TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 55, Issue III, 2012

THE ANALYSIS OF WAVES PROPAGATION IN STRUCTURES UTILIZES IN DESIGN AND CONSTRUCTION OF ULTRASOUND GENERATING TRANSDUCERS

Grigore Liviu ODOBESCU

Abstract: *The high energy ultrasound it is generated by piezoelectric elements which are excited from electric signal obtained from an electronic generator. The transmission of this high energy it is obtained by transmission element or concentrator element in function of what we want to do with this energy especial in no conventional technology. The efficiency work is a very important aspect because the energy utilizes has high values. In this paper it is presented a method to calculate the elements components necessary to generate and transmission the high ultrasound energy: piezoelectric transducers and transmission / concentrator elements. Are presented the experimental results obtained with the theory presented. Original contribution consists by method used and suggestive graphics for appreciation of variations parameters.*
Key words: *piezoelectric transducer, ultrasound power, dimension design.*

1. INTRODUCTION

In practice [1] we have two important cases. The case of the transmission/concentrator elements, when the ultrasound waves pass through one medium (usual aluminum or titan) and the case of ultrasound piezoelectric transducer, when the ultrasound waves pass through three mediums (usual steel - piezoelectric element- aluminum alloy).

2. THE PRINCIPLE OF OPERATION

In case of a piezoelectric transducer [2] formed by 2 pastilles which are catch between 2 metallic blocs (director and reflector) with help of central screw. The reflector is made [3] from steel, aluminum, bronze, naval brass. The director, fig.1, is made from titanium, aluminum or magnesium composite. It is noted:

$$K_i = \frac{2\pi}{\lambda_i} = \frac{2\pi f}{v_i} = \frac{\omega}{v_i} ... K_C = \frac{2\pi}{\lambda_C} = \frac{2\pi f}{v_C} = \frac{\omega}{v_C} \quad (1)$$

where K_i – referrer to the end piece and K_C to the ceramic material; - λ_i and λ_C represents the waves long of ultrasound waves through end material and ceramic material; - v_i and v_C represents propagation speed through end material and ceramic material and - f represents frequencies of waves and $\omega=2\pi f$ its pulsation

The speed v depends by: ρ- the material density; Y- the elasticity module and σ-the Poisson coefficient.

It is considered [4] the piezoelectric transducer presented in fig.2. In fig.2 it is represented the evolution of vibration amplitude y(x) in function of distance x. We have: $y_m (l_C + l_m) = 0$ and $y_P (-l_P) = Y_{PM}$

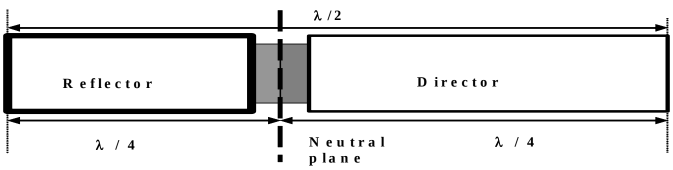

Figure 1-The piezoelectric transducer.

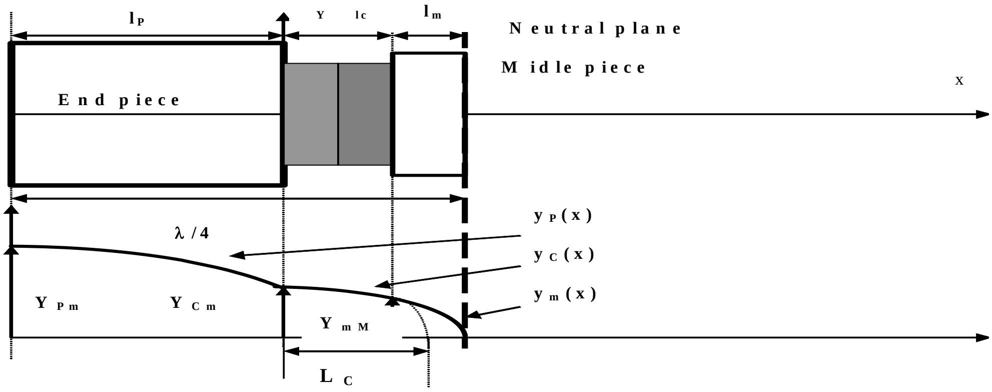

Figure 2 - Semi-transducer with three propagation mediums.

It noted the wave factors: $K_i = \frac{2\pi}{\lambda_i} = \frac{\omega}{v_i}$ where i=p, c, m referrer to end, ceramic and middle pieces

We note amplitude of vibration into end, ceramic, respectively medium pieces:

$$y_P(x)=Y_{PM}\cos[K_P(x+l_P)];\; y_P(-l_P)=Y_{PM}$$

$$y_C(x)=Y_{CM}\sin[K_C(x-L_C)]\;;\; y_C(L_C)=0 \qquad (2)$$

$$y_m(x)=Y_{mM}\sin[K_m(x-l_C-l_m)];\; y_m(l_C+l_m)=0$$

where L_C represents the distance on x axe where the material points will have null displacement if we have only ceramic material. So, we have $y_C(L_C) = 0$

The conditions [5] at separation limit between of these three propagation mediums will be:

a) for displacements:

1) $y_P(0) = y_C(0)$ and 2) $y_C(l_C) = y_m(l_C)$

b) for forces:

3) $F_P(0) = F_C(0)$ and 4) $F_C(l_C) = F_m(l_C)$

We have the relation: $F(x)=Y\cdot A\cdot \frac{dy(x)}{dx}$. So:

$$F_P(x)=Y_P\cdot A\cdot \frac{dy_P(x)}{dx} = -Y_P\cdot A_P\cdot Y_{PM}\cdot K_P\cdot\sin[K_P(x+l_P)] \text{ - at end piece}$$

$$F_C(x)=Y_C\cdot A_C\cdot \frac{dy_C(x)}{dx} = Y_C\cdot A_C\cdot Y_{CM}\cdot K_C\cdot\cos[K_C(x-L_C)] \text{ - at ceramic piece} \qquad (3)$$

$$F_m(x)=Y_m\cdot A_m\cdot \frac{dy_m(x)}{dx} = Y_m\cdot A_m\cdot Y_{mM}\cdot K_m\cdot\cos[K_m(x-l_C-l_m)] \text{ - at middle piece}$$

It put the limit conditions for displacements - a) and for forces b)- and it obtained:

1) $y_P(0)=y_C(0)$ sau $Y_{PM}\cdot\cos K_P\cdot l_P=-Y_{CM}\cdot\sin K_C\cdot l_C \Rightarrow \frac{Y_{PM}}{Y_{CM}} = -\frac{\sin K_C\cdot l_C}{\cos K_P\cdot l_P}$

2) $y_C(l_C)=y_m(l_C)$ sau $Y_{CM}\cdot\sin[K_C(l_C-L_C)]=Y_{mM}\cdot\sin[K_m(-l_m)]$ (4)

$$\Rightarrow \frac{Y_{CM}}{Y_{mM}} = -\frac{\sin K_m\cdot l_m}{\sin[K_C\cdot(l_C-L_C)]}$$

3) $F_P(0)=F_C(0)$.

It obtained:

$$-Y_P\cdot A_P\cdot Y_{PM}\cdot K_P\cdot\sin(K_P l_P)= Y_C\cdot A_C\cdot Y_{CM}\cdot K_C\cdot\cos(-K_C L_C) \qquad (5)$$

$$\Rightarrow -\frac{Y_{PM}\cdot\sin K_P l_P}{Y_{CM}\cdot\cos K_C L_C} = \frac{Y_C\cdot A_C\cdot K_C}{Y_P\cdot A_P\cdot K_P}$$

Replacing the ratio $\frac{Y_{PM}}{Y_{CM}}$ with the value find it at 2) condition, relation (4):

$$-\left(-\frac{\sin K_C l_C}{\cos K_P l_P}\right)\cdot\frac{\sin K_P l_P}{\cos K_C L_C} = \frac{Y_C\cdot A_C\cdot K_C}{Y_P\cdot A_P\cdot K_P} \qquad (6)$$

Taken $l_C \approx L_C$ it obtained:

$$tgK_P l_P\cdot tgK_C L_C = \frac{Y_C\cdot A_C\cdot K_C}{Y_P\cdot A_P\cdot K_P} = \frac{\rho_C\cdot v_C\cdot A_C}{\rho_P\cdot v_P\cdot A_P} = \frac{Z_C\cdot A_C}{Z_P\cdot A_P} = R_{CP} \qquad (7)$$

We have: $v=\sqrt{\frac{Y}{\rho}}$ (bars); $K_i=\frac{\omega}{v_i}=\frac{2\pi}{\lambda_i}$ ($v_i=f\cdot\lambda_i$);

where $Z_C=\rho_C \cdot v_C$ and $Z_P=\rho_P \cdot v_P$ represents the acoustic impedances for ceramics and end pieces. It puts the 3) conditions for forces: $F_C(l_C)=F_m(l_C)$. It obtained:

$$Y_C \cdot A_C \cdot Y_{CM} \cdot K_C \cdot \cos[K_C(l_C - L_C)] = Y_m \cdot A_m \cdot Y_{mM} \cdot K_m \cdot \cos[K_m(-l_m)] \tag{8}$$

$$\frac{Y_{CM}}{Y_{mM}} \cdot \frac{\cos[K_C(l_C - L_C)]}{\cos K_m l_m} = \frac{Y_m \cdot A_m \cdot K_m}{Y_C \cdot A_C \cdot K_C}$$

Replacing $\frac{Y_{CM}}{Y_{mM}}$ with the value finding at 3) condition:

$$\frac{(-)\cdot \sin K_m l_m}{\sin[K_C(l_C - L_C)]} \cdot \frac{\cos[K_C(l_C - L_C)]}{\cos K_m l_m} = \frac{Y_m A_m K_m}{Y_C A_C K_C}$$

$$- \text{tg}\, K_m l_m \cdot \text{ctg}\,[K_C(l_C - L_C)] = \frac{Y_m \cdot A_m \cdot K_m}{Y_C \cdot A_C \cdot K_C} = R_{mC} \tag{9}$$

In conclusion we have obtained:

$$\text{tg}K_C L_C \cdot \text{tg}K_P l_P = R_{CP} \quad \text{and} \quad - \text{tg}K_m l_m \cdot \text{ctg}[K_C(l_C - L_C)] = R_{mC} \tag{10}$$

where:

$$R_{CP}=\frac{Y_C K_C S_C}{Y_P K_P S_P} = \frac{\rho_C v_C S_C}{\rho_P v_P S_P} = \frac{Z_C S_C}{Z_P S_P}; \quad R_{mC} = \frac{Y_m K_m S_m}{Y_C K_C S_C} = \frac{\rho_m v_m S_m}{\rho_C v_C S_C} = \frac{Z_m S_m}{Z_C S_C} \tag{11}$$

It notes: $K_C L_C=\alpha_L$; $K_P l_P=\alpha_P$; $K_m l_m=\alpha_m$; $K_C l_C=\alpha_C$. The equations become:

$$\text{tg}\, \alpha_L \cdot \text{tg}\alpha_P=R_{CP} \quad \text{and} \quad \text{tg}\alpha_m \cdot \text{ctg}(\alpha_C-\alpha_L)=- R_{mC} \tag{12}$$

$$\text{tg}\alpha_L \cdot \text{tg}\alpha_P=R_{CP} \Rightarrow \text{tg}\, \alpha_L=\frac{R_{CP}}{tg\alpha_P};$$

$$g\alpha_m \cdot \frac{1+tg\alpha_C \cdot tg\alpha_L}{tg\alpha_C - tg\alpha_L} = -R_{mC} \tag{13}$$

Replacing tg α_L in the second equation, it obtained:

$$tg\alpha_m \cdot \frac{1 + tg\alpha_C \cdot \frac{R_{CP}}{tg\alpha_P}}{tg\alpha_C - \frac{R_{CP}}{tg\alpha_P}} = -R_{mC} \tag{14}$$

Or relation:

$$\text{tg}\, \alpha_m \cdot \text{tg}\, \alpha_P + R_{CP} \cdot \text{tg}\, \alpha_m \cdot \text{tg}\, \alpha_C + R_{mC} \cdot \text{tg}\, \alpha_C \cdot \text{tg}\, \alpha_P = R_{mC} \cdot R_{CP} \tag{15}$$

From R_{mC} si R_{CP} expressions it obtained:

$$R_{mC} \cdot R_{CP} = \frac{\rho_m \cdot v_m \cdot S_m}{\rho_C \cdot v_C \cdot S_C} \cdot \frac{\rho_C \cdot v_C \cdot S_C}{\rho_P \cdot v_P \cdot S_P} = \frac{\rho_m \cdot v_m \cdot S_m}{\rho_P \cdot v_P \cdot S_P} = R_{mP} \tag{16}$$

This relation it is noted with R_{mP} and shows the transition from metal (m) to end piece (p)–fig. 4, and permits to calculate the dimensions of semi-transducer having three propagation mediums. It starts from neutral plane which it is determined (l_m it is determined). The determination of l_m , l_C or l_P dimensions when other sizes are know, may be done from one relations given down, which are obtained from the same (15) equation.

$$l_m = \frac{1}{K_m} \cdot arctgR_{mC} \cdot \frac{R_{CP} - tg\alpha_C \cdot tg\alpha_P}{tg\alpha_P + R_{Cp} \cdot tg\alpha_C}$$

$$l_P = \frac{1}{K_P} \cdot arctgR_{CP} \cdot \frac{R_{mC} - tg\alpha_m \cdot tg\alpha_C}{tg\alpha_m + R_{mC} \cdot tg\alpha_C} \tag{17}$$

$$l_C = \frac{1}{K_C} \cdot arctg \cdot \frac{R_{mP} - tg\alpha_m \cdot tg\alpha_P}{R_{CP} \cdot tg\alpha_m + R_{mC} \cdot tg\alpha_P}$$

Where: l_C represents the total thickness of whole package ceramics from semi-transducer considered.

Because ceramics isn’t placed into nodal plane (where we have the null displacements y=0 and maximum mechanical tensions T=max) for to obtain the same piezoelectric effect it shall to advance the thickness of piezoelectric elements package. This is disadvantageous by economic point of view. To calculate the transversal dimensions necessary for a given level power we take in account two parameters: the stress and dynamic deformations.

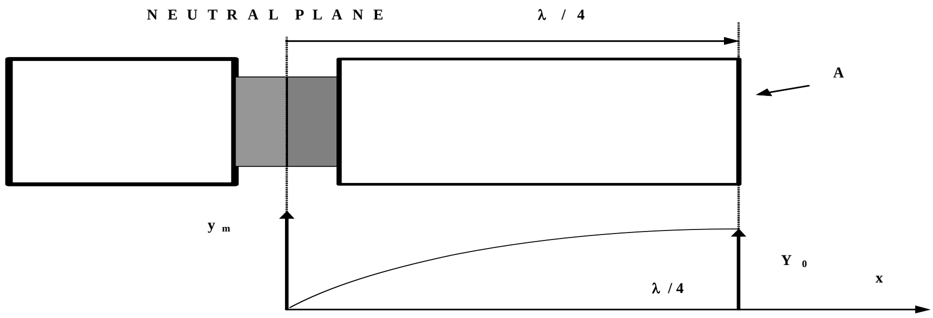

Figure 3 - Transversal dimensioning.

The stress (T), respectively dynamic deformations ($S=\Delta l/l$) into a resonant mechanical element are in function of end element displacement. These displacements are determined by acoustic charge. Let find out a relation between displacement of end element and maximum tension which take place at displacement node from the same element. Let's note: - Y_0 - the maximum amplitude of displacement; - y_m- the displacement amplitude at x distance from displacement node.

The displacement node represents the central nodal section of transducer where the displacements are nulls and mechanical tensions are maximums. It can writes, fig. 3:

$$y_m(x)=Y_0 \sin \frac{2\pi}{\lambda}\cdot x . \quad (18)$$

The value of relative deformation S for any x value:

$$S(x)=\lim_{\Delta x\to 0}\frac{y_{m}(x+\Delta x)-y_m(x)}{\Delta x}=y'_m(x) \quad (19)$$

Results:

$$S(x)=y_m'(x)=Y_0\frac{2\pi}{\lambda}\cdot\cos\frac{2\pi}{\lambda}\cdot x \quad (20)$$

The maximum relative deformation S(x) in displacement node (x=0):

$$S_{max}(x=0)=Y_0\cdot\frac{2\pi}{\lambda}=Y_0\cdot\frac{2\pi}{v}\cdot f=\frac{\omega\cdot Y_0}{v} \quad (21)$$

Where: v – represents the speed sound through material; $\omega=2\pi f$ – vibrations pulsation and $\lambda=v/f$ – the length wave of vibrations.

The maximum amplitude of dynamic vibrations T_m (the maximum amplitude of dynamic pressure P_m) will give by Hooke law: $T_m=Y\cdot S_m=\frac{F}{A}$, where: Y - represents the elasticity module of material; F - represents the force; A - represents the area.

Also, we have: $v=\sqrt{\frac{Y}{\rho}}$ in the case of bars (having the same diameter from it along), where: v- represents the sound speed through material and ρ - represents the material density.

The maximum amplitude of speed (v_m) at end of element: $y_m(x)=Y_0\sin\omega x \Rightarrow v(x)=y_m'(x)=Y_0\cdot\omega\cdot\cos\omega x$

Results:

$$v_m=\omega\cdot Y_0=v\cdot S_{max}=v\cdot\frac{T_m}{Y}=v\cdot\frac{T_m}{v^2\cdot\rho}=\frac{T_m}{\rho\cdot v}$$

or: $T_m = P_m = \rho\cdot v\cdot\omega\cdot Y_0$

So, the mechanical tension (T_m) or acoustic pressure (P_m) in displacement node it is given from product from maximum speed of particles at his end and specific acoustic impedance ($Z=\rho\cdot v$): $T_m=P_m=\rho\cdot v\cdot v_m=Z\cdot v_m$.

The acoustic intensity I, represents the acoustic energy flux which pass through surface unit perpendicular on propagation direction of waves. It is given by relation: $I=\frac{1}{2}\cdot v_m\cdot P_m$ where: v_m – represents the speed amplitude and P_m- represents the pressure amplitude.

To overtaking of I_{max} may destroy the transducer via: to overtaking of T_m [daN/cm^2] or to overtaking of temperature T [^{0}C]. We have the relations:

$$v_m = \frac{T_m}{\rho \cdot v} = \frac{P_m}{\rho \cdot v} \quad ; \quad Z = \rho \cdot v \qquad (22)$$

Replacing, results:

$$I = \frac{1}{2} \cdot \frac{P_m}{\rho \cdot v} \cdot P_m = \frac{1}{2} \cdot \frac{P_m^2}{\rho \cdot v} \quad \text{(by } P_m\text{)};$$

$$I = \frac{1}{2} \cdot v_m \cdot v_m \cdot \rho \cdot v = \frac{1}{2} \cdot \rho \cdot v \cdot v_m^2 \quad \text{(by } v_m\text{)} \qquad (23)$$

where: - $Z_1=\rho_1 v_1$- represents the transducer impedance and - $Z_2=\rho_2 v_2$- represents the medium impedance where it is transmit the energy flux. At separation limit transducer/medium these two acoustic intensities are equals: $I_1=I_2=I$. Replacing will obtain:

$$I = \frac{1}{2} \cdot \rho_2 \cdot v_2 \cdot v_m^2 = \frac{1}{2} \cdot \rho_2 \cdot v_2 \cdot \left(\frac{T_m}{\rho_1 \cdot v_1} \right)^2 \qquad (24)$$

The relation is valid for a homogeneous transducer.

In case of composite transducer, we have an amplification coefficient G_1 given by relation:

$$I = \frac{1}{2} \rho_2 v_2 v_m^2 G_1 = \frac{1}{2} \rho_2 v_2 \left(\frac{T_m}{\rho_1 \cdot v_1} \right)^2 G_1 \qquad (25)$$

The debited power by transducer through end surface A will be:

$$P = I \cdot A = \frac{1}{2} \cdot \rho_2 \cdot v_2 \cdot \left(\frac{T_m}{\rho_1 \cdot v_1} \right)^2 \cdot G_1 \cdot A \qquad (26)$$

This is the power in function of material parameters. For to calculate we have:

- It is imposing: → T_m - not pass the admissible maximum tensions (of fatigue); → P - debited power.

- It is known: →$\rho_1 v_1$ – the end of transducer (director); →$\rho_2 v_2$ – the work medium; →G_1 - the amplification.

Results A – the transversal area of transducer. The power which may be debits from transducer it is depending from acoustic impedance charge $Z_2=\rho_2 \cdot v_2$. For a small acoustic $Z_2 \to 0$, the director will vibrate with a big amplitude and so no will appliqué high power. So we have: for water like acoustic charge→ $Z_2=\rho_2 v_2=1{,}5 \cdot 10^6$ Kg / m^{2}s; -for air like acoustic charge → $Z_2 = \rho_2 v_2 = 420$ Kg / m^{2}s. So, the admissible power, in case or air, is much small. The transversal dimension isn't being too big. It is impose to be ≤ ½ from longitudinal dimension (λ/2) for not permit transversal resonance close from longitudinal resonance.

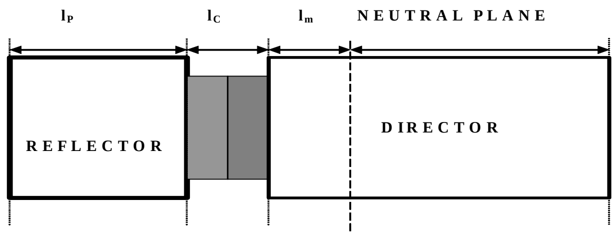

Figure 4 - 500W / 20 kHz transducer.

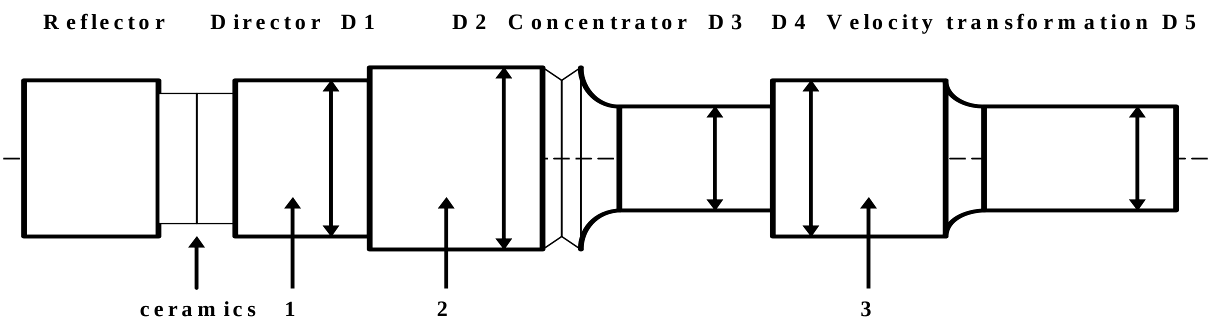

Figure 5 - Acoustic chain - transducer (1), concentrator (2) and velocity transformation (3).

3. EXPERIMENTAL RESULTS.

A piezoelectric transducer, having 500W power at 20kHz frequency it is given in fig. 4. In these situations it is preferable [6] to use the velocity transformations for to magnify the vibration amplitude. These velocity transformations are formed from alloy-aluminum resonators having λ/2 long (λ=256 mm) - fig. 5.

It is built 2 acoustic systems and with $1000V_{vv}$ excitation signal it is obtained:

The I acoustic system having D_2=52mm; D_3=25mm diameters. The vibration amplitude was 50 μm.

The II acoustic system having D_2=59mm; D_3=22 mm diameters. The vibration amplitude was 80 μm.

In both systems it is utilizes an acoustic transformation having D_4=60mm and D_5=26mm as diameters.

The amplification obtained was: $G = \left(\frac{D_2}{D_3} \cdot \frac{D_4}{D_5} \right)^2$ in conformities with general formulas $G = \left(\frac{D_{max}}{D_{min}} \right)^2$

Replacing it is obtained:

- The I acoustic system ⇒ $G_1 = 23$;
- The II acoustic system ⇒ $G_2 = 39,3$

These coefficients will represent the amplifications realized from acoustic impedances from these two cases. In both cases we have an increase of acoustic impedance in same time with an increase of transducer charge.

4. CONCLUSIONS.

With help of those presented in paper we can calculate the dimensions of a piezoelectric transducer used to produce the power ultrasonic field [7] necessary at unconventional technology washes, solders etc. The results will be verified in practice [8] and help us to quickly find the dimensions and these influence on global performances.

REFERENCES

[1] Odobescu G., Rugina I., Lala C. *"Metode de creştere a eficienţei sistemelor acustice de mare putere."* Sesiunea de comunicări ştiinţifice – Comisia de Acustica – Academia Română Bucureşti, 12-13 octombrie 2000, pp. 31-36.

[2] Odobescu G., Lala C. *"The comparative study about energetic efficiency for miscellaneous types of adaptation schemes generator-piezoelectric transducers",* The Annual Symposium of the Institute of Solid Mechanics, SISOM 2002, (16-17) May, Bucharest 2002, pp 183 ÷188.

[3] Odobescu G.. *"The propagation study of ultrasound high intensity waves through resonance bodies transformation with a new adaptation scheme with acoustic charge",* First Conference of the Romanian Acoustical Society, October 15-17, 2001, Bucharest , Romania, pp 175-182.

[4] Odobescu G.. *"The propagation study of ultrasound high intensity waves through resonance bodies transformation/adaptation with acoustic charge",* The 17th International Congress on Acoustic, September 2-7 2001, Rome, Italy, abstract ID 1522, Oral session on 7A.02.07 " Nonlinear acoustic", Book of Abstracts-pp 351; Scientific programme-pp89, abstract-pp 71.

ANALIZA PROPAGARII UNDELOR IN STRUCTURI CU UTILIZARE IN PROIECTAREA SI CONSTRUCTIA TRADUCTOARELOR GENERATOARE DE ULTRASUNETE

Rezumat: Energia ultrasonora este generata de elemente piezoelectrice care sunt excitate de un semnal electric obtinut de la un generator electronic. Transmisia acestei energii inalte este obtinuta prin elemente de transmisie sau elemente de concentrare in functie de ceea ce vrem sa facem cu aceasta energie in special in tehnologiile neconventionale. Lucrul cu eficienta este un foarte important aspect pentru ca energia utilizata are valori mari. In aceasta lucrare este prezentata o metoda de a calcula elementele componente necesare de a genera si transmite energia ultrasonora de mare energie: traductorul piezoelectric si elementele de transmisie/concentrare. Sunt prezentate rezultatele experimentale obtinute cu teoria prezentata. Contributia originala consta in metoda utilizata si graficile sugestive pentru aprecierea variatiei parametrilor.

Grigore Liviu Odobescu, Institute of Solid Mechanics, Romanian Academy – Bucharest, Constantin Mille street, no. 15, Bucharest, Romania, e-mail: g_odobescu@yahoo.com