TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue III, 2012

SIZE EFFECT ON THE DYNAMICAL BEHAVIOUR OF ELECTROSTATICALLY ACTUATED MEMS RESONATORS

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Abstract: *The size effect on the dynamical response of electrostatically actuated MEMS resonators is analyzed and presented in this paper. The interest is to measure the frequency response of microresonators and to estimate the energy loss coefficient. The loss of energy in a vibrating structure is experimentally estimated measuring the quality factor. The quality factor is a resonator parameter that gives information about the energy dissipated during one cycle of oscillation. The total energy dissipation depends on the air damping and on the structural thermoelastic effect. MEMS resonators under investigations are polysilicon microcantilevers fabricated in different geometrical dimensions.*
Key words: *Multiphysics coupling, MEMS resonators, frequency response, quality factor, loss of energy.*

1. INTRODUCTION

The microresonator represents currently one of the important research areas of Microelectromechanical Systems (MEMS). The usual applications of microresonators are: radio-frequency MEMS devices, MEMS gyroscopes and vibrating MEMS sensors used in seismic or mass detection applications [1-4].

The microresonators can be dynamically characterized based on their resonant frequency response and quality factor [2, 5]. The quality factor is an expression of the cyclic energy loss in an oscillating system. These mechanical resonators are essential components in communication circuits because they generally exhibit orders of magnitude higher quality factor than electrical components. Such devices can be designed to vibrate over a very wide frequency range (higher than 1GHz), making them ideal for ultra stable oscillator and low filter functions for a wide range of transceiver types. Energy losses in high-frequency resonators are critical to designers [5].

Changes in the environmental conditions have influence on the dynamic response of resonators by modifying the structural damping and the squeeze film coefficient [2, 4]. Indeed for the same loading conditions, the amplitude and velocity of oscillations increase if the temperature decreases [2]. In the same way for high temperature operating conditions, the amplitude and velocity of oscillations decrease due to the heat energy dissipation called thermal damping, and the material heat softening called temperature relaxation [1].

The geometrical configurations and dimensions have a big influence on the response sensitivity of microresonators. Sensitivity depends on the resonator stiffness that is strongly influenced by the beam length. Resonant frequency of resonators is changed as a function of geometrical dimensions.

The effect of geometrical dimensions on the dynamical response of electrostatically actuated MEMS cantilevers is analyzed and discussed in this paper. Electrostatic actuation is the most common type of electromechanical energy conversion scheme in MEMS for a wide range of applications [1, 2].

2. THEORETICAL FORMULATION

2.1 Stiffness and resonant frequency

MEMS resonators considered here are polysilicon microcantilevers fabricated in different geometrical dimensions. The samples are loading by an electrostatic force set up when a voltage is applied between the vibrating cantilever and the lower electrode (Fig.1).

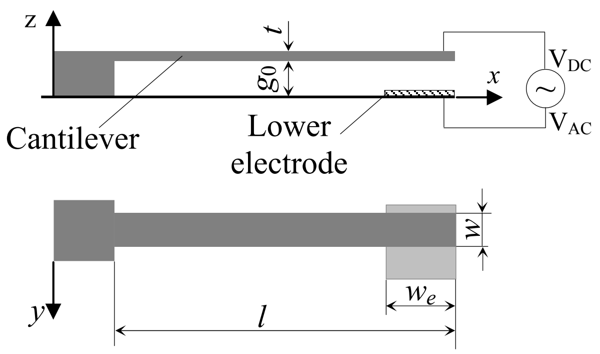


Fig. 1. Schematic representation of an electrostatically actuated MEMS cantilever.

When a DC voltage (V_{DC}) is applied between electrode and cantilever, the cantilever bends downwards and come to rest in a new position. To drive the resonator at resonance, an AC harmonic load of amplitude V_{AC} vibrates the cantilever at the new deflected position.

A single degree of freedom model is used to analyze the dynamic response of microresonator due to the V_{DC} and V_{AC} electric loadings as presented in figure 2.

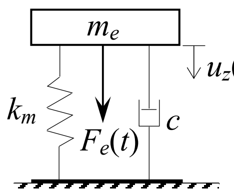


Fig. 2. A single degree of freedom model for electrostatically actuated MEMS cantilever.

In this model the proof mass of microcantilever is modeled as a lumped mass m_e, and its stiffness is considered as a spring constant k_m. This part forms one side of a variable capacitor - the movable part. The bottom electrode is fixed and considered as the second part of the sensor. If an voltage composed of DC and AC terms as

$$V(t) = V_{DC} + V_{AC}\cos(\omega t) \tag{1}$$

is applied between electrodes, the electrostatic force applied on the structure has a DC component as well as a harmonic component with the driving frequency ω such as

$$F_e(t) = \frac{\varepsilon A V(t)^2}{2[g_0 - u_z(t)]^2} \tag{2}$$

where ε is the permittivity of the free space, $A=w_e \times w$ is the effective area of the capacitor, g_0 is the initial gap between flexible plate and substrate, and $u_z(t)$ is the displacement of the mobile plate.

When only a DC voltage is applied across the plates (V_{AC} = 0), the static force balance equation, including the electrostatic force and the spring force is

$$k_m u_z = \frac{\varepsilon A V_{DC}^2}{2(g_0 - u_z)^2} \tag{3}$$

where u_z is the static displacement of the beam under a DC signal and k_m is the mechanical stiffness given by

$$k_m = \frac{3EI_y}{l^3} \tag{4}$$

where E is the Young's modulus of cantilever material, I_y is the axial moment of inertia and l is the length of beam.

The equivalent stiffness k_e and resonant frequency ω_0 of microresonator under a DC actuation is obtained by linearizing the electric system around an equilibrium position $\tilde{u}_z$ as

$$k_e = \frac{3EI_y}{l^3} - \frac{\varepsilon A V_{DC}^2}{(g - \tilde{u}_z)^3} \tag{5}$$

$$\omega_0 = \frac{1}{2\pi}\sqrt{\frac{k_m - \dfrac{\varepsilon A V_{DC}^2}{(g - \tilde{u}_z)^3}}{m_e}} \tag{6}$$

where m_e, the equivalent mass of system can be calculated using m_e=33m/140, with m - the effective mass of beam.

The free response of a mechanical resonator determines the resonant frequency in either the presence or the absence of damping. The forced response reveals the behavior of an undamped or damped mechanical system under the action of a harmonic excitation. In mechanical resonators, the phenomenon of resonance is important, and in such situation the excitation frequency matches the resonant frequency of the system.

2.2 Dynamic response and quality factor

The dynamic response of the microcantilever resonator presented in figure 1 subjected to a harmonic electrostatic force $F_e(t)$ with a driving frequency ω given by an AC voltage is governed by the equation of motion:

$$m \cdot \ddot{u}_z(t) + c \cdot \dot{u}_z(t) + k_m \cdot u_z(t) = F_e(t) \qquad (7)$$

where $u_z(t)$ is the amplitude of beam oscillations and c is the damping factor.

The system response under DC and AC voltages is given by the equation [1, 2]

$$u_z(t) = \frac{u_z}{\sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_0}\right)^2}} \qquad (8)$$

where ξ is the damping ratio and ω_0 is the resonant frequency of microcantilever given by equation (6).

The damping ratio ξ is any positive real number. For value of the damping ratio $0 \le \xi < 1$, the system has an oscillatory response. The system damping controls the amplitude and velocity of the response when is excited at resonance.

Usually, the response is plotted as a normalized quantity $u_z(t)/u_z$. When the driving frequency equals the resonant frequency $\omega = \omega_0$, the amplitude ratio reaches a maximum value. At resonance, the amplitude ratio becomes

$$\frac{u_z(t)}{u_z} = \frac{1}{2\xi} \qquad (9)$$

An important qualifier of mechanical microresonators is the quality factor Q. At resonance, the quality factor is expressed as [1]

$$Q_r = \frac{1}{2\xi} \qquad (10)$$

and the normalized response given by equation (9) is exactly equal with Q_r.

The quality factor is also called sharpness at resonance, which is defined as the ratio

$$Q_r = \frac{1}{\Delta\omega} = \frac{\omega}{\omega_2 - \omega_1} \qquad (11)$$

where $\Delta\omega = \omega_2 - \omega_1$ is the frequency bandwidth corresponding to $u_z(t)_{\max}/\sqrt{2}$ on the amplitude (velocity) versus frequency curves.

The loss coefficient of energy can be estimated using the quality factor. The total loss coefficient of a microresonator is influenced by two components as

$$Q_{total}^{-1} = Q_e^{-1} + Q_i^{-1} \qquad (12)$$

where the subscripts e denotes the extrinsic losses given by air damping and i is the intrinsic losses corresponding to thermoelastic damping.

The total loss coefficient is experimentally determined when the sample is oscillated in air.

3. EXPERIMENTAL INVESTIGATIONS

The scopes of experimental investigations are to measure the resonant frequency responses of investigated MEMS cantilevers, to estimate the quality factor and the energy dissipated during oscillations. The samples for experiments are polysilicon microcantilevers fabricated in different lengths (150µm, 175µm and 200µm), width of 30µm and 20µm, and a thickness of 1.9µm. The gap between flexible part and substrate is 2µm. Two different geometrical types of cantilevers, with holes and without holes, are chosen (Fig.3 and Fig.4). The holes effect on the dynamical response of microcantilevers changes the amplitude and velocity of oscillations based on air damping decreases. The holes diameter is 3µm.

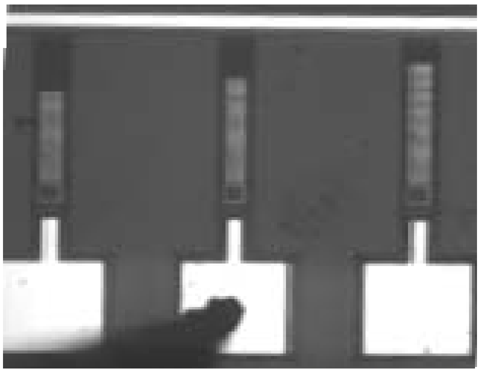


Fig. 3. MEMS cantilever with different lengths (without holes).

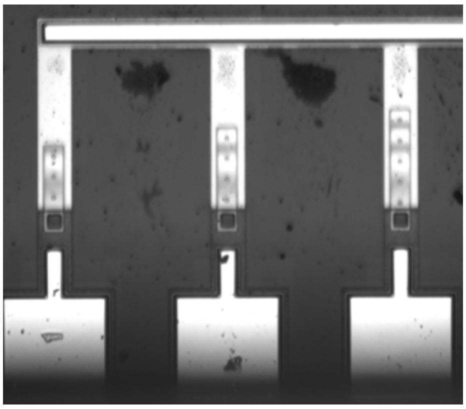


Fig. 4. MEMS cantilever with holes and different lengths.

Using a Vibrometer Analyzer and a white noise signal of 5V offset current and 5V peak-to-peak amplitude of the driving current, frequency responses of investigated microcantilevers were determined. The average value of the distributed input voltage per Hertz is approximately 28mV (Fig.5).

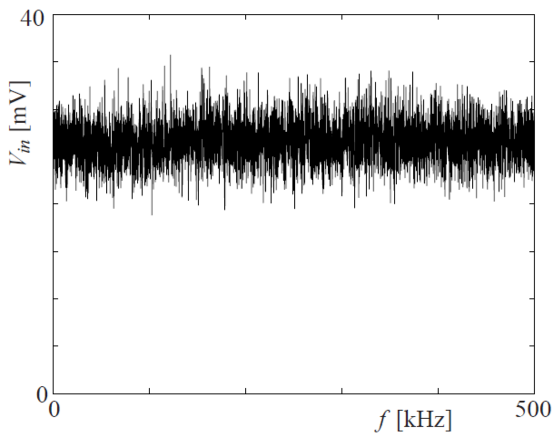
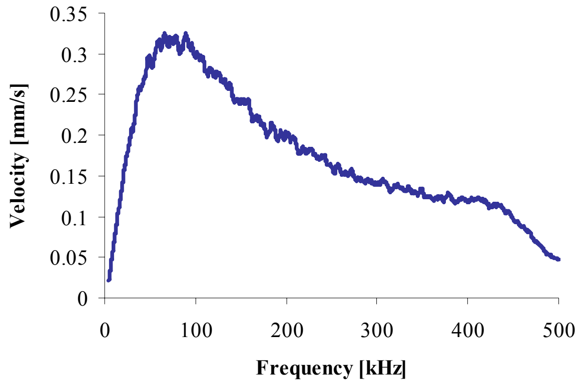
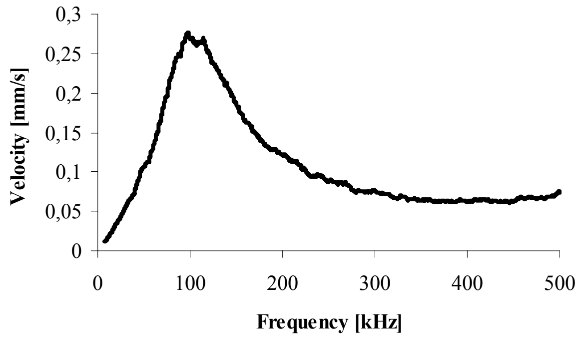


Fig. 5. Input signal (white noise) in the frequency domain (0…500 kHz): Offset voltage 5V, Peak-Peak signal 5V.



(a)



(b)

Fig. 6. Frequency response of microcantilever with length of 150µm: (a) width of 30µm; (b) width of 20µm.

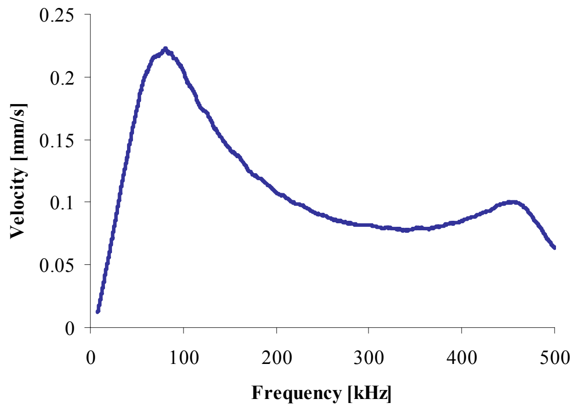


Fig. 7. Frequency response of microcantilever with length of 175µm and width of 30µm.

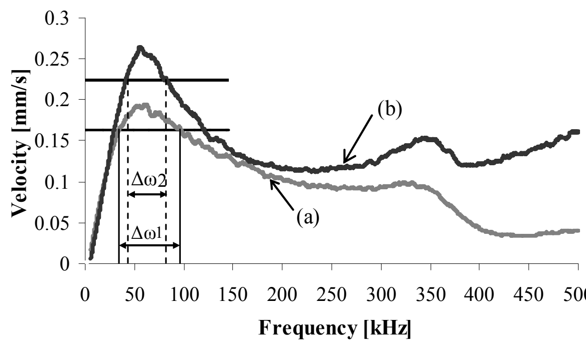


Fig. 8. Frequency responses of microcantilevers with length of 200µm and width of 30µm: (a) without holes; (b) with holes.

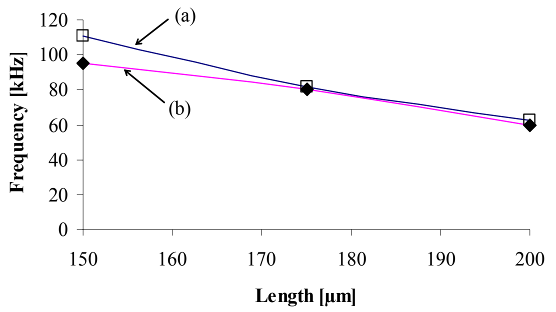


Fig. 9. Frequency response of microcantilevers (without holes) as function of their lengths: (a) theoretical dependence; (b) experimental values.

Table 1

Velocity, quality factor, and loss coefficient of energy of investigated microcantilevers.

Length [µm]	Velocity [mm/s]	Quality factor Q	Total energy loss coefficient
Microcantilever without holes (w=20µm, t=1.9µm)			
150	0,28	1,45	0,68
Microcantilevers without holes (w=30µm, t=1.9µm)			
150	0,33	0,91	1,09
175	0,22	0,76	1,30
200	0,18	0,59	1,68
Microcantilever with holes (w=30µm, t=1.9µm)			
200	0,27	0,73	1,36

Firstly, using a white noise exciting signal, the resonant frequency responses of investigated microcantilevers tested in air are measured. The resonant frequency decreases if the microcantilever length increases, respectively. If the experimental resonant frequency of the microcantilever with a length of 150µm is determined of 95kHz (Fig.6), it decreases to 80kHz for the microcantilever with a length of 175µm (Fig.7) and to 60µm for the microcantilever with a length of 200µm (Fig.8). The experimental results of resonant frequency are in good agreement with theoretical results (Fig.9) obtained using equation (5).

Secondly, using the band width $\Delta\omega$ given by the frequency response experimental curves, the quality factor of vibrating structures and the loss coefficient of energy is estimated based on equation (11) for microcantilevers with different lengths and widths. Table 1 shows the effect of the beam length and width on the dynamical response. If the length of microcantilevers increases, the quality factor decreases. On the other hand, if the width of microcantilevers decreases, the quality factor increases, respectively. The quality factor of microcantilever with a length of 150µm increases from 0,91 to 1,45 if the width of beam decreases from 30µm to 20µm.

Moreover, the effect of holes on the dynamical responses of beam is monitored. The holes increases the amplitude and velocity of oscillations because the damping given by air decreases. Figure 8 shows the effect of holes on the dynamical response of the microcantilever with a length of 200µm and a width of 30µm. The bandwidth of microcantilever without holes $\Delta\omega_1$ is different from the bandwidth $\Delta\omega_2$ of microcantilever with holes (Fig.8) that increasing the quality factor and decreasing the loss energy coefficient as it can be observed in table 1.

4. NUMERICAL SIMULATION

In order, to estimate the holes effect on the frequency response of investigated microcantilevers, modal analysis is performed.

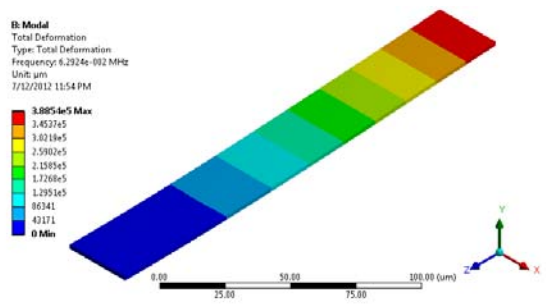


Fig. 10. Modal analysis of microcantilever with a length of 200µm (without holes)

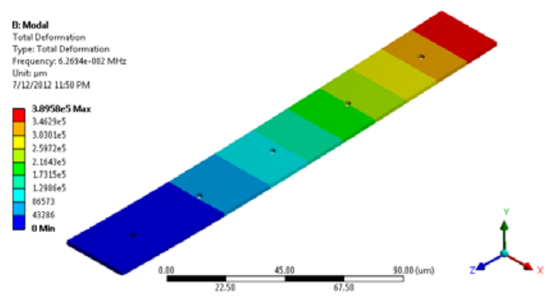


Fig. 11. Modal analysis of microcantilever with a length of 200µm (with holes)

Modal analysis of investigated microcantilever with a length of 200µm, a width of 30µm and a thickness of 1,9µm without holes is presented in figure 10. The numerical value of resonant frequency is 62,92kHz close to the resonant frequency of the same microcantilever but with holes (Fig. 11).

5. CONCLUSIONS

The scale effect on the dynamical behavior of polysilicon microcantilevers was investigated and presented in this paper. If the microcantilever length increases, the resonant frequency decreases and the energy loss increases, respectively. The loss of energy is estimated based on quality factor measured using the frequency response experimental curves. During experiments the total loss coefficient is monitored. Decreasing the beam width, the quality factor can be improved. In order to decrease the damping effect and to improve the dynamical response of beam operating in air, microresonators with small length and width are recommended to be used. The experimental results are in good agreement with the theoretical values.

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Acknowledgment

This work was supported by a grant of the Romanian National Authority for Scientific Research, CNCS-UEFISCDI, project number PN-II-RU-TE-2011-3-0106.

Efectul dimensional asupra comportarii dinamice la rezonatoarele MEMS actionate electrostatic

Efectul dimensiunilor geometrice asupra comportarii dinamice a rezonantoarele MEMS este analizat si prezentat in acest articol. Scopul lucrarii este de a determina frecventa de raspuns a rezonatoarelor MEMS si de a estima pierderea de energie. Pierderea de energie intr-un rezonator poate fi determinata experimental prin masurarea factorului de calitate pe curba frecventei de raspuns. Factorul de calitate este un parametru care ia in considerare pierderea de energie disipata pe perioada oscilatiei. Microrezonatoarele studiate sunt realizate din polisiliciu in diferite dimensiuni geometrice.

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