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ON THE STRUCTURE OF THE STABILITY ZONES OF THE DYNAMICAL SYSTEMS THAT DEPEND OF PARAMETERS

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Abstract: *The subject of this paper is focused on the stability analysis, in sense of Liapunov, for the evolution of the dynamical systems that depend of parameters. In this paper we emphasize new mathematical aspects on a property of separation referred to stable and unstable zones, in the plane of principal parameters, and on the possible existence of the isolate (singular) points of stability or instability, in the plane of principal parameters of the dynamical system. A numerical test on a particular case of dynamical system that depends of parameters, is described in the paper.*

Key words: *dynamical system, stability, principal parameters, structure of stability zones.*

1. INTRODUCTION NOTIONS

We mention, in the paper, some results and notions in domain [1]-[4].

Theorem 1 (Peano). Let $h: R_+ \times R^n \to R^n$, $R_+ = [0, \infty)$, $n \in N$, $n \neq 0$, be the continuous function and the system:

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, t} = h(t, y) \tag{1}$$

The system (1), for each $t_0 \geq 0$, $y_0 \in R^n$, has a local solution $y(t, t_0, y_0)$, précised by the value $t_1 \in R_+$, $t_1 > t_0$, solution that verifies the system (1) for all $t_0 \leq t < t_1$.

Theorem 2 (Cauchy). Let $h: R_+ \times R^n \to R^n$, $R_+ = [0, \infty)$, $n \in N$, $n \neq 0$, be as in Theorem 1 and supplementary there exist partial derivatives of h in rapport with components of y that are supposed continuous (h of C^1 class in rapport with components of y). Then, for each $t_0 \geq 0$, $y_0 \in R^n$, there exist $t_1 \in R_+$, $t_1 > t_0$, and a unique solution $y(t, t_0, y_0)$ that verifies the system (1) for all $t_0 \leq t < t_1$ and the initial condition $y(t_0) = y_0$.

Let $\tilde{y}$ be a solution of Eq. (1), with h fulfilling the conditions from Theorem 2, defined on the interval $[t_0, t_1)$, $t_0 < t_1$ with $\tilde{y}(t_0) = \tilde{y}_0$ and $y(t)$ other solution of Eq. (1) with $y(t_0) = y_0 \neq \tilde{y}_0$.

Definition 1. The solution $\tilde{y}$ of the equation (1) is called Liapunov stable in t_0 if for each $\varepsilon > 0$ there exists $\delta(\varepsilon, t_0)$ such that if $|y_0 - \tilde{y}_0| < \delta(\varepsilon, t_0)$ then $|y(t, t_0, y_0) - \tilde{y}(t, t_0, \tilde{y}_0)| < \varepsilon$ for all $t \geq t_0$.

If δ does not depend on t_0, the stability is “uniformly”.

The stability, in other words, means that small deviation from the basic evolution is produced for sufficiently small deviations of the initial conditions.

The study of the stability for a solution of the Eq. (1) can be reduced to one, corresponding to a solution that identically equals to zero. This is the reason for which, without loss of generality, we comment only the stability properties of the solution identically equal to zero. The definition of the stability is formulated as:

Definition $1'$. The solution $\tilde{y} \equiv 0$, in the case that verifies the equation (1), is called Liapunov stable in t_0 if for each $\varepsilon > 0$ there exists $\delta(\varepsilon,\ t_0)$ such that if $|y_0| < \delta(\varepsilon,\ t_0)$ involve the solution $|y(t,t_0,y_0)| < \varepsilon$ for all $t \ge t_0$.

One criterion for identify the stability of the solution $\tilde{y} \equiv 0$, with the initial condition $\tilde{y}_0$ and $|\tilde{y}_0| = 0$, referred to Eq. (1), will be formulated using the notions of oscillation of a function on a set and in the point.

Let $V_n = \left\{ \tilde{y}(t,t_0,y_0^*) \middle|\ |\tilde{y}_0^*| < 1/n,\ n \in N \right\}$, for each fixed n, be the neighborhood of solutions for Eq. (1), around the solution $\tilde{y} \equiv 0$ of the Eq. (1), defined using the neighborhood of the initial condition $\tilde{y}_0$ with $|\tilde{y}_0| = 0$. The sets $V_n,\ n \in N,$ satisfy the inclusions $V_1 \supseteq V_2 \supseteq \ldots \supseteq V_n \supseteq \ldots$.

The oscillation of the function précised by the correspondence $(t_0,\tilde{y}_0) \to \tilde{y}(t,t_0,\tilde{y}_0)$, with $\tilde{y}(t,t_0,\tilde{y}_0)$ solution of the Eq. (1) (deduced to each fixed initial condition) on the set $V_n,\ n \in N,$ is defined by the formula $\omega_y(V_n) = \sup\left\{ \left|\tilde{y}(t,t_0,\tilde{y}_0^*) - y(t,t_0,\tilde{y}_0^{**})\right|, |\tilde{y}_0^*| < 1/n, |\tilde{y}_0^{**}| < 1/n,\ t > t_0 \right\}$ where $\tilde{y}(t,t_0,\tilde{y}_0^*),\ \tilde{y}(t,t_0,\tilde{y}_0^{**})$ are solutions of the Eq. (1). There are the inequalities:

$$\omega_{\tilde{y}}(V_1) \ge \omega_{\tilde{y}}(V_2) \ge \ldots \ge \omega_{\tilde{y}}(V_n) \ge \ldots \ge 0 \tag{2}$$

The oscillation of the function $(t_0,\tilde{y}_0) \to \tilde{y}(t,t_0,\tilde{y}_0)$, with $\tilde{y}(t,t_0,\tilde{y}_0)$ solution of the Eq. (1), in the point $\tilde{y}_0$ with $|\tilde{y}_0| = 0$, is the limit of the following convergent sequence $\omega_{\tilde{y}}(V_1),\ \omega_{\tilde{y}}(V_2),\ \ldots, \omega_{\tilde{y}}(V_n), \ldots$.

Theorem 3. The solution $\tilde{y} \equiv 0$, of the Eq. (1) is stable in t_0 if and only if the sequence $\omega_{\tilde{y}}(V_1),\ \omega_{\tilde{y}}(V_2), \ldots, \omega_{\tilde{y}}(V_n), \ldots$ has zero limit.

Proof:

Suppose that the solution $\tilde{y} \equiv 0$, of the Eq. (1) is stable in t_0. Then, for $\varepsilon > 0$, there is $\delta(\varepsilon,\ t_0)$ such that if $|y_0| < \delta(\varepsilon,\ t_0)$ involve the solution $|y(t,t_0,y_0)| < \varepsilon$ for all $t \ge t_0$. For the set $V_n = \{ \tilde{y}(t,t_0,\tilde{y}_0)|,\ |\tilde{y}_0| < 1/n,\ n \in N \}$ and $\varepsilon > 0$ with corresponding $\delta(\varepsilon,\ t_0)$, we select the rank N_δ such that if $n > N_\delta$ than the value $1/n < \delta(\varepsilon,\ t_0)$ and estimate $\omega_{\tilde{y}}(V_n) = \sup\{ |\tilde{y}(t,t_0,\tilde{y}_0^*) - \tilde{y}(t,t_0,\tilde{y}_0^{**})|,\ |\tilde{y}_0^*| < 1/n,\ |\tilde{y}_0^{**}| < 1/n,\ t \ge t_0 \}$. Because $|\tilde{y}_0^*| < 1/n < \delta(\varepsilon,t_0)$ and $|\tilde{y}_0^{**}| < 1/n < \delta(\varepsilon,t_0),\ n > N_\delta$ that involve, from the hypothesis, the inequalities $|\tilde{y}(t,t_0,\tilde{y}_0^*)| < \varepsilon$, $|\tilde{y}(t,t_0,\tilde{y}_0^{**})| < \varepsilon$ and thus $|\tilde{y}(t,t_0,\tilde{y}_0^*) - \tilde{y}(t,t_0,\tilde{y}_0^{**})| \le |\tilde{y}(t,t_0,\tilde{y}_0^*)| + |\tilde{y}(t,t_0,\tilde{y}_0^{**})| < \varepsilon + \varepsilon = 2\varepsilon$ for all $t \ge t_0$. It follows the inequality $\omega_{\tilde{y}}(V_n) \le 2\varepsilon$, for $n > N_\delta$ and arbitrary $\varepsilon > 0$, such that $\omega_{\tilde{y}}(V_n) \to 0$ for $n \to \infty$.

Conversely, we suppose that the oscillation of the function $(t_0,\tilde{y}_0) \to \tilde{y}(t,t_0,\tilde{y}_0)$, with $\tilde{y}(t,t_0,\tilde{y}_0)$ solution of the Eq. (1), is zero in the point $\tilde{y}_0$ with $|\tilde{y}_0| = 0$, namely that the limit of the convergent sequence $\omega_{\tilde{y}}(V_1),\ \omega_{\tilde{y}}(V_2), \ldots, \omega_{\tilde{y}}(V_n),\ \ldots$, is equal with zero. In these conditions we shall verify that the solution $\tilde{y} \equiv 0$, of the equation (1) is stable in t_0, with other words that for each $\varepsilon > 0$ there exists $\delta(\varepsilon,\ t_0)$ such that if $|\tilde{y}_0| < \delta(\varepsilon,\ t_0)$ involve the inequality $|\tilde{y}(t,t_0,\tilde{y}_0)| < \varepsilon$ for all $t \ge t_0$, referred to Eq. (1) solution. Let $\varepsilon > 0$ be as small as chosen.

The hypothesis $\omega_{\tilde{y}}(V_n) \to 0$ for $n \to \infty$ involves that for fixed $\varepsilon > 0$ there is the rank N_ε such that if $n > N_\varepsilon,\ \omega_{\tilde{y}}(V_n) < \varepsilon$. It follows $|\tilde{y}(t,t_0,\tilde{y}_0^*) - \tilde{y}(t,t_0,\tilde{y}_0^{**})| < \varepsilon$ for $|\tilde{y}_0^*| < 1/n < 1/N_\varepsilon$ and $|\tilde{y}_0^{**}| < 1/n < 1/N_\varepsilon,\ t > t_0$. We choose $\delta(\varepsilon,t_0) = 1/N_\varepsilon$, $\tilde{y}_0^* = y_0$, $|y_0| < 1/N_\varepsilon$, $|y_0^{**}| = |\tilde{y}_0| = 0$, such that $|\tilde{y}(t,t_0,\tilde{y}_0^{**})| = 0$ that follows $|y(t,t_0,y_0)| < \varepsilon$ for $|y_0| < 1/N_\varepsilon$, $t \ge t_0$.

The stability of the solution $\tilde{y} \equiv 0$, of the Eq. (1), is thus verified in t_0.

Definition 2. The solution $\tilde{y} \equiv 0$ of the Eq. (1), is named asymptotically stable in t_0 if it is stable in t_0 and moreover, there exists $\delta_0 > 0$ such that for each other solution $y(t)$ of the Eq. (1) with $y(t_0) = y_0$ and $|y_0| < \delta_0$ involve $|y(t, t_0, y_0)| \to 0$ for $t \to \infty$

We mention a possible particular case of the Eq. (1) for which function h does not depend explicit on the first argument. In this case the Eq. (1) is named autonomous. The function $\tilde{y} \equiv 0$ is supposed to be a solution of the autonomous Eq. (1). We mention the notion of the "stability by the first approximation" for the autonomous systems. The function $h(y)$ is developed in Taylor series, around the solution $\tilde{y} \equiv 0$, such that the Eq. (1) can be substituted by the approximate system:

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, t} = A(t) y + B(y) \tag{3}$$

In the Eq. (3), matrix $A(t)$ is of dimension $n \times n$ and is compounded from the first order partial derivatives of the function h components, in rapport with the function y components.

The stability study by the first approximation is the stability study for the system:

$$\frac{\mathrm{d}\, y}{\mathrm{d}\, t} = A(t) y \tag{4}$$

The stability of the equation of type (4) can be analyzed also independent of their possible origin. If the matrix A is not dependent of time t the Eq. (4) is also named autonomous. The solution of the Eq. (4) in the case of constant matrix A of real values is described using the eigenvalues and eigenvectors of the matrix. The real matrix A of degree n has n eigenvalues, distinct or multiples. The eigenvectors are determined from the equation $Ay = \lambda y$ for each eigenvalue λ (real or complex) of the matrix A.

The eigenvectors of the matrix A , denoted $f_k,\ k \in N$, are linearly independents if the eigenvalues are distinct. The general solution of the Eq. (4), in this case, is of the form below.

$$y = \sum_{k=1}^{n} c_k f_k e^{\lambda_k t} \tag{5}$$

Theorem 4. The solution $y \equiv 0$ of the system (4), in the case of matrix A with constant real values components is asymptotic stable if all eigenvalues of matrix A have negative real parts, is instable if at list one eigenvalue of matrix A has positive real part and is with uncertain stability (stable or instable) if at list one eigenvalue has zero real part.

The propriety that the "most" matrices have distinct eigenvalues is explicated below.

Definition 3. The subset U of R^n is "dense" set in R^n if there are points in U arbitrarily close to each point in R^n. More precisely, if $X \in R^n$ and ε>0 is as small as chosen, there exist some $Y \in U$ such that $|Y - X| < \varepsilon$.

Definition 4. Let $L(R^n)$ be the set of $n \times n$ matrices, or, equivalently, the set of linear maps of R^n. The distance between two matrices of dimension $n \times n$ is defined using the rule for the distance between two vectors in R^n where the matrices of $L(R^n)$ are assimilated as vectors from R^m with $m = n^2$.

The space $L(R^n)$, with the distance defined above, is a linear normed space.

Hirsch, Smale and Devaney have demonstrated, using the transformed Jordan form of any matrix of $L(R^n)$, the theorem formulated in the following [1].

Theorem 5. The set M from $L(R^n)$ of matrices that have n distinct eigenvalues is open and dense in $L(R^n)$. Another theorem, about canonical Jordan form of the real matrix A, is formulated below [1].

Theorem 6. Consider the system $X' = A X$ where real matrix A has distinct eigenvalues $\lambda_1, ..., \lambda_{k1} \in R$ and $\alpha_1 + i\beta_1, ...,\ \alpha_{k2} + i\beta_{k2} \in C$, where a real numbers set is R and a complex numbers set is C. Let T be the matrix that puts matrix A in the canonical form $T^{-1}AT$, denoted FJ. The Jordan form FJ has, on the diagonal, successively, the eigenvalues $\lambda_1, ..., \lambda_{k1} \in R$ and the matrices, of the order two, denoted $B_1, ..., B_{k2}$, of the form

$$B_j = \begin{pmatrix} \alpha_j & \beta_j \\ -\beta_j & \alpha_j \end{pmatrix}, j = 1,...,k2 \qquad (6)$$

2. ANALYSIS OF THE STABILITY

The analysis of the dynamical system stability, for some fixed parameters and two free parameters, is performed in many cases of dynamical systems, on the matrix A with continuous components or continuous on piecewise as functions of free parameters. From the algorithms of type defined by Rutishauser [5] or others, in specific conditions, which are not analyzed in this paper, can be concluding that *the analytical expressions of the matrix A eigenvalues are continuous functions or continuous on piecewise, referred to free parameters of the dynamical system*. In these conditions, the property of separation between the stability and instability zones, in the plane of principal parameters, is deduced from the theorem 4 and from the theorem of the type below.

Theorem 7. If the function $f: E \to R$ is continuous in point $x_0 \in E$ and $f(x_0)$ satisfies the inequalities $\alpha < f(x_0) < \beta;\ \alpha, \beta \in R$, then there exist a neighborhood $V(x_0)$ such that $\alpha < f(x) < \beta$ for all $x \in V(x_0) \cap E$.

In this paragraph we analyze the possible structure of the stable and unstable zones, in the plane of principal parameters, for the dynamical systems that depend of parameters. Firstly we define the pantograph - catenary dynamical system [6]-[9] with physical model in fig.1.

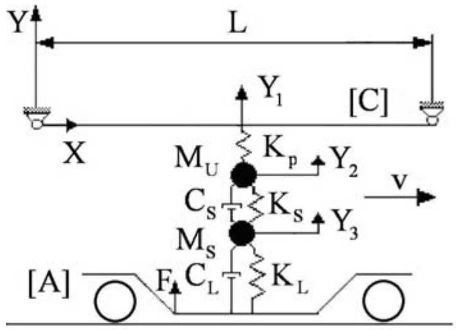

Fig. 1. Physical model.

The oscillating system is moving with a constant speed v at the same time with the vehicle $[A]$. The values y_1, y_2, y_3 are respectively, the deflections of the wire compressed by the oscillating system, the deflections of the centroid for the masses M_u and M_s from the equilibrium position ($M_s + {}$ ${}+ M_u = M_{su}$), $y(x,t)$ is the deflection of the wire, EI is the bending stiffness of the wire, T is horizontal tension in the wire, β is viscous damping of the wire, m is the mass per unit length of the wire, c_s and c_L are damping coefficients, k_s, k_L and k_p are stiffness elements of the system, $y_0 = y(x,0)$ is the micro-irregularity in x, d is the length of the contact zone between pantograph and catenary wire, L is the length of the span for the wire, g is gravity acceleration, t is the time.

We introduce dimensionless variables and parameters (where the fundamental natural frequency of the wire span is ω_1, the maximum displacement of a simply supported wire when a unit load is applied in its midpoint is y_{st}), specified as follows (see also [9]).

$$\begin{gathered}
\xi = \frac{\pi x}{L},\ \tau = \frac{\pi v t}{L},\ \tilde{v} = \sqrt{m/T} v = \tilde{v}_T, \\
\tilde{M} = M_{su} / m / L,\ \Delta = \pi\, d / L, \\
y_{st} = \frac{g\, L^3}{48\, EI},\ \tilde{y} = y / y_{st},\ \tilde{g} = \frac{48}{\pi^4 \tilde{M}\ \tilde{v}^2}, \\
\mu = M_u / M_{su},\ \tilde{\omega}_{n\,s} = \frac{L}{\pi v} \sqrt{k_s / M_{su}}, \\
\tilde{\omega}_{n\,L} = \frac{L}{\pi v} \sqrt{k_L / M_{su}},\ \tilde{v}_\beta = \frac{m\,\pi}{\beta\, L}\ v, \\
\tilde{\Omega}_n = \frac{L}{\pi v} \sqrt{k_p / M_{su}},\ \zeta_s = \frac{c_s / \sqrt{k_s M_{su}}}{2}, \\
\zeta_L = \frac{c_L / \sqrt{k_L M_{su}}}{2},\quad \tilde{v}_{EI} = \frac{L}{\pi} \sqrt{\frac{m}{EI}}\ v.
\end{gathered} \qquad (7)$$

The matrix form of the equations system, which approximates the evolution of the physical model from figure1, has the unknown vector components described below.

$$\begin{gathered}
X_1(\tau) = \mu\ \tilde{y}_2,\ X_2(\tau) = \mu\ \dot{\tilde{y}}_2, \\
X_3(\tau) = (1 - \mu)\ \tilde{y}_3,\ X_4(\tau) = \\
= (1 - \mu)\ \dot{\tilde{y}}_3, \\
X_5(\tau) = \frac{1}{2M\ \sin\tau}\ T_1(\tau), \\
X_6(\tau) = \frac{1}{2M\ \sin\tau}\ \dot{T}_1(\tau).
\end{gathered} \qquad (8)$$

The function $T_1(\tau)$ intervenes in system (8) for describing the displacement of the contact point between pantograph and catenaries wire.

The matrix A, in the equation $\mathrm{d}X/\mathrm{d}\tau =$, $= AX$, where the vector X is précised by their components in (8), has the components a_{ij} described below.

$$
\begin{aligned}
&a_{12} = 1,\ a_{21} = -\tilde{\omega}_{sn}^2/\mu - \tilde{\Omega}_n^2/\mu,\\
&a_{22} = -2\zeta_s\tilde{\omega}_{sn}/\mu, a_{23} = \tilde{\omega}_{sn}^2/(1-\mu),\\
&a_{24} = 2\zeta_s\tilde{\omega}_{sn}/(1-\mu),\ a_{25} = 2M\ \tilde{\Omega}_n^2\ \sin^2(\tau),\\
&a_{34} = 1,\ a_{41} = \tilde{\omega}_{sn}^2/\mu,\ a_{42} = 2\zeta_s\tilde{\omega}_{sn}/\mu,\\
&a_{43} = -(\tilde{\omega}_{sn}^2 + \tilde{\omega}_{nL}^2)/(1-\mu),\\
&a_{44} = -2(\zeta_s\tilde{\omega}_{sn} + \zeta_L\tilde{\omega}_{nL})/(1-\mu),\\
&a_{56} = 1,\ a_{61} = \tilde{\Omega}_n^2/\mu,\ a_{65} = -1/\tilde{v}_{EI}^2 - 1/\tilde{v}_T^2 -\\
&-2M\ \tilde{\Omega}_n^2\ \sin^2(\tau),\ a_{66} = -1/\tilde{v}_\beta.
\end{aligned}
\tag{9}
$$

The components a_{ij}, which are not appeared in formulas (9), have value zero.

The stability of this dynamical system has analyzed in the following particular case:

$$
\begin{aligned}
&\tilde{\Omega}_{\mathrm{n}} = 3.185,\ \ \tilde{v}_{\beta} = 19.8,\ \ \tilde{\omega}_{\mathrm{nL}} = 0.48,\\
&\tilde{v}_{\mathrm{EI}} = 90.96,\ \ \varsigma_s = 0.3,\ \ \varsigma_L = 0.3
\end{aligned}
\tag{10}
$$

The free parameters for the plane of principal parameters are chosen dimensionless variables $\tilde{\omega}_{ns}$ and $\tilde{v}_T$. We analyze the stability of motion for the dimensionless displacement $\tilde{y}_2$ of the concentrated mass M_u in the free vibration of the system, in specified domain of interest. In fig.2 are plotted, the stable and unstable zones of the displacement $\tilde{y}_2$, in the two chosen parameters plane, separated by the line of periodic solutions.

For identify the stability zones we have used an algorithm defined by us and explicated in some previous papers ([6], [7]).

The transformed Jordan form of the matrix $A(\tau)$, of our mathematical model, precise the eigenvalues of the matrix for which we can discuss the sign of eigenvalues real part. The sign is decided for continuous functions or continuous on piecewise functions of values $(\tilde{\omega}_n,\ \tilde{v})$ and fixed time τ_0 such that for each sign there is a neighborhood of values pair $(\tilde{\omega}_n,\ \tilde{v})$ where the sign is preserved.

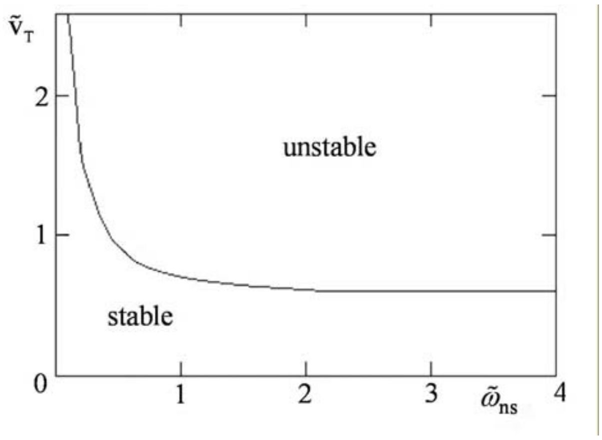

Fig.2. Stable zones in the plane of two free parameters.

The stability and instability zones, in the plane of principal parameters are separated by the values of neutral curves. This can be seeing in the fig.2. Another question can be discussed here, namely, for bounded zone of stability (or of instability), in the plane of principal parameters, identified from the continuity of the real part of the matrix eigenvalues that intervene in characterization of the dynamical system, we can analyze, using the theorem 4, the case when the real part of the complex eigenvalue is zero (in the case that exist précised values of the principal parameters that annul the real part of the complex eigenvalue). In this case, of précised values of parameters, the solution of the dynamical system can be stable or instable, indifferent of the characterization in neighborhood of the précised point of the dynamical system. Such that we underline a possible alternative on the structure of the stable and unstable zones in the plane of principal parameters: the existence of the separated stable and unstable zones in the plane of principal parameters, delimited by the line of periodic solutions, but also possible isolate (singular) points of stability or of instability.

3. CONCLUSION

In the paper some fundamental notions and results in the domain are presented, referred to stability in sense of Liapunov for the dynamical systems that depend of parameters. The authors have analyzed the possible structure of the stable and unstable zones in the plane of principal parameters for dynamical systems that depend of parameters, exemplified on the pantograph - catenary dynamical system.

Theoretical considerations are performed by the authors referred to the possible alternative on the structure of the separate stable and unstable zones in the plane of principal parameters, and the possible isolate (singular) points of stability (or instability) in the plane of principal parameters of the dynamical system.

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ASUPRA STRUCTURII ZONELOR DE STABILITATE ALE SISTEMELOR DINAMICE CARE DEPIND DE PARAMETRI

Rezumat: Subiectul acestei lucrari consta in analiza stabilitatii, in sensul lui Liapunov, privind evolutia sistemelor dinamice care depind de parametri. Se reliefeaza aspecte matematice noi, referitoare la proprietatea de separare a zonelor de stabilitate si a celor de instabilitate, in planul parametrilor principali si referitoare la posibila existenta a punctelor de stabilitate sau instabilitate izolate. Se prezinta un test numeric pentru un caz concret de sistem dinamic.

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