TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue III, 2012

A 3D MODEL FOR TIRE/ROAD DYNAMIC CONTACT

Ligia MUNTEANU, Cornel BRIŞAN, Veturia CHIROIU, Ştefania DONESCU

Abstract: The paper presents a 3D model for tire/road dynamic contact by using the Sneddon dual integral equations. The tire tread is modeled as an incompressible elastic half-space and the road surface is supposed to be a rigid rough surface.

Key words: tire/road contact, business problem, dual integral equations.

1. INTRODUCTION

The dynamic contact or impact is not a continuous process over a finite interval of time. The impact occurs in a short interval of time and determines the slipping, sticking and reverse motion after stopping. The interaction tire/road expressed by forces distributed on the tireprint is a dynamic contact which includes brief duration of impacts, high forces, large acceleration and decelerations and fast dissipation of energy. Also, the compression and restitution, rolling resistance, skid resistance, wear and noise generation, are important in the modeling of the interaction between the tire and the road surface [1, 2]. The impact may generate discontinuities in geometry and the material properties [3, 4]. The literature in the field of contact/impact analysis is rich and broaches many subjects and disciplines [5-7].

A 3D model for tire/road dynamic contact by using the Sneddon dual integral equations is presented in this paper. The tire tread is modeled as an incompressible elastic half-space and the road surface is supposed to be a rigid rough surface with irregularities (See Fig.1).

2. BASIC THEORY

The contact between the elastic half-space and the road surface is reduced to the following Boussinesq equations [8]

$$\delta = \iint_D \frac{p(\xi,\eta)(1-\nu^2)}{\pi E\sqrt{(x-\xi)^2+(y-\eta)^2}}\mathrm{d}\xi\mathrm{d}\eta \ \text{ in } D\,, \qquad (1)$$

where δ is the displacement at the surface of the half-space, D is the surface of half-space, p is the normal contact pressure, E the Young's modulus of the half-space and ν the Poisson's ration of the half-space.

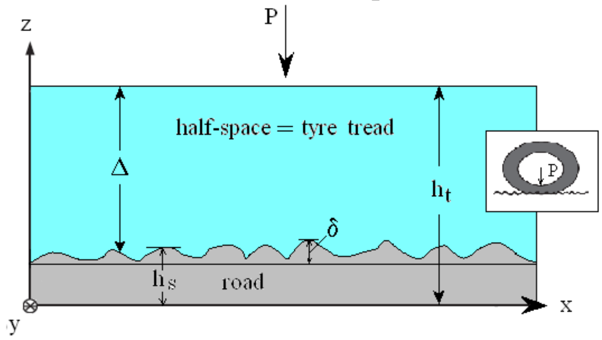


Fig.1. The model of contact.

$$-P = \iint_D p(\xi,\eta)\mathrm{d}\xi\mathrm{d}\eta\,, \qquad (2)$$

where P is the total load applied to the tire.

$$\delta = h_s(x,y) - h_t(x,y) - \Delta\,,\ \ p(x,y) > 0 \ \text{ contact}\,, \qquad (3)$$

with $(x,y)\in D_c$, where D_c is the contact area, h_s is the height of the road surface, h_t is the height of the tire and Δ is the total penetration between the elastic half-space and the road surface.

$$\delta > h_s(x,y) - h_t(x,y) - \Delta\,,\ p(x,y) = 0 \ \text{ separation}\,, \qquad (4)$$

with $(x,y)\in \bar{D}_c$, where $\bar{D}_c$ is the non-contact area. The equation (2) represents the equilibrium condition, and the unilateral contact conditions (2)-(3) describe the non-penetration and the separation outside the contact area, respectively.

One way to solve the problem is to express the pressure p from (1) as a function of δ and other geometrical parameters that characterise the unknown contact domain D_c, and to determine these parameters, δ and D_c from (2)-(4). Finally, the pressure $p(\xi,\eta)$ is determined. For circular and elliptic contact domains, the elastic contact problem can be solved exactly [9, 10].

In this article, the surface of the unknown contact domain D_c is described by the Lamé curve [12]

$$\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=1,\ n>0, \qquad (5)$$

where a and b are the radii of the oval shape.

The parametric representation of (5) is

$$x(\theta)=a\cos^{2/n}\theta,\ y(\theta)=b\sin^{2/n}\theta. \qquad (6)$$

By applying the Sneddon dual integral equations for elastic contact problem [8], [11], the solution of (1)-(4) can be written as

$$p=\left[\frac{3PH^2E}{(1-\nu)}\right]^{1/3}\left(\frac{1}{2\bar{E}(\varphi,r\bar{k})}\right)^{2/3}\left(1-r^2\bar{k}^2\right)^{1/6}, \quad (7)$$

where H is the mean curvature of the contact domain, $\varphi=\arcsin(1-r)$, $\varphi\le\frac{\pi}{2}$, $\bar{k}=\frac{\left(n\tilde{r}^n-2b^n\right)^{1/n}}{a}$, $a=\max(a,b)$ and $F(\varphi,\bar{k}r)$, $\bar{E}(\varphi,\bar{k}r)$ are the elliptic integrals of the first and of the second kind, respectively .

The maximum value of pressure is given by

$$p_{\max}=\left[\frac{3PH^2E}{(1-\nu)}\right]^{1/3}\left(\frac{1}{2\bar{E}(\pi/2,\bar{k})}\right)^{2/3}\left(1-\bar{k}^2\right)^{1/6}. \quad (8)$$

The displacement δ at the surface of the half-space is expressed as

$$\delta=\left[\left(\frac{3}{2}\right)^2\frac{(1-\nu)^2}{4\pi^2E^2}P^2H\right]^{1/3}\frac{\left(1-\bar{k}^2\right)^{1/3}K(\bar{k})}{\bar{E}^{1/3}(\bar{k})}. \quad (9)$$

where $F(\pi/2,\bar{k})=K(\bar{k})$ and $\bar{E}(\pi/2,\bar{k})=\bar{E}(\bar{k})$ are the complete elliptic integrals of the first and of the second kind, respectively

When the tire runs along the rough surface of the road, both longitudinal and bending vibrations are produced. Impulsive forces that develop at the asperities of the road have components both in tangential and normal directions. Assuming that impulsive contact forces dominate over the friction forces, only the normal components of the forces are taking into considerations [15]

$$F_c=\sum_{n=1}^{m}F_n\sin\varphi_n\Pi\left((n\Delta x-vt)/b\right)\delta(x-n\Delta x), \qquad (10)$$

where $n=\text{integer}(L/\Delta x)$, m is the number of simultaneous contacts of length b traveling with the velocity v. The function F_n represents the discrete impulsive point forces that appear during the motion at an angle φ_n with the axis of the road.

3. TIRE/ROAD INTERACTION

Two cases are examined for the tire motion with the velocity v on a single bump road and an irregular road, respectively. The initial reference pressure distribution is used to estimate the deformation of the surface of the half-space, and also the pressure distribution.

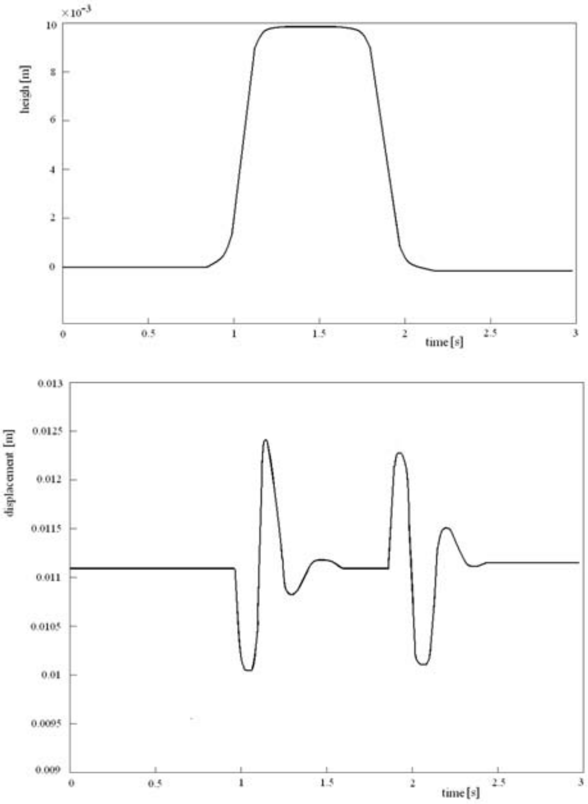
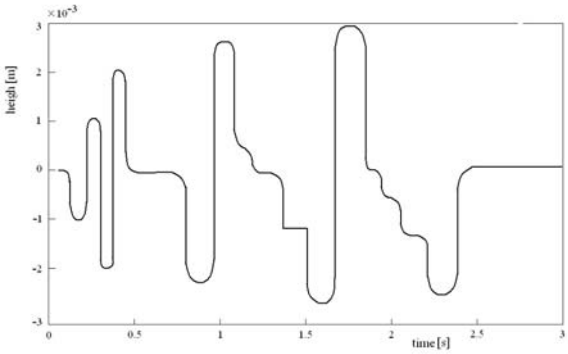


Fig.2. Displacements at the surface of the half-space for the single bump road.

The displacement δ at the surface of the half-space is plotted in Figs.2 and 3 for both cases. The road surface profile represented in figures acts on the tire as exciting signals [14].

Displacements for both cases describe the tire-road deformation and determine the final contact pressure.

The final pressure $p / p_{\max}$ is plotted in Figs.5 and 6 with respect to a reference initial contact pressure $p_0 / p_{0\max}$.



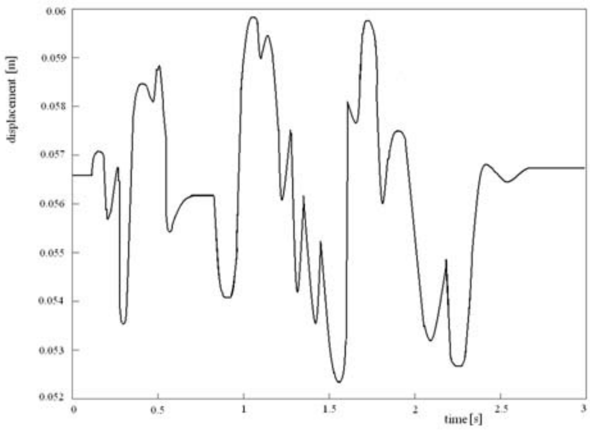


Fig.3. Displacements at the surface of the half-space for the irregular road.

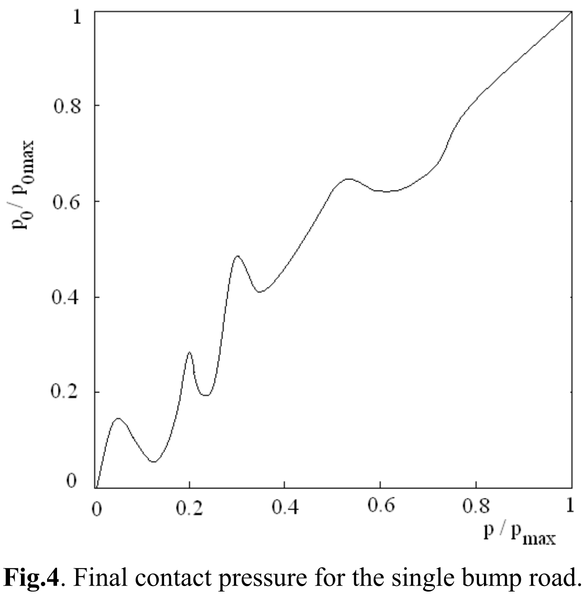


Fig.4. Final contact pressure for the single bump road.

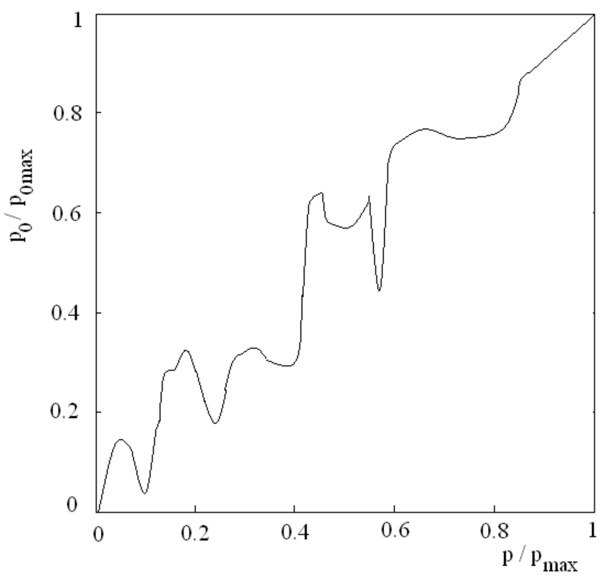


Fig.5. Final contact pressure for the irregular road.

4. CONCLUSION

The pressure distribution on two simulated road surfaces was evaluated in this paper, together with the displacement of the surface of the half-space which defines the tire tread.

The numerical results for road irregularities show the effect of the uncertainty in the road data upon the forces in the vehicle's suspension. This aspect is crucial for the development of the road response simulator which can be designed as a reconfigurable haptic interface that includes real-time dynamic contact simulators [15, 16]. The paper excluded many other topics related to the interaction tire/road as rolling resistance produced by rotation and adherence of the tires to road, the nonsmoothness of friction forces and torques, generation of noise.

Acknowledgement This research was elaborated through the PN-II-PT-PCCA-2011-3.1-0190 Project of the National Authority for Scientific Research (ANCS, UEFISCSU), Romania.

REFERENCES

[1] Jazar, R.N., *Vehicle Dynamics. Theory and applications*, Springer, 2008.

[2] Dubois, G, Cesbron, J., Yin, H.P., Anfosso-Lédée, F. *Numerical evaluation of tyre/road*

contact pressure using a multi-asperity approach, International Journal of Mechanical Sciences, 54, 84–94, 2012.
[3] Cesbron, J., Anfosso-Lédée, F., Duhamel, D., Yin, H.P., Le Houédec, D, *Experiment study of tyre/road contact forces in rolling conditions for noise prediction,* Sound Vibrations, 320, 125–144, 2009.
[4] Gilardi, G., Sharf, I., *Literature survey of contact dynamics modelling,* Mechanism and Machine Theory, 37, 1213–1239, 2002.
[5] Stronge, W.J., *Energy dissipated in planar collision*, Journal of Applied Mechanics, 59(3), 681–682, 1992.
[6] Baldovin, D., Dumitriu, D., Chiroiu, V., Solomon, L., *On the elastic-plastic contact modelling,* Research Trends in Mechanics, vol.2, 9–32, Editura Academiei, 2008.
[7] Dumitriu, D., Chiroiu, V., *On the modelling of nanocontacts*, Revue Roumaine des Sciences techniques, serie de mecanique Appliquee, 52(1), 2008
[8] Dumitriu, D., Chiroiu, V., *On the dual equations in contact elasticity,* Revue Roumaine des Sciences techniques, serie de mecanique Appliquee, 2006.
[9] Solomon, L., *Une solution approchée du problème du poinçon rigide à base plane bornée convexe non elliptique*, Compt. Rend. Acad. Sc. Paris, 258, 64–66, 1964.
[10] Solomon, L., Zamfirescu, I., *Some approximative formulae and solutions in the contact problem for punches with a plane bounded non-elliptical basis.* Proc. of the Int. Symp. Tbilisi, sept. 17-23 (eds. Muskhelishvili N.I., Sedov L.I., Mikhailov G.K.), 400–409, 1963.
[11] Sneddon, I.N., *Dual equations in elasticity, in Applications of the theory of functions in continuum mechanics*, vol.1 Mechanics of Solids (eds. N.I. Muskhelishvili, L.I. Sedov, G.K. Mikhailov), Nauka Publishing House, 76–94, 1965.
[12] L.Munteanu, V.Chiroiu, *Shell Buckling of Carbon Nanotubes Using Nanoindentation*, CMES: Computer Modeling in Engineering & Sciences, 48(1), 27–41, 2009.
[13] Akay, A., *Acoustics of friction*, Journal of Acoustical Society of America, 111(4), 1525–1548, 2002.
[14] Addi, K., Goeleven, D., Rodić, *A., Mathematical analysis and numerical simulation of a nonsmooth road-vehicle spatial model*, ZAMM – Z. Angew. Math. Mech. 86 (3), 185–209, 2006.
[15] Brisan**,** **C.**, Csiszar, A., *Computation and Analysis of the Workspace of a Reconfigurable Parallel Robotic System*, Mechanism and Machine Theory Journal, 46, 1647–1668, 2011.
[16] Vasiu, R.V., Brisan**,** C**.**, *Aspects regarding Modular Road Design in Virtual Reality*, ASME 2011 World Conference on Innovative Virtual Reality, 27-29 June, 2011, Milan, Italy, 2011.

Modelul 3D de contact dinamic roată/drum

Rezumat: Lucrarea propune un model 3D de contact dinamic pentru interacţiunea roată/drum cu ajutorul ecuaţiilor integrale duale de tip Sneddon. Banda de rulare a cauciucului este modelată ca un semi-spaţiu incompresibil şi elastic, iar suprafaţa drumului, ca o suprafaţă rigidă cu asperităţi.

Munteanu Ligia, PhD., Senior Researcher CS1, Institute of Solid Mechanics, Position, Institution, Dept. of Deformable Media and Ultrasonics, E-mail: ligia_munteanu@hotmail.com, Office Phone: 021 3153810, Home Phone: 0214212550.

Brişan Cornel, PhD., Professor, Technical University of Cluj-Napoca, Dept. of Mechatronics and System Dynamics, E-mail: Cornel.Brisan@mmfm.utcluj.ro, Office Phone: 0264401680, Home Phone: 0264411320.

Chiroiu Veturia, PhD**.**, Senior Researcher CS1, Institute of Solid Mechanics, Dept. of Deformable Media and Ultrasonics, E-mail: veturiachiroiu@yahoo.com, Office Phone: 021 3153810, Home Phone: 0216885181.

Donescu Ştefania, PhD., Associate Professor, Technical University of Civil Engineering, Bucharest, Dept. of Mathematics and Computer Science, E-mail: stefania.donescu@yahoo.com, Office Phone: 0212407330, Home Phone: 0212105836 .