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STATIC ANALYSIS AND STIFFNESS OF A 2-DOF PARALLEL DEVICE FOR ORIENTATION

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Abstract: *In the paper is studied a simple orientation mechanism with two degrees of freedom (DOF). The kinematics, workspace and this singularity of the mechanism are presented. In order to determine the generalized static forces from the legs, beside oriented panel weight, the static pressure of the wind considered perpendicular to the plane of the mobile platform has been taken into consideration. Based on the kinematic model, two performance indices have been evaluated and graphically represented regarding the control and stiffness of the mechanism: global conditioning index and global stiffness index.*
Key words: *Parallel device, Orientation, Kinematics, Workspace, Stiffness*

1. INTRODUCTION

The design of the orientation mechanisms was boosted by necessity to their application as support for radio antennas, satellite TV antennas, telescopes, cameras, solar panels etc. A lot of mechanisms for this task have serial architecture. Lately, the mechanisms with parallel architecture started to impose, taking into account their characteristics: high stiffness and kinematic accuracy, advantageous ratio between payload and its own weight, possibility of installing of the actuators on the base. The drawbacks of parallel mechanisms are their more limited workspace and in many case the presence of the overmobility singularities.

In this paper the assessment of generalized static actuating forces and stiffness of a 2-DOF parallel manipulator is studied without taking into account the elements elasticity and actuators compliance. The values of generalized forces are necessary for sizing telescopic legs and command linear actuators.

Stiffness is defined as the ability to retrieve loads without excessive deformations [1]. Stiffness is a very important performance indicator of a parallel kinematic structure.

There are three main methods to derive the stiffness model of parallel manipulators. These methods are based on the calculation of the Jacobian matrix, the Finite Element Analysis (FEA) and the Matrix Structural Analysis (MSA) [2]. The stiffness analysis of a parallel mechanism has been studied by many researches [9]-[15]. The study of [Arai] et al.[3] did not include elastic deformations. Gosselin [4] studied the stiffness considering only the stiffness of each actuator. Complex studies about stiffness can be found in papers [5], [6], [7].

The paper is organized as follows: Section 2 deals with kinematics of the parallel mechanism, Section 3 presents its workspace and singularities. The evaluation of the static generalized forces are exposed in the Section 4. In Section 5 is studied the stiffness of parallel mechanism. The conclusions are described in section 6.

2. KINEMATICS

In the figure 1 the kinematic scheme of a symmetrical parallel mechanism is shown [8]. It consists in a fixed base OB_1B_2 and a mobile platform OA_1A_2 connected by a main universal joint in O and two telescopic legs B_1A_1, B_2A_2 ended with spherical joints.

The rotation matrix, corresponding to the β and α angles, has the form in this case:

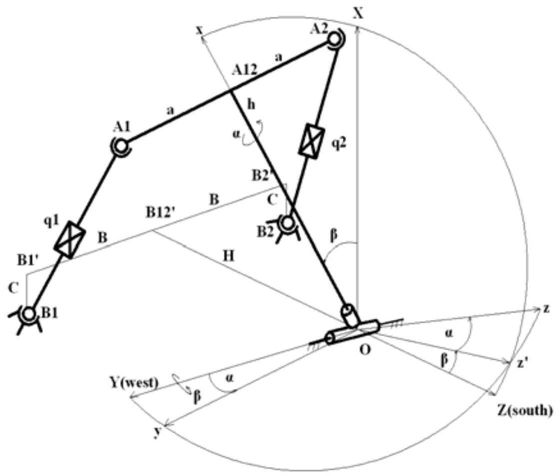

Fig. 1. The kinematic scheme of parallel robot

$$\left[R_{y(\beta),x(\alpha)}\right]= \\ =\begin{bmatrix} \cos(\beta) & \sin(\alpha)\sin(\beta) & \cos(\alpha)\sin(\beta) \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ -\sin(\beta) & \sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta) \end{bmatrix} \quad (1)$$

The rotation matrix, corresponding to the φ and θ angles, has the following form:

$$\left[R_{x(\varphi),y(\theta)}\right]= \\ =\begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ \sin(\varphi)\sin(\theta) & \cos(\varphi) & -\sin(\varphi)\cos(\theta) \\ -\cos(\varphi)\sin(\theta) & \sin(\varphi) & \cos(\varphi)\cos(\theta) \end{bmatrix} \quad (2)$$

The following conditions should be accomplished, in order to the Oz axis is directed to the same point in space in both cases:

$$\begin{bmatrix} \cos(\alpha)\sin(\beta) \\ -\sin(\alpha) \\ \cos(\alpha)\cos(\beta) \end{bmatrix} = \begin{bmatrix} \sin(\theta) \\ -\sin(\varphi)\cos(\theta) \\ \cos(\varphi)\cos(\theta) \end{bmatrix} \quad (3)$$

As $-90^{o} \le \varphi \le 90^{o}$ and $0^{o} \le \theta \le 90^{o}$, results:

$$\begin{cases} \beta = a\tan(\cos(\varphi)\tan(\theta)) \\ \alpha = a\tan(\sin(\varphi)\cos(\theta)) \end{cases}; \begin{cases} \varphi = a\tan(\dfrac{\tan(\alpha)}{\cos(\beta)}) \\ \theta = a\sin(\cos(\alpha)\sin(\beta)) \end{cases} \quad (4)$$

The relationships between the position vectors of guided points A_i (i=1,2) in two systems of reference are:

$$\overline{P_i} = [R]\cdot\overline{p_i}; \quad i=1,2 \quad (5)$$

where

$$\overline{P_i} = \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix}; \ \overline{p_i} = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}; \ i=1,2; \ x_1 = x_2 = h;$$

$$y_1 = -y_2 = a; \ z_1 = z_2 = 0 \quad (6)$$

The position of the fixed points B_i (i=1,2) is known:

$$\overline{P_{Bi}} = \begin{bmatrix} X_{BI} \\ Y_{Bi} \\ Z_{Bi} \end{bmatrix}; \ i=1,2; \ X_{B1} = X_{B2} = -C;$$

$$Y_{B1} = -Y_{B2} = B; Z_{B1} = Z_{B2} = -H \quad (7)$$

Using the above relationships, the generalized coordinates q_1 and q_2 of the mechanism, represented by the lengths of legs B_iA_i (i=1,2), can be calculated:

$$q_i = \sqrt{(\overline{P_i} - \overline{P_{Bi}})^2} = \sqrt{(X_i - X_{Bi})^2 + (Y_i - Y_{Bi})^2 + (Z_i - Z_{Bi})^2}; \\ i=1,2 \quad (8)$$

The kinematic model is obtained by deriving the square of equation (8) with respect to the time:

$$\dot{\overline{X}} = [J]\dot{\overline{q}} \quad (9)$$

where [J] is the Jacobi matrix, $\dot{\overline{X}}$ is vector of the mobile platform angular velocities and $\dot{\overline{q}}$ is vector of active joint velocities:

$$[J] = \begin{bmatrix} \dfrac{\left(\overline{P_1} - \overline{P_{B1}}\right)}{q_1}\cdot\dfrac{\partial\overline{P_1}}{\partial\alpha} & \dfrac{\left(\overline{P_1} - \overline{P_{B1}}\right)}{q_1}\cdot\dfrac{\partial\overline{P_1}}{\partial\beta} \\ \dfrac{\left(\overline{P_2} - \overline{P_{B1}}\right)}{q_2}\cdot\dfrac{\partial\overline{P_2}}{\partial\alpha} & \dfrac{\left(\overline{P_2} - \overline{P_{B2}}\right)}{q2}\cdot\dfrac{\partial\overline{P_2}}{\partial\beta} \end{bmatrix};$$

$$\dot{\overline{X}} = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix}; \dot{\overline{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad (10)$$

The scalar components and the module of the mobile platform angular velocity can to be obtained:

$$\omega_x = \dot{\alpha}\,;\; \omega_y = \dot{\beta}\cos(\alpha)\,;$$

$$\omega_z = -\dot{\beta}\sin(\alpha)\,;\; \omega = \sqrt{\dot{\alpha}^2 + \dot{\beta}^2} \qquad (11)$$

3. WORKSPACE AND SINGULARITIES

We consider that in ideal case, the shape of workspace is a rectangle located in the plane defined by φ and θ coordinates with the sides π and π/2. In real case, the workspace area and her shape are affected by the constraints due to:

- allowed spherical joints angles
- actuated joints stroke
- transmission angles
- presence of singularities

The upper spherical joints restraints could be avoided by replacing them with universal joints.

For the actuated joints the conditions are imposed:

$$q_{\min} \le q_i \le q_{\max}\,;\; i = 1,2 \qquad (12)$$

For transmission angles:

$$\psi_i = a\sin\frac{\left|\left(\overline{P}_i - \overline{P}_{Bi}\right)\cdot\left(\overline{P}_1 \times \overline{P}_2\right)\right|}{\left|\overline{P}_i - \overline{P}_{Bi}\right|\cdot\left|\overline{P}_1 \times \overline{P}_2\right|}\,;\; i = 1,2 \qquad (13)$$

the condition is imposed:

$$\psi_i \ge 20^o \qquad (14)$$

The curves of singularity result from the condition:

$$Det[A] = 0 \qquad (15)$$

In this case the reachable workspace can be determined imposing the condition:

$$Det[A] < 0 \qquad (16)$$

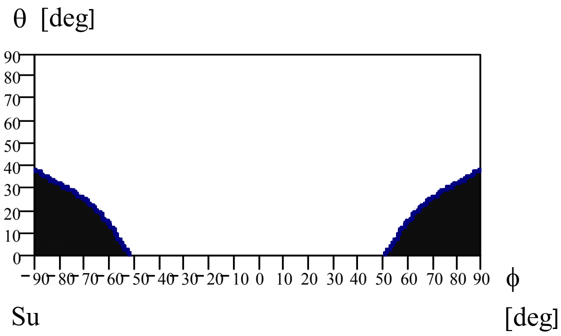

Fig.2. The reachable workspace

For the input data: a=0.25m; h=0.3m; H=0.8m; B=0,05m; C=0.9m; q_{min}=1.0m; q_{max}=1.6m, the constraints (12) and (14) yields to the workspace shown in figure 2. Area of reachable workspace (white zone) is A = 4.20 rad^2; (ideal area=4.93 rad^2).

Checking the condition (16), it results that in ideal workspace we do not have singularities.

4. STATIC GENERALIZED FORCES

The actuating generalized forces should overcome the weight of the panel or antenna mounted on the mobile platform (Gp) and the resultant wind pressure (Pv).The two forces are considered applied in the panel center and the force (Pv) is always perpendicular to the panel (Fig.3.)

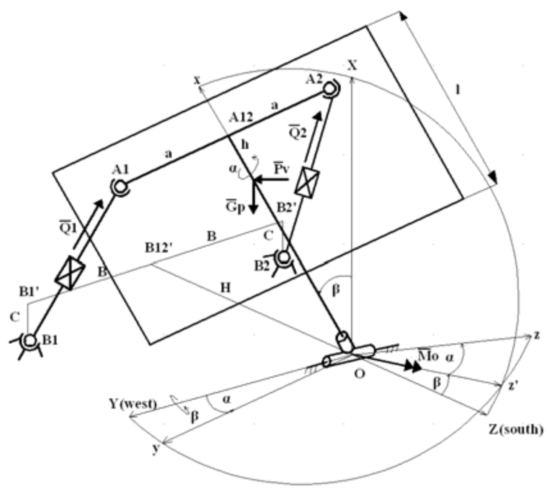

Fig.3. Forces scheme

For evaluation of the generalizaed forces Q1 and Q2, the virtual power principle is applied.

$$\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = -[J]^T \begin{bmatrix} M_\alpha \\ M_\beta \end{bmatrix} \quad (17)$$

where,

$$M_\alpha = M_x = 0$$

$$M_\beta = M_y = (P_v \cos\alpha + G_p \sin\beta)\cdot\frac{l}{2}$$

l is the panel height

For P_v=100daN, G_p=19,6daN, l=1m the izo-generalized forces curves of Q1 and Q2 in ideal workspace are obtained.

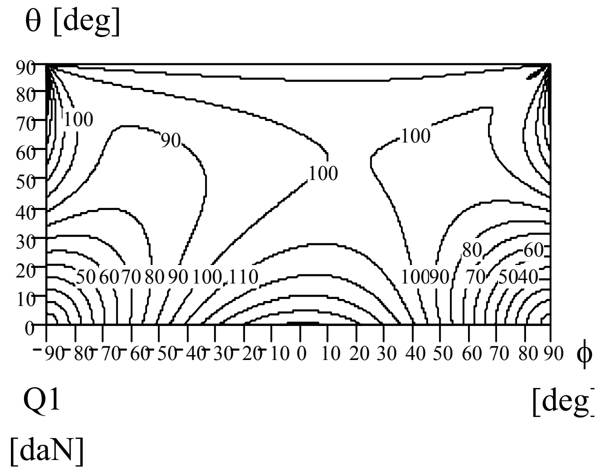

Fig.4. Izo-generalized forces curves of Q1

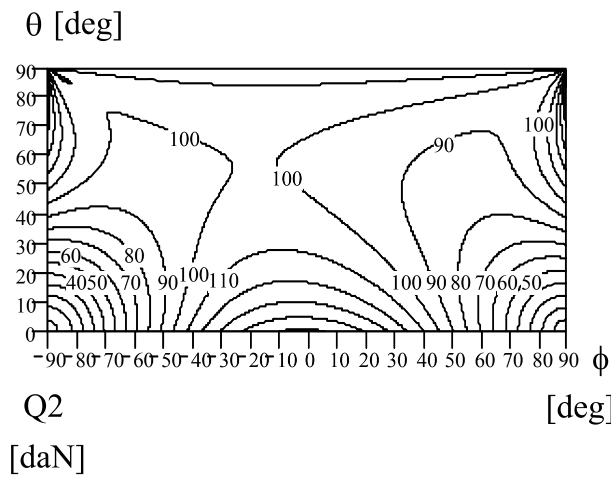

Fig.5. Izo-generalized forces curves of Q2

The maximum value of the generalized forces Qmax=151,73 daN is recorded when the panel is vertical and facing south.

A measure of the mechanism performance is the torque capacity of the universal joint:

$$M_0 = \\ = \left[\overline{OA_1} \times Q_1 \overline{\ell}_1 + \overline{OA_2} \times Q_2 \overline{\ell}_2 + \overline{OC} \times \left(\overline{P_v} + \overline{G_p}\right)\right] \cdot \overline{k_{z'}} \quad (18)$$

where:

$\overline{e_1}$, $\overline{e_2}$ are the unit vectors ; C is the panel center; $\overline{k_{z'}}$ the unit vector of the Oz' axis perpendicular on the main universal joint axes.

The contour plot of Mo is shown in figure 6.

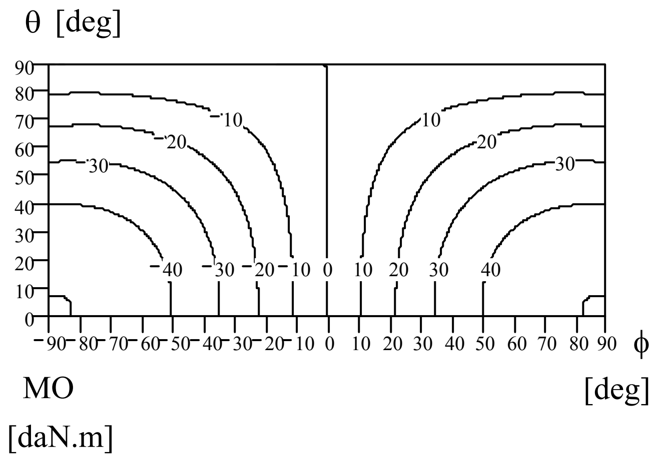

Fig.6. Moment from main universal joint

Maximum, respectively minimum values

$$M_{o_{\max,\min}} = \pm 50{,}121 daN\cdot m$$

are registered in unused areas of the workspace.(Θ=0°, φ=±90 °)

5. CONDITIONAL NUMBER AND STIFFNESS ASSESSMENT

The stiffness of the mechanism has a direct influence on the accuracy of positioning and orientation. The studied case does not take into account the elasticity and actuators compliance of the active joints.

Control precision of the parallel device depends on the conditional Jacobi matrix, defined by:

$$C = norm[J]\cdot norm[J]^{-1} \quad (19)$$

Figure 7 illustrates the map of level curves of conditional number. Performance is better when the condition number is lower.

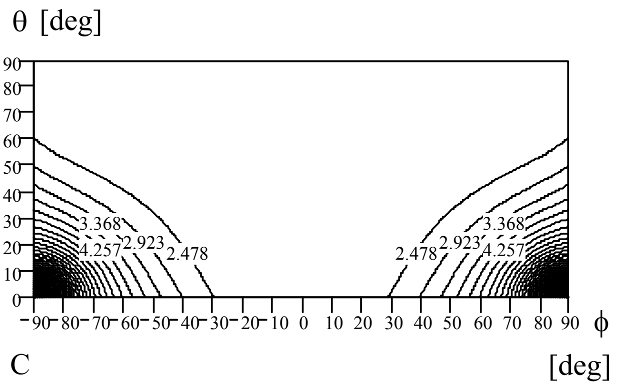

Fig.7 Conditional number

Starting from the conditional number, Gosselin and Angeles [13] defined a performance index called the Global Conditioning Index:

$$\eta_C = \frac{1}{C} \tag{20}$$

given in figure 8.

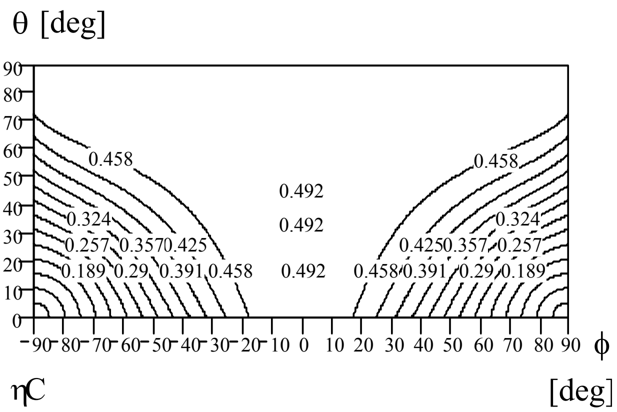

Fig.8 Global Conditioning Index:

Note that in the reachable workspace the device is quite well conditioned.

With the stiffness matrix

$$[K] = k[J]^T \cdot [J];\ k = 1 \tag{21}$$

the stiffness index can be calculated:

$$K\dot{I} = norm[K]^{-1} \cdot norm[K] \tag{22}$$

and the Global Stiffness Index [15]:

$$\eta = \frac{1}{KI} \tag{23}$$

whose maps are given in figures 9 and 10.

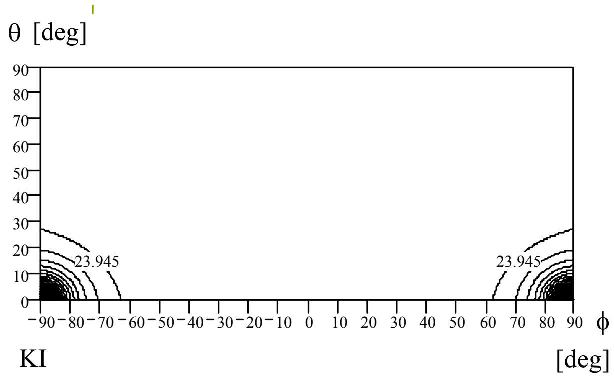

Fig.9 Stiffness index

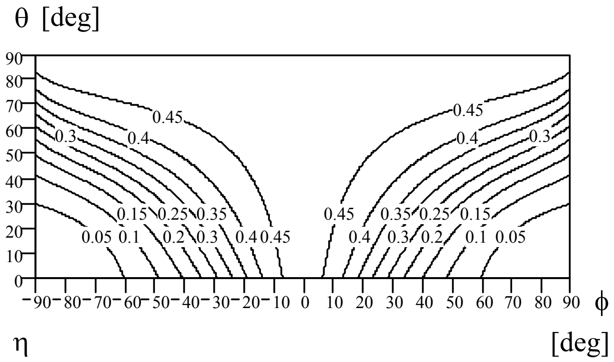

Fig.10 Global Stiffness Index

Figure 10 shows that η<0,05 corresponds to the unused areas of the workspace where the transmission angles are less than 20º.

6. CONCLUSION

In this paper, a simple orientation parallel structure with two degrees of freedom was analyzed, its kinematics, represented workspace, generalized driving forces have been evaluated under static conditions and the global stiffness index and global conditioning index were calculated.

The numerical results indicate that, in reachable workspace, the mechanism is well conditioned, especially in the central area which is also characterized by high stiffness.

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ANALIZA STATICA SI STUDIUL RIGIDITATII A UNUI MECANISM PARALEL CU 2 GRADE DE MOBILITATE PENTRU ORIENTARE

In lucrare este studiat un mecanism simplu de orientare cu doua grade de libertate. Sunt analizate cinematica, spatiul de lucru si singularitatile. Fortele generalizate motoare au fost determinate in conditii statice luandu-se in considerare, pe langa greutatea panoului care este orientat si presiunea vantului. Pe baza modelului cinematic au fost evaluati si reprezentati doi indici de performanta privind controlul si rigiditatea mecanismulu: indicele global de conditionare si indicele gobal de rigiditate.

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