jet de plasmă

Lucrarea de faţă prezintă modelul matematic al interactiunilor dintre plasmă şi pulberile injectate în jetul de plasmă. Este prezentată viteza particulelor antrenate de jetul de plasmă cât şi schimbul de căldură de la plasmă la particule.

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