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NEW LEM ESTIMATIONS FOR THE UNDAMPED DÜFFING OSCILLATOR

P.P. TEODORESCU, Ileana TOMA

Abstract: *In a previous paper, the authors deduced a LEM representation for the damped Düffing oscillator with harmonic forcing, in null Cauchy conditions; the undamped case could not be obtained from this by canceling the damping coefficient. Therefore, in this paper they consider directly the harmonically forced undamped Düffing oscillator with nonzero initial conditions, establishing for it analytic LEM approximate solutions tested by using classical numerical methods. Some representative phase portraits are also presented.*
Keywords: *Düffing oscillator, linear equivalence method.*

1. INTRODUCTION

Düffing's oscillator is mathematically modeled as [1]

$$\ddot{x} + \delta\dot{x} + \gamma x + \alpha x^3 = A\cos\omega t\,, \qquad (1)$$

where δ is a positive damping constant. For positive values of γ, this can be physically interpreted as a forced oscillator with a spring of non-linear restoring force; for positive α, one has a hardening, while for negative α – a softening spring. For $\beta < 0$, it can be regarded as a describing the dynamics of a point mass in a double well potential [14][25][26].

The linear equivalence method (LEM) was previously introduced by I. Toma to the purpose of getting and studying – both numerically and qualitatively – the solutions of nonlinear dynamical systems. The normal LEM representations the emphasize dependence on parameters and is particularly fitted for the study of long term behaviour of the solution. LEM has many applications in mechanics, physics, chemistry, biology (see e.g. [2]-[4], [6],[8][22]. In a previous paper [13], LEM was applied to a damped Düffing oscillator with harmonic forcing, in null Cauchy conditions; yet, the undamped case could not be obtained by plainly canceling δ in this LEM representation.

Therefore here we applied LEM to undamped Düffing oscillator harmonically forced in more general Cauchy conditions, thought as to find also periodical solutions.

Let us note that Düffing's model is an algebraically simple equation involving time-dependent acceleration (jerks) that have chaotic solutions, as previously shown by Ueda [23].

In what follows, we take $\delta = 0$ and $\alpha > 0, \gamma > 0$, hence

$$\ddot{x} + b^2 x + a x^3 = A\cos\omega t\,, \qquad (2)$$

where $\gamma = b^2$. For this equation we get the LEM solution, including third order effects. A parallel with the Runge-Kutta method shows that the analytic LEM formula holds on large time intervals; this recommends its application to a qualitative study of the solutions. For a better insight into the behaviour of the solutions, we give several representative phase portraits.

2. AN OUTLINE OF LEM

The main idea of LEM consists of an exponential mapping which transports a nonlinear ODS in a linear frame, making it easier tackled and controlled than in its primary form.

While LEM can be applied to more general ODSs, as the involved model studied here is polynomial and with constant coefficients, we will restrict to this case. Consider therefore the polynomial ODS

$$\mathcal{P}\mathbf{y} \equiv \frac{\mathrm{d}\mathbf{y}}{\mathrm{d}t} - \mathbf{P}(\mathbf{y}) = \mathbf{0}, \quad \mathbf{P} = \left[P_j(\mathbf{y})\right]_{j=\overline{1,n}}, \tag{3}$$

where

$$P_j(\mathbf{y}) \equiv \sum_{|\eta| \le p_j} a_{j\eta}\mathbf{y}^{\eta}, \quad a_{j\mu} \in \Re,\ j = \overline{1,n}, |\mu| \le p_j, \quad j = \overline{1,n}. \tag{4}$$

As it was mentioned – firstly in [15] – the LEM mapping is

$$v(t,\xi) = \mathrm{e}^{\langle \xi, \mathbf{y} \rangle},\ \xi = (\xi_1, \xi_2, ..., \xi_n) \in \Re^n, \tag{5}$$

where ξ are newly introduced parameters. This mapping associates to the nonlinear ODS two linear equivalents [17][21]:

- a linear PDE, always of first order with respect to t
$$\mathcal{L}\, v(t,\xi) \equiv \frac{\partial v}{\partial t} - \langle \xi, \mathbf{P}(D) \rangle v = 0, \tag{6}$$

and

- a linear, while infinite, first order ODS, that may be also written in matrix form
$$\mathcal{S}\mathbf{V} \equiv \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} - \mathbf{A}\mathbf{V} = \mathbf{0}, \quad \mathbf{V} = (\mathbf{V}_j)_{j \in \mathfrak{N}}, \quad \mathbf{V}_j = (v_\gamma)_{\|\gamma\| = j}. \tag{7}$$

The second LEM equivalent, the system (7), is obtained from the first one, by searching the unknown function v in the class of analytic in ξ functions

$$v(t,\xi) = 1 + \sum_{|\gamma|=1}^{\infty} v_\gamma(t)\frac{\xi^j}{j!}. \tag{8}$$

The LEM matrix A is row and column-finite as the differential operator is polynomial. It has a cell-diagonal structure.

The involved cells $\mathbf{A}_{k,k+s}$ are generated by those $a_{j\mu}$ with $|\mu| = s+1$ only; for instance, $\mathbf{A}_{11}$ is the linear part of the operator. This special form of $\mathbf{A}$ allows the calculus by block partitioning.

Let us associate to (3) the initial conditions

$$\mathbf{y}(t_0) = \mathbf{y}_0, \quad t_0 \in \mathrm{I}. \tag{9}$$

By LEM, they are transferred to

$$v(t_0,\xi) = \mathrm{e}^{\langle \xi, \mathbf{y}_0 \rangle}, \quad \xi \in \Re^n, \tag{10}$$

a condition that must be associated to (6), and

$$\mathbf{V}(t_0) = \left(\mathbf{y}_0^{\gamma}\right)_{|\gamma| \in \mathfrak{N}}, \tag{11}$$

indicating an initial condition for the second LEM equivalent (7).

The linear equivalents are consistent on Exp-type spaces [21].

The following result holds true

Theorem 1. [16][19][21] *The solution of the nonlinear initial problem* (1), (9)

i) coincides with the first n components of the infinite vector

$$\mathbf{V}(t) = \mathrm{e}^{\mathbf{A}(t-t_0)}\mathbf{V}(t_0), \tag{12}$$

where the exponential matrix, defined as for finite matrices, can be computed by block partitioning, each step involving finite sums;

ii) coincides with the series

$$y_j(t) = y_{j0} + \sum_{l=1}^{\infty} \sum_{|\gamma|=l} u_{j\gamma}(t) y_0^{\gamma}, \quad j = \overline{1,n}, \tag{13}$$

where $u_{j\gamma}(t)$ satisfy the finite linear ODSs

$$\frac{\mathrm{d}\mathbf{U}_k^j}{\mathrm{d}t} = \mathbf{A}_{1k}^{\mathrm{T}}\mathbf{U}_k^j + \mathbf{A}_{2k}^{\mathrm{T}}\mathbf{U}_k^j + \ldots + \mathbf{A}_{kk}^{\mathrm{T}}\mathbf{U}_k^j, \quad k = \overline{1,l}, \quad \mathbf{U}_s^j(t) = \left[u_{j\gamma}(t)\right]_{|\gamma|=s}, \tag{14}$$

and the Cauchy conditions

$$\mathbf{U}_1^j(t_0) = \mathbf{e}_j^n \equiv \left[\delta_i^j\right]_{i=\overline{1,n}}, \quad \mathbf{U}_s^j(t_0) = \mathbf{0},\ s = \overline{2,l}, \tag{15}$$

T *standing for transpose matrix and* δ_i^j *– for the Kronecker delta.*

The representation (13) was called *normal* by analogy with the linear case [21]. The eigenvalues of the diagonal cells $\mathbf{A}_{kk}$ are always known [17][21]; this was important in many applications requiring the qualitative behavior of the solution and in stability problems, in general (see e.g. [3],[4],[8], where it was used along with other LEM representations).

3. NORMAL LEM SOLUTIONS FOR THE HARMONICALLY FORCED UNDAMPED DÜFFING OSCILLATOR

We establish here the normal LEM solutions corresponding to third order effects.

Introducing three auxiliary functions $y = \dot{x}$, $u = A\cos\omega t$, $v = A\sin\omega t$, Düffing's equation may be written in the form of a homogeneous polynomial first order ODS

$$\begin{aligned} &\dot{x} = y,\\ &\dot{y} = -b^2 x - a x^3 + u,\\ &\dot{u} = -\omega v,\\ &\dot{v} = \omega u. \end{aligned} \tag{16}$$

If we associate to (2) the Cauchy conditions

$$x(0) = \alpha, \quad \dot{x}(0) = 0, \tag{17}$$

they become for (16)

$$\begin{aligned} &x(0) = \alpha, \quad \dot{x}(0) = 0,\\ &u(0) = A, \quad v(0) = 0. \end{aligned} \tag{18}$$

The transposed of the associated LEM matrix is then

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{A}_{13}^{\mathrm{T}} & \mathbf{A}_{33}^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{A}_{35}^{\mathrm{T}} & \mathbf{A}_{55}^{\mathrm{T}} & \mathbf{0} & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}, \quad \mathbf{A}_3 = \begin{bmatrix} \mathbf{A}_{11}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{A}_{13}^{\mathrm{T}} & \mathbf{A}_{33}^{\mathrm{T}} \end{bmatrix}, \tag{19}$$

where

$$\mathbf{A}_{11}^{\mathrm{T}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -b^2 & 0 & 0 \\ 0 & 0 & 0 & \omega \\ 0 & 0 & -\omega & 0 \end{bmatrix}, \quad \mathbf{A}_{13}^{\mathrm{T}} = \left[a_{jk}\right]_{\substack{j=\overline{1,20},\\ k=1,4,}} \quad a_{jk} = -a\delta_{j+1}^2 \delta_k^2 . \tag{20}$$

If we stick to third order effects, then we truncate the LEM matrix to

$$\mathbf{A}_3 = \begin{bmatrix} \mathbf{A}_{11}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{A}_{13}^{\mathrm{T}} & \mathbf{A}_{33}^{\mathrm{T}} \end{bmatrix}. \tag{21}$$

The eigenvalues of $\mathbf{A}_{11}^{\mathrm{T}}$ are

$$0, \pm \mathrm{i}\beta, \pm \mathrm{i}\omega; \tag{22}$$

according to the general LEM results [21], the eigenvalues of $\mathbf{A}_{33}^{\mathrm{T}}$ will be

$$\begin{aligned} &\pm \mathrm{i}\omega, \pm \mathrm{i}\omega, \pm \mathrm{i}\beta, \pm \mathrm{i}\beta, \pm 3\mathrm{i}\omega, \pm 3\mathrm{i}\beta,\\ &\pm \mathrm{i}(\omega - 2\beta), \pm \mathrm{i}(\omega + 2\beta),\\ &\pm \mathrm{i}(2\omega - \beta), \pm \mathrm{i}(2\omega + \beta). \end{aligned} \tag{23}$$

The associated LEM ODS for up to third order effects will have two blocks; using the Laplace transformation [17][18][21], we find the normal LEM solution for the Cauchy data (18) in the form

$$\begin{aligned} x(t) = {} & \alpha\cos bt + \frac{A}{d_1}(\cos\omega t - \cos bt)\\ & + \frac{a\alpha^3}{32b^2}(\cos 3bt - \cos bt - 12bt\sin bt)\\ & - 36ab^4 A\alpha^2 [c_1\cos\omega t + c_2\cos 3bt\\ & + c_3\cos(\omega - 2b)t + c_4\cos(\omega + 2b)t\\ & + c_5\, bt\sin bt + +c_6\cos bt. \end{aligned} \tag{24}$$

The coefficients $c_j, j = \overline{1,6}$, have the following expressions

$$\begin{aligned} &c_1 = \frac{3}{16b^2 d_1 d_2}, \quad c_2 = \frac{3b^2 + \omega^2}{32b^4 d_1^2 d_2 d_3},\\ &c_3 = \frac{b - 3\omega}{32b^4\omega c(b-\omega)(3b-\omega)^2(5b-\omega)},\\ &c_4 = -\frac{b + 3\omega}{32b^4\omega c(b+\omega)(3b+\omega)^2(5b+\omega)},\\ &c_5 = -\frac{1}{8b^4 d_1 d_2},\\ &c_6 = \frac{-51b^4 + 18b^2\omega^2 + \omega^4}{32b^4 d_1^2 d_2^2}, \end{aligned} \tag{25}$$

where

$$\begin{aligned} &d_1 = b^2 - \omega^2, d_2 = 9b^2 - \omega^2,\\ &d_3 = 25b^2 - \omega^2. \end{aligned} \tag{26}$$

The derivative $\dot{x}$ can be obtained applying again theorem 1, with

$$\mathbf{U}_1^2 = [0 \quad 1 \quad 0 \quad 0], \tag{27}$$

or by straightford differentiation of with respect to *t*; eitherwise, we get

$$\begin{aligned} \dot{x}(t) &= -b\alpha \sin bt + \frac{A}{d_1}(b \sin bt - \omega \sin \omega t) \\ &- \frac{a\alpha^3}{32b}(3\sin 3bt + 11 \sin bt + 12bt \cos bt) \\ &+ 36ab^4 A\alpha^2 [\omega c_1 \sin \omega t + 3bc_2 \sin 3bt \\ &+ (\omega - 2b)c_3 \sin(\omega - 2b)t \\ &+ (\omega + 2b)c_4 \cos(\omega + 2b)t \\ &- c_5\, bt \cos bt + b(c_6 - c_5)\cos bt. \end{aligned} \tag{28}$$

These two formulae can be applied to get (approximate) solutions for the above considered non-linear model.

4. NUMERICAL STUDY AND CONCLUSIONS

As getting periodic solutions is an important goal in every non-linear study with applications to mechanics, let us note that if such solutions of period ω are expected, then their amplitude α must satisfy the algebraic equation [25]

$$\alpha^6 + \frac{64}{9}\frac{(\omega - b)^2}{a^2}\alpha^2 - \frac{16}{9} = 0, \tag{29}$$

that can be transformed into a cubic one

$$r^3 + \frac{64}{9}\frac{(\omega - b)^2}{a^2} r - \frac{16}{9} = 0, \quad r = \alpha^2. \tag{30}$$

This equation always allows only one real root, given by the well known Cardan's (Tartaglia's) formula

$$r = \frac{2}{\sqrt[3]{9}}\sqrt[3]{1 + \sqrt{1+B}} + \sqrt[3]{1 - \sqrt{1+B}}, \quad B = 3\left[\frac{4(\omega - b)}{3a}\right]^6. \tag{31}$$

The following table gives relative errors between the numerical solution, deduced by Runge-Kutta method, and the LEM formulae (24),(28), computed for initial data $(\alpha, 0)$, α satisfying (29). We firstly note that the relative errors are the same for both formulae. Table 1 is built up for $b = 1, a = 1, A = 1$ and increasing ω. The relative errors are established on the interval $[0, 2\pi]$.

Table 1

Comparison between the LEM formulae (25),(28) **and the numerical solution (** $b = 1$ **)**

ω	α	relative error/step	ω	α	relative error/step
6	0.0999	$4.9\cdot10^{-2}$	120	0.0042	$4.5\cdot10^{-6}$
10	0.0555	$1.1\cdot10^{-2}$	150	0.0033	$2.6\cdot10^{-6}$
15	0.0357	$2.0\cdot10^{-3}$	170	0.0029	$1.6\cdot10^{-6}$
20	0.0263	$1.0\cdot10^{-3}$	200	0.0025	$1.1\cdot10^{-6}$
30	0.0172	$3.6\cdot10^{-4}$	248	0.0020	$5.3\cdot10^{-7}$
40	0.0128	$1.3\cdot10^{-4}$	250	0.0020	$5.1\cdot10^{-7}$
45	0.0113	$3.1\cdot10^{-4}$	252	0.0019	$5.1\cdot10^{-2}$
50	0.0101	$7.0\cdot10^{-5}$	298	0.00168	$3.0\cdot10^{-7}$
80	0.0063	$1.6\cdot10^{-5}$	300	0.00167	$2.9\cdot10^{-7}$
100	0.0050	$8.1\cdot10^{-6}$	302	0.00166	$3.0\cdot10^{-7}$

It is easily seen that, for increasing ω, α decreases, as well as the relative error; in fact, further computation shows that, from $\omega = 20$ on, formulae (24) and (28) are concordant with the numerical solution on much larger time intervals, like $[0, 608]$, with a relative error per step of order 10^{-2}. Also, the contribution of the non-linear term seems to diminish as ω increases; this remains true even for large *a*.

Same conclusions can be drawn by taking $\omega = 1, a = 1, A = 1$ and increasing *b*. The results of this analysis are listed in table 2.

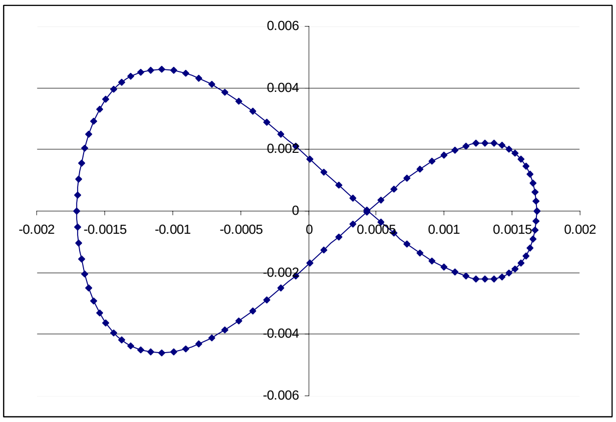
Table 2

Comparison between the LEM formulae (24),(28) **and the numerical solution (** $\omega = 1$ **)**

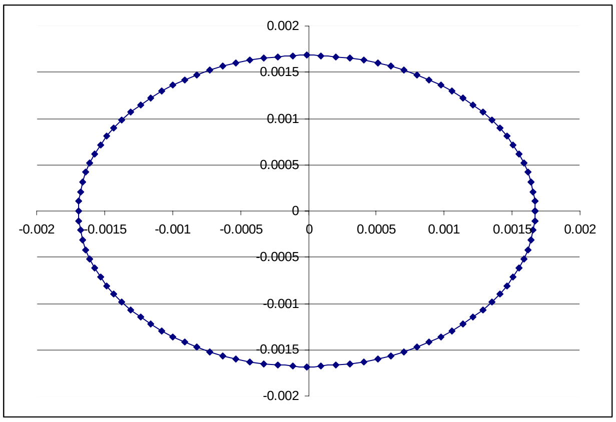
b	α	relative error/step	*b*	α	relative error/step
3	0.2499	$2.0\cdot10^{-2}$	80	0.0063	$2.6\cdot10^{-8}$
4	0.1666	$5.0\cdot10^{-3}$	10 0	0.0050	$1.2\cdot10^{-8}$
5	0.1249	$4.3\cdot10^{-4}$	120	0.0042	$1.8\cdot10^{-8}$
10	0.0555	$3.5\cdot10^{-5}$	150	0.0033	$1.4\cdot10^{-8}$
20	0.0263	$2.4\cdot10^{-6}$	170	0.0029	$9.4\cdot10^{-9}$
30	0.0172	$5.5\cdot10^{-7}$	200	0.0025	$2.9\cdot10^{-8}$
40	0.0128	$2.1\cdot10^{-7}$	250	0.0020	$3.6\cdot10^{-9}$
50	0.0102	$1.2\cdot10^{-7}$	300	0.0016	$1.3\cdot10^{-8}$

We conclude by presenting several phase portraits illustrating the above considerations.

A study of the phase portraits shows that for $b = 1, a = 1, A = 1$ one finds ellipses every 50 units, starting with $\omega = 50$.



i)



ii)

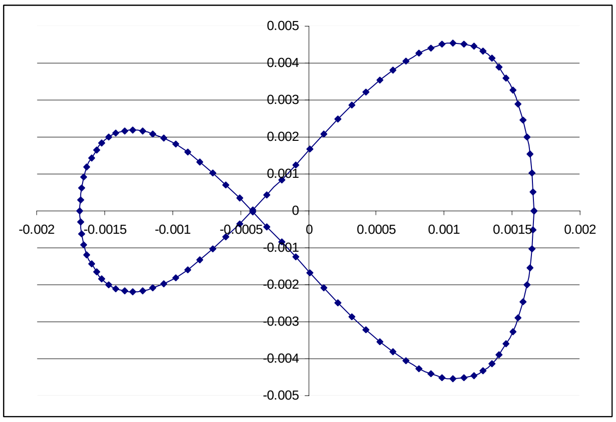


Figure 2. Phase portraits for $b = 1, a = 1, A = 1$

i) $\omega = 298, \alpha = 0.00168$

ii) $\omega = 300, \alpha = 0.00167$

iii) $\omega = 302, \alpha = 0.00166$

The above figures seem to be topologically equivalent as follows: for $\omega = 50n$ the phase portrait is an ellipse, like in figure 2*ii)*; in slight vicinity before this value one has figure 2*i)* and slightly after – figure 2*iii)*. This topological behaviour is not spoiled if one increases a – the coefficient of the nonlinear term – to values exceeding several times ω.

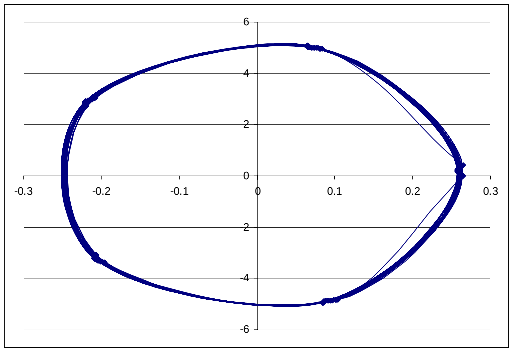


Figure 2. Phase portrait
$b = 20, \omega = 1, a = -10, A = 1, \alpha = 0.263$

For numerical applications we considered large values of the parameters and focused our study mainly on ω because, in order to obtain a less predictable behaviour of the solution one must obviously strengthen the forcing. This is not easily done in practice, for instance, for mechanical devices modelled by Düffing's equation. Moreover, we observe that formulae (24) and (28) contain denomimators vanishing for $b = \omega$, therefore one expects a weaker convergence for values of b and ω close to each other.

One can also obtain LEM solutions for Düffing's equation (1) with $\delta \neq 0$, whose solution may show a chaotic behaviour. This and a study of LEM representations for models with softening ($a < 0$) forms the objects of some future preoccupations.

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NOILE ESTIMARI LEM PENTRU OSCILATORUL DÜFFING NEUMED

***Rezumat**: In prezenta lucrare, autorii deduc o reprezentare LEM pentru un oscilator umed Düffing prevăzut cu forță armonică, in condiții nule Cauchy; cazul neumed nu poate fi obținut pornind de la coeficientul de umiditate anulat. Astfel, in această lucrare se consideră direct forțarea armonică neumedă, dată de oscilatorul Düffing cu condiții inițiale nenule, stabilind pentru acesta o aproximare analitică LEM ale soluțiilor folosind metode numerice clasice. Câteva imagini reprezentative sunt prezentate.*

P.P.Teodorescu, Prof. dr. rer. nat., University of Bucharest, petre_teodorescu@hotmail.com, Bucharest, Popa Soare 34, C.P.023984, sector 2, tel.021/3264373

Ileana Toma, Prof. dr., Technical University of Civil Engineering of Bucharest, Dept. of Mathematics and Computer Science, icvtoma@yahoo.com, Bucharest, Plantelor 78, C.P.023976, sector 2, tel.021/2532491