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A 2 STAGE COAXIAL HELICAL SPEED REDUCER GEARINGS OPTIMAL DESIGN WITH GENETIC ALGORITHMS

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Abstract: *The full description of the gearings corresponding on a 2 stage coaxial helical speed reducer generally requires a large number of design variables (typically, well over ten), resulting a very large and heavily constrained design space. Considering these we propose a Genetic Algorithm (in a formulation that can be extended to include additional stages or different layouts) to solve this complex gearings design problem. The objective is the minimization of the volume bounded by the inner surface of the speed reducer housing. It can be observed that the proposed optimal design with GAs has the potential to yield considerably better solutions than the traditional design.* ***Key words:*** *Automated optimal design, Genetic Algorithms, 2 stage coaxial helical speed reducer gearings.*

1. INTRODUCTION

Designing a power transmission such as a 2 stage coaxial helical speed reducer is a complex task. The complexity arises from strong and often intractable interdependencies between the design variable describing its subsystems. Also, it is know that designing of a reducer is an iterative process in which it is necessary to make some tentative choices, and to determine which parts of the design are critical. Of course, in a few trivial cases from all of them, it is almost impossible to tell what the first compromise should have been done. Moreover, for solving such complex design problem, conventional optimization techniques are very difficult to consider, taken into account the large number of design variables and the complexity of the interactions between them and the highly non-linear nature of the constraints and the objectives. For the last decades or so, Genetic Algorithms (GAs) are getting increasing attention to solve the complex mechanical power transmission design problems among the scientific and engineering community at the expense of simple trial and error type methods which were used to tackle this design problem. The potential replacements have begun in the shape of computer programs and expert systems. Thus, Madhusudan and Vijayasimha [11] presented a computer program in order to design a required type of gear under a specified set of working conditions. Lin et al. in [12] described a new computer-aided method for automated gearbox design. Huang [10] developed an interactive physical programming in order to optimize a three-stage spur gear reduction unit. Aberšek et al. [1] described an expert system to design and manufacture a gearbox. In [14] Li X. et al. carried out a study for minimizing the centre distance of a helical gear using American Gear Manufacturers Association (AGMA) procedures. Yokota et al. [21] solved an optimal weight design problem of a gear with an improved GA. In [13] Li R. et al. presented an adaptive genetic algorithm based on a fuzzy controller in order to solve the multi-objective optimization design of a reducer. Deb and Sachin [5] used a non-dominated sorting genetic algorithm (NSGA-II) in order to solve a multi-objective optimization of a multi-speed gearbox. Thompson et al. in [19] presented a generalized optimal design of two-stage and three-stage spur gear reduction units in a formulation with multiple objectives. Ray and Saini [15] illustrated the benefits of the particle swarm searches in resolving different engineering designs. Savsani et al. [17] presented two advanced optimization algorithms known as particle swarm optimization (PSO) and simulated annealing (SA) in order to minimize the weight of a spur gear train. The results of the proposed algorithms were compared with the results obtained by Yokota et al. in [21]. Gologlu and Zeyveli in [7] applied GA to minimize the volume of a two stage helical gear train. Tudose et al. in [20] presented a complete automated optimal design of a two-stage helical gear reducer using a two-phase evolutionary algorithm. The studies referenced above have been instrumented in order to highlight the importance of using modern global optimization techniques in mechanical power transmission design (as opposite to conventional, trail and error type methods), even when considering certain components or intermediate assemblies. In the following section a brief description of traditional gearing design corresponding on a 2 stage coaxial helical speed reducer is presented.

We shall now introduce the traditional speed reducer design method (currently used for designing the gearings of a multi-stage power transmission–Section 2), after which we describe the general principle of the proposed GA (Section 3), followed by a detailed discussion regarding the statement of the optimal design problem (the objective function, the design variables and the constraints–Section 4). The fifth Section contains an effective example and a detailed presentation and comparison of the numerical results for optimal and traditional design (i.e. a commonly trial and cut error procedure) solution. Eventually, some suggestions regarding the possible extensions of the results of this study are presented.

2. TRADITONAL DESIGN OF A 2 STAGE COAXIAL HELICAL SPEED REDUCER GEARINGS BRIEF OVERVIEW

The traditional speed reducer design process depends on the designer's intuition, experience, and skills [2]. The flowchart of a traditional design for the *gearings* of a 2 stage speed reducer is presented in Fig. 1. The chief difficulty that arises in traditional gear set design lies in the fact that it is necessary to know all the dimensions of the gears, as well as the tooth form and size, before the loads and stresses may be accurately determined.

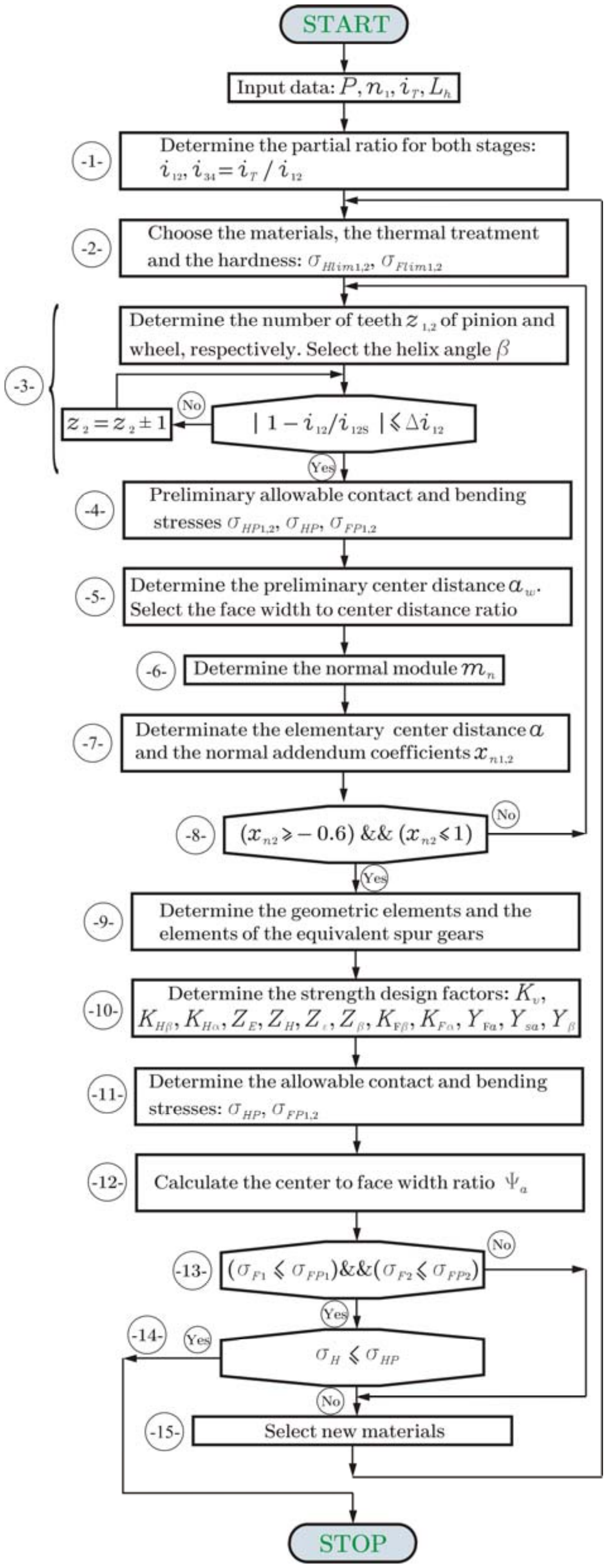

Fig. 1. The flowchart of traditional helical gear design

This makes it necessary to estimate the size of the gears [18] (i.e. steps -1- through -9- from Fig. 1), using simplified methods, and then to check this estimate, using the various design factors in conjunction with the more exact equations (i.e. steps -10- through -15- from Fig. 1). The estimated size and tooth form are then altered in accordance with the information obtained from the exact equations. Let us now present the steps (Fig. 1.) used in gear design [4], [6] and [8]. It is well known that in a speed reducer unit initial input speed n_1 and output speed n_3 determine the total gear ratio i_T. Thus, the process of speed reducer design begins with determination of the partial ratios for both stages (-1-). Then, the gearing of the 1st stage is computed starting by choosing the materials, the hardness, and the thermal treatment for pinion and wheel, respectively (-2-). Next, the number of teeth on pinion and gear are determined. The actual value of the gear ratio should not differ from the nominal one by more than 4% (for double reduction units [8]) (-3-). The estimated allowable contact and bending stresses for the gears are then computed (-4-). In the following step is determined the preliminary center distance aw and is selected the gear width to center distance ratio ψ_a (i.e. 0.25…0.4, depending on the position of gears [8]) (-5-). Then, is determined the normal module m_n (-6-), the elementary centre distance a and the normal teeth addendum coefficients $x_{n1,2}$ (-7-)(-8-). Then, the elements of the helical gears and of the equivalent spur gears are determined (-9-). All the strength design factors are exactly computed (-10-). Next are calculated the allowable contact and bending stresses (-11-) and then the face width to center distance ratio (-12-). Then the gearing is checked on bending (-13-) and contact (-14-) stresses. If the calculated values are lower then the allowable ones, then the earlier parameters of gearing are taken as final. Otherwise recalculation is required (-15-).

A traditional design for the gearings of a multi-stage speed reducer (as it can be seen from the above brief description) involves computations based on contact stress, bending strength, interference etc., leading to a complex and time-consuming process. Moreover the design so obtained may not be the optimum [17]. To overcome these drawbacks we proposed an optimal design with GA. We consider that GAs represent the direction of choice at the present in solving such complex design problem. In the following Section a short description of the GA will be presented.

3. THE GENETIC ALGORITHM

The GA is perhaps the most well known of all evolution-based search algorithms [3], suited for solving complex optimization problems [16]. The basic concepts of GAs were developed by John Holland [9] in an attempt to explain and describe the biological processes that can be appreciated in Nature, and to design new artificial evolutionary systems based upon these natural processes. GAs maintain a population of solutions, then allow the fitter individuals to reproduce, and let the less fit individuals die off [16]. Each individual consists of a genotype (i.e. the search space of coded solutions) and a corresponding phenotype (i.e. the solution space). Phenotypes usually are collections of parameters (for example, in the optimal design problem presented in this paper, such parameters might define the number of teeth on pinions, the centre distance, etc.). Genotypes consist of coded versions of these parameters. A coded parameter is normally referred to as a gene. A collection of genes in one genotype is often held internally as a string, and is known as a chromosome [16] (the chromosome of our 2 stage coaxial helical speed reducer is composed from 12 genes–as it can be seen in Section 4). This GA works as follow: (a) the genotype of every individual in the population is randomly initialized. Then the main loop of the algorithm begins. (b) The phenotype of every individual from the initial population is evaluated using the fitness function. Next, (c) two parents are randomly selected for reproduction based on the fitness values of the individuals. Offspring are created (d, e) by applying the genetic operators: crossover (merges information from two parents into one or two offspring) and mutation (acts on a single offspring and works by applying some variation to one or more genes in the offspring's chromosome). The new generated individuals are evaluated (f) using the fitness measure. After evaluation the offspring replace some/all of the individuals in the current population (g). This entire process of evaluation and reproduction continues until either a satisfactory solution emerges or the GA has run for a specified number of generations.

4. THE PROBLEM FORMULATION

The first step in the formulation of the optimal design problem is to identify the set of ***genes*** (design variables) that uniquely describe the ‘genotype’ of the optimal design problem. The 12 design variables that unequivocally define the objective function are presented in Table 1.

Table 1

The 12 genes describing the gearings of the 2 stage coaxial helical speed reducer

Gene symbol	Range
Gear ratio i_{12S} of the 1st stage, standardized, discrete real values	{1.12 ,…, 40}
Center distance a_w of the 1st, standardized, discrete, real values	{56 ,…, 315}
Number of teeth $z_{1,2}$ of 1st and 2nd stage pinions, respectively. Integer values.	{14 ,…, 21}
Gear width to center distance ratio coefficient $\psi_{a1,2}$ of the 1st and 2nd stage respectively. Real values	[0.2,…,0.8]
Helix angle $\beta_{1,2}$ measured at the pitch diameters of the 1st and 2nd stage, respectively. Discrete real values.	[7.2°, 19.8°]
Normal tooth addendum coefficients $x_{n1,3}$ of the 1st and 2nd stage pinions, respectively. Discrete, real values.	{–0.6 ,…, 1}
Geometric dimensions $y_{1,2}$. Real values.	{0 ,…, 31}

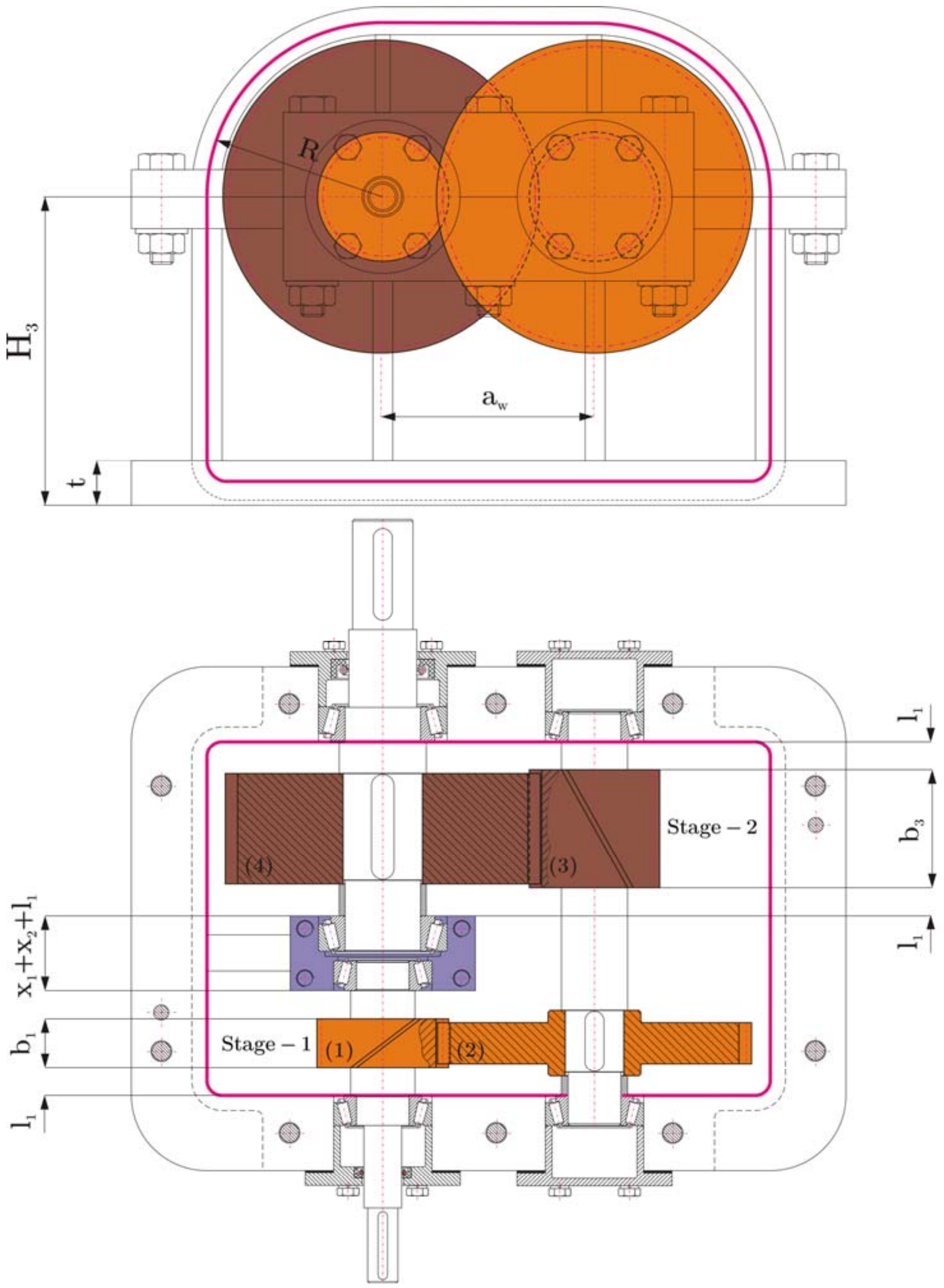

Fig. 2. The sketch of the 2 stage coaxial helical speed reducer

The objective of our optimal design is the minimization of the volume bounded by the inner surface of the speed reducer housing (Fig. 1). Thus, the ***objective function*** is:

$$Obj: F(x) = V = (5l_1 + b_1 + y_1 + y_2 + b_3)[(H_3 - -t + \delta)(2R + a_w) + \pi R^2/2 + a_w R] \rightarrow \min \quad (1)$$

where: $b_{1,3}$ is the width of the 1st and 2nd stage pinion, in mm, H_3 is a standardized dimension, $l_1 = 5$, $t = 2.25\delta$, $R = \max(d_{a2}/2, d_{a4}/2,) + 10\ldots20$, $\delta = 0.025(a_w + 5)$.

The constraints. In this sub-section the constraints of the speed reducer optimal design problem are presented. These constraints are all of inequality type, involving strength, geometrical or structural considerations. There are a total of 37 constraints (involving strength, geometrical and structural considerations) typically encountered in practical design of multi-stage speed reducers gearings. For conciseness we shall not dwell on the details of their calculation. The interest reader may find all the details of the gearings calculations in the relevant industrial standard document DIN 1987 [17]. Obviously, a feasible solution of the optimization problem should satisfy those 37 constraints (all the values of these constraints have to be negative or at least zero). The value g_i of a constraint is defined as $g_i = a_i / b_i - 1 \leq 0$, where the constraints is of the form $a_i \leq b_i$ (a_i, $b_i > 0$). With reference to the sketch presented in Fig. 2, the following list of constraints should be viewed.

***C*1, 2**–*The relative difference between the required and the actual gearing ratio must be within the range of* [-2.5% … +2.5%] *on the both stages*. ***C*3, 4**–*The Hertzian contact pressure on the teeth of gears on both stages must not exceed a specified value*. ***C*5–8** *The bending stress on the teeth of gears on both stages must not exceed a specified value*. ***C*9–12** *The teeth of all 4 gears of the two-stage coaxial helical speed reducer must not be undercut*. ***C*13–16** *The top land on the teeth on gears* 1 *through* 4 *must not vanish*. ***C*17, 18** *The contact ratio on first and second stage must be greater than a specified value*. ***C*19, 20**–*The addendum coefficient of the wheel* (2) *and* (4) *should be in the range of* [-0.6, 1]. ***C*21, 22**–*The numbers of teeth on both stages must be relative primes*. ***C*23–34** *A set of measurability constraints of the gears on both stages*. ***C*35**–*Lubrication constraint—the margin between the minimum and maximum allowable lubricant levels should be no less than* 10 *mm*. ***C*36, 37**–*Geometrical constraints regarding the space between the* 1*st and* 2*nd stage*.

5. A PRACTICAL EXAMPLE

Consider the following optimal design problem. A 2.75 kW two-stage coaxial helical speed reducer (Fig.2) is to be designed, given an input speed of 1000 rpm and a total transmission ratio of 11.2. The gears should be based on an ISO 53 basic rack profile ($\alpha_n = 20^\circ$, $h_{an} = 1$, $c_{sa} = 0.4$) with the pinions and wheels made of case hardened alloy steel 17CrNiMo6 and 17Cr3, respectively. Running the GA (described earlier in Section 3) led to a speed reducer with a 3.55 and 3.15 standardized transmission ratio (on 1st and 2nd stage, respectively), a centre distance of 100 mm on both stages, having a volume of $9.071 \cdot 10^{-3}$ m^3. The values of all considered genes, after optimization, are given in Table 2.

Table 2

The genes values obtained after optimization

No	Symbol	Value
1	i_{12S}	3.55
2	a_w (mm)	100
3	z_1	19
4	z_3	15
5	Ψ_{a1}	0.2
6	Ψ_{a2}	0.5921
7	β_1 (°)	7.2°
8	β_2 (°)	19.4°
9	x_{n1}	0.5465
10	x_{n3}	0.4583
11	y_1 (mm)	16.25
12	y_2 (mm)	18.25

In Table 3, the main characteristics of the speed reducer gearings (*traditional* and *optimal* design) are shown side-by-side. Also an overlap image representing these two solutions is presented in Fig. 3.

Table 3

***Traditional* and *optimal* design solutions**

No	Traditional design	Optimal design
	1st stage	
1	Transmission ratio, i_{12}	
	4.5	3.55
2	Standardized center distance, a_w, (mm)	
	112	100
3	Normal module, m_{nI}, (mm)	
	2.5	2.25
4	Number of teeth on pinion, z_1	
	16	19
5	Number of teeth on wheel, z_2	
	71	67
6	Pinion width, b_1, (mm)	
	32	24
7	Wheel width, b_2, (mm)	
	28	20
8	Root diameters, $d_{f1,2}$ (mm)	
	35.8231	39.249
	174.4777	148.5836
9	Outside diameters, $d_{a1,2}$, (mm)	
	47.5222	49.6163
	186.1768	158.9509
	2nd stage	
10	Transmission ratio, i_{34}	
	2.5	3.15
11	Normal module, m_{nII}, (mm)	
	3	3
12	Number of teeth on pinion, z_3	
	20	15
13	Number of teeth on wheel, z_4	
	51	47
14	Pinion width, b_3, (mm)	
	54	63
15	Wheel width, b_4, (mm)	
	50	59
16	Root diameters, $d_{f3,4}$, (mm)	
	54.9324	42.0585
	152.8589	141.2695
17	Outside diameters, $d_{a3,4}$, (mm)	
	68.741	56.3304
	166.6675	155.5414
18	Objective function, (m^3)	
	$11.4621 \cdot 10^{-3}$	$9.071 \cdot 10^{-3}$

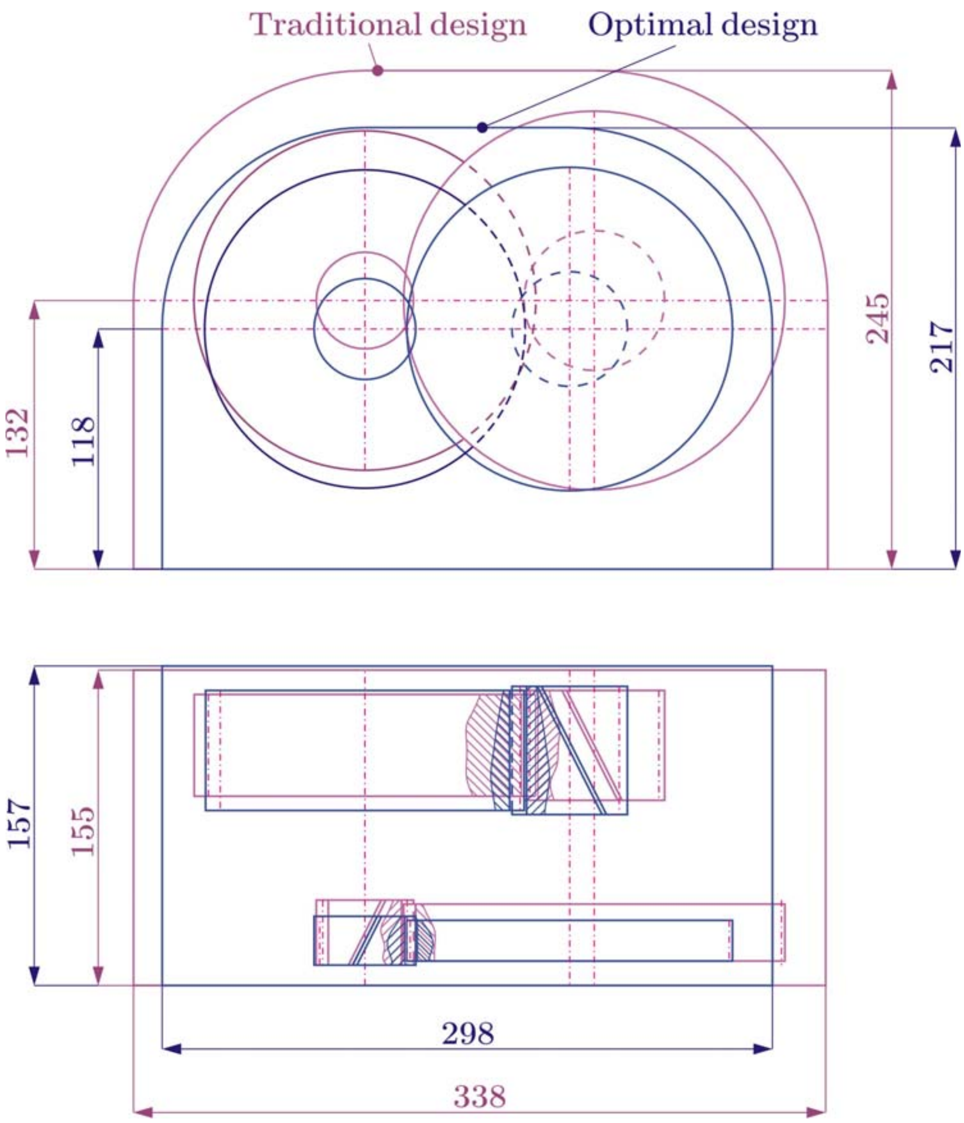

Fig. 3. *Traditional* and *optimal design* solutions.

As it can be observed from Table 2, the volume of the inner surface calculated with the traditional design is $11.4621 \cdot 10^{-3}$ m^3 while the optimal design solution offers a smaller volume, equal to $9.071 \cdot 10^{-3}$ m^3, i.e. 20.86% reduction. In the case of a large series of production, the advantages are obviously, and manufacturing costs are significantly diminished. For example at 5 speed reducers produced, 1 is for free taking into account the material.

6. CONCLUSIONS AND THE WAY FORWARD

This paper presents how a GA, can be used to solve a complex structural design problem of a 2 stage coaxial helical speed reducer gearings. The objective is the minimization of the volume bounded by the inner surface of the speed reducer housing. The objective function was subjected to a set of 37 constraints. The design variables considered in the optimization are of mixed nature i.e., continuous, integer, and discrete in total of 12. The optimal design solution was compared to traditional design (i.e. a trial and cut error procedure). The results obtained by using GA show significant improvement over the results obtained by traditional design (i.e. 20.86% volume reduction). The proposed GA could be easily modified to suit multi-objective design optimization. Also additional stages and other objective functions (the manufacturing cost is merely a possible example) could be considered. This optimization example illustrates the effectiveness of the proposed approach and also serves as further evidence of the power and versatility of GAs in designing multi-stage power transmissions.

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PROIECTAREA OPTIMALĂ A UNUI REDUCTOR COAXIAL CU DOUĂ TREPTE CU ROȚI DINȚATE CILINDRICE CU DINȚI ÎNCLINAȚI

Descrierea completă a angrenajelor corespunzătoare unui reductor coaxial cu două trepte cu roți dințate cilindrice cu dinți înclinați necesită un număr mare de variabile de proiectare (de obicei peste 10) rezultând un spațiu de căutare a soluției optime extrem de vast. Luând în considerând aceste aspecte pentru rezolvarea acestei complexe probleme de proiectare am propus un algoritm genetic (într-o formulare care poate fi modificată cu uşurință). Obiectivul este minimizarea volumului delimitat de suprafața interioară a carcasei reductorului. În urma optimizării s-a constat că uilizarea algoritmilor genetici a condus la obținerea unei soluții mult îmbunătățită față de soluția obținută în urmă proiectării clasice.

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