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THE CALCULATION OF ACTUATORS FOR THE TRTR1 ROBOT

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Abstract: *In this paper, the authors propose an original method of calculation and choosing the actuators for the robot's modules. The method consist in using the dynamic equations of the robot along with relations established for driving forces and moments, taking into consideration the mechanical structure of each module. For this, by following the kinematic scheme of the serial robot TRTR1 and by using the Lagrange's equations of second type, the differential equations of motion were determined. Forwards, the authors present relations for the driving forces and moments. These expressions contain the constructive mechanical characteristics for the robot's modules as: power, speed and driving moment. By introducing these relations in the dynamic equations of the robot, the expressions for the driving moments were determined. Carrying on, by imposing trapezoidal laws of variation, with parabolic variation, for the accelation and braking sections and by specifying numerical data for the constructive mechanical characteristics, the maximal driving moments were obtained. Then, after consulting a specific catalogue, the actuators (that have the value of the driving moment superior to the one calculated) were chosen.*

Key words: *industrial robot, kinetic energy, generalized forces, differential equations, modules, actuators.*

1. INTRODUCTION

In the production process at S.C. ROMBAT S.A. Bistriţa, for the palletization of car batteries is used an articulated robot with five degrees of freedom.

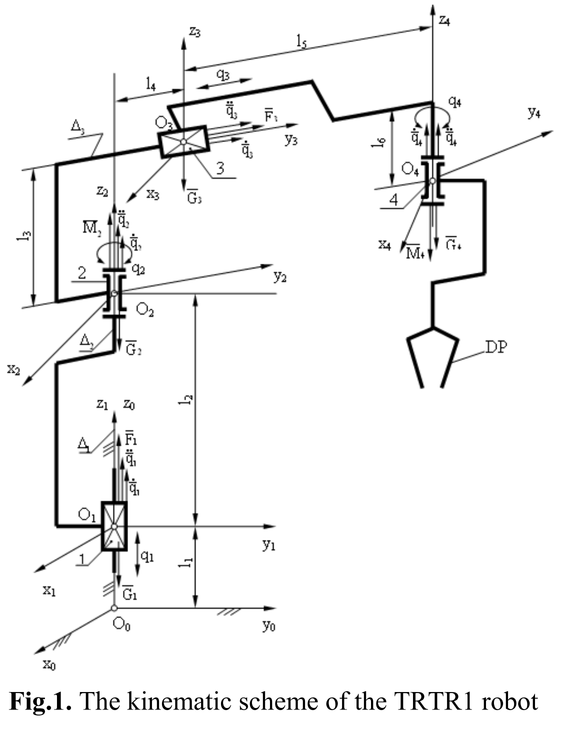

Fig.1. The kinematic scheme of the TRTR1 robot

Following the palletization process, it can be seen that the robot is used inefficiently because during each manipulation all five actuators are working. After a closer analysis, the existing robot can be replaced by a robot with four degrees of freedom (TRTR1). The kinematics scheme of the robot is presented in fig. 1.

Starting from the base of the robot to the endeffector, the following modules can be identified: module 1, of translation on vertical (MTV), module 2 - MRB (rotation of the robot's arm around the vertical axis), module 3 of translation on horizontal (MT) and module 4, of orientation for the gripper (MO). It can be seen that the module 4 is vertical, so the gripper and the car battery hold by it, rotates around an vertical axis parallel to the vertical axis of the robot noted with Δ_1.

2. THE DYNAMIC EQUATIONS OF ROBOT

In order to determine the actuators of the robot, according to [1], by using the Lagrange's equations of second type, the dynamic study of the robot was made. Thus, the following relation can be written:

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_k}\right)-\frac{\partial E_c}{\partial q_k}=Q_k, k=1\div 4. \quad (1)$$

The kinetic energy of the robot is:

$$E_c=\frac{1}{2}\left(\sum_{i=1}^{4} m_i\right)\dot{q}_1^2+\frac{1}{2}[J_{\Delta_2}^{(2)}+J_{z_3}^{(3)}+J_{z_4}^{(4)}+ \\ +m_3(l_4+q_3)^2+m_4(l_4+l_5+q_3)^2]\dot{q}_2^2+ \quad , \quad (2) \\ +\frac{1}{2}\left(\sum_{i=3}^{4} m_i\right)\dot{q}_3^2+\frac{1}{2}J_{z_4}^{(4)}\dot{q}_4^2 \pm J_{z_4}^{(4)}\dot{q}_2\dot{q}_4$$

and the generalized driving forces Q_k are:

$$Q_1=F_1-\sum_{i=1}^{4} G_i, Q_2=M_2, Q_3=F_3, Q_4=M_4. \quad (3)$$

The differential equations of movement for the robot TRTR1 were obtained by introducing (2) and (3) in expression (1). In this way, the system of differential equations is obtained:

$$\left(\sum_{i=1}^{4} m_i\right)\ddot{q}_1=F_1-\sum_{i=1}^{4} G_1 \\ [J_{\Delta_2}^{(2)}+J_{z_3}^{(3)}+J_{z_4}^{(4)}+m_3(l_4+q_3)^2+ \\ +m_4(l_4+l_5+q_3)^2]\ddot{q}_2 \pm J_{z_4}^{(4)}\ddot{q}_4+2[m_3(l_4+q_3)+ \\ +m_4(l_4+l_5+q_3)]\dot{q}_2\dot{q}_3=M_2 \\ \left(\sum_{i=3}^{4} m_i\right)\ddot{q}_3-\left[m_3(l_4+q_3)+m_4(l_4+l_5+q_3)\right]\dot{q}_2^2=F_3 \\ J_{z_4}^{(4)}(\pm\ddot{q}_2+\ddot{q}_4)=M_4. \quad (4)$$

In system (4) the following notations were made:

l_i, i=4 $\div$ 5, the design parameters of the robot;

q_k, k=1 $\div$ 4, the generalized coordinates of the robot (the geometric and kinematic parameters);

$\dot{q}_k$, $\ddot{q}_k$, k=1 $\div$ 4, the generalized velocities and accelerations (linear and angular);

m_i, i=1 $\div$ 4, the masses for the modules (m_4 also includes the gripper mass and the car battery mass);

$\overline{G}_i$, i=1 $\div$ 4, the gravity forces of the modules (G_4 also includes the gripper and the car battery gravity);

$\overline{F}_1$, $\overline{F}_3$, $\overline{M}_2$, $\overline{M}_4$, the driving forces and moments;

$J_{\Delta_2}^{(2)}$, - the mechanical moment of inertia for the mobile crew of the module MRB, determined in relation with Δ_2 axis;

$J_{z_3}^{(3)}$, - the mechanical moment of inertia for the module MT, determined in relation with O_3z_3 axis;

$J_{z_4}^{(4)}$, - the mechanical moment of inertia for the mobile crew of the module MO, determined in relation with O_4z_4 axis.

3. THE CALCULATION OF THE ACTUATORS

Taking into consideration the dynamics of the robot and the structure of each module, an original method of calculation and choice of actuators were obtained .

In the beginning, this method involves establishing relations between the driving forces and moments from the exit of the modules and the driving moment of each module separately. Thereby,

For the vertical translation module MTV, according to [2], the following relations were established:

$$F_1=\frac{191\cdot 10^5\cdot P_{m_1}\eta_{c_1}\eta_{r_1}\eta_{m_1}i_{c_1}i_{m_1}}{n_{m_1}d_{0_1}tg\left(\psi_1+arctg\frac{s_1}{d_{b_1}\sin\theta_1}\right)},$$

$$M_{m_1}=9550\frac{P_{m_1}}{n_{m_1}}, \quad \psi_1=arctg\frac{p_1}{\pi d_{0_1}}, \quad (5)$$

in which:

P_{m_1}, n_{m_1}, represent the actuator's power and speed;

M_{m_1}, is the driving moment;

η_{c_1}, η_{r_1}, η_{m_1}, are the yields for the intermediate cylindrical gear, for a pair of bearings and for the worm gear;

i_{c_1}, i_{m_1}, represent the gear ratio for the cylindrical gear and for the worm gear;

d_{01}, is the diameter for the cylinder on which are placed the centers of balls for the ball screw;

ψ_1, is the closing angle of the propeller on average cylinder;

s_1, is the rolling friction coefficient;

d_{b_1}, represents the ball diameter;

θ_1, is the contact angle;

p_1, is the ball screw pitch.

For the rotation module MRB, according to [3], the following expressions were determined:

$$M_2 = 9550 \frac{P_{m_2} \eta_{c_2} \eta_{r_2} \eta_{m_2}}{n_{m_2}} i_{c_2} i_{m_2} i_{s_2},$$

$$M_{m_2} = 9550 \frac{P_{m_2}}{n_{m_2}}, \qquad (6)$$

where:

P_{m_2}, M_{m_2}, n_{m_2}, represent the power, the moment and the speed for the MRB's actuator;

η_{c_2}, η_{r_2}, η_{m_2}, are the yields for the cylindrical gear, for a pair of bearings and for the worm gear;

i_{c_2}, i_{m_2}, i_{s_2}, represent the gear ratio for the cylindrical gear, for the worm gear and for the ball and roller screw.

For the translation module MT, according to [2] and [4], the following relations were established:

$$F_3 = \frac{191 \cdot 10^5 \cdot P_{m_3} \eta_{c_3} \eta_{r_3} \eta_{m_3} i_{c_3} i_{m_3}}{n_{m_3} d_{0_3} \operatorname{tg}\left(\psi_3 + \operatorname{arctg} \frac{s_3}{d_{b_3} \sin \theta_3} \right)},$$

$$M_{m_3} = 9550 \frac{P_{m_3}}{n_{m_3}}, \ \psi_3 = \operatorname{arctg} \frac{p_3}{\pi d_{0_3}}. \qquad (7)$$

The significance of the notations from (7) is the same as the ones in relations (5), the only difference is that it refers to the MT module.

For the module of orientation MO, according to [3], the following expressions were set:

$$M_4 = 9550 \frac{P_{m_4}}{n_{m_4}} \eta_{c_4} i_{c_4}, \ M_{m_4} = 9550 \frac{P_{m_4}}{n_{m_4}}, \qquad (8)$$

in which M_{m_4}, P_{m_4}, n_{m_4}, represent the moment, the power and the speed for the MO's actuator, η_{c_4} is the yield for the cylindrical gear and i_{c_4} is the gear ratio for the cylindrical gear.

Forwards, the expressions for $\overline{F}_1$, $\overline{F}_3$, $\overline{M}_2$ and $\overline{M}_4$, written in (5) - (8), are introduced in system (4), from which the relations $\frac{P_{m_i}}{n_{m_i}}$, i=1 $\div$ 4 are extracted. Thereby:

$$\frac{P_{m_1}}{n_{m_1}} = \frac{d_{0_1} \operatorname{tg}\left(\psi_1 + \operatorname{arctg} \frac{s_1}{d_{b_1} \sin \theta_1} \right)}{191 \cdot 10^5 \eta_{c_1} \eta_{r_1} \eta_{m_1} i_{c_1} i_{m_1}} \cdot \left[\left(\sum_{i=1}^{4} m_i \right) \ddot{q}_1 + \sum_{i=1}^{4} G_i \right] \qquad (9)$$

$$\frac{P_{m_2}}{n_{m_2}} = \frac{1}{9550 \cdot \eta_{c_2} \eta_{r_2} \eta_{m_2} i_{c_2} i_{m_2} i_{s_2}} \{ [J_{\Delta_2}^{(2)} + J_{z_3}^{(3)} + J_{z_4}^{(4)} + m_3 (l_4 + q_3)^2 + m_4 (l_4 + l_5 + q_3)^2 \ddot{q}_2 \pm \pm J_{z_4}^{(4)} \ddot{q}_4 + 2[m_3(l_4 + q_3) + m_4(l_4 + l_5 + q_3)] \dot{q}_2 \dot{q}_3 \} \qquad (10)$$

$$\frac{P_{m_3}}{n_{m_3}} = \frac{d_{0_3} \operatorname{tg}\left(\psi_3 + \operatorname{arctg} \frac{s_3}{d_{b_3} \sin \theta_3} \right)}{191 \cdot 10^5 \cdot \eta_{c_3} \eta_{r_3} \eta_{m_3} i_{c_3} i_{m_3}} \left\{ \left(\sum_{i=3}^{4} m_i \right) \ddot{q}_3 - - [m_3 (l_4 + q_3) + m_4 (l_4 + l_5 + q_3)] \dot{q}_2^2 \right\} \qquad (11)$$

$$\frac{P_{m_4}}{n_{m_4}} = \frac{J_{z_4}^{(4)}}{9550 \cdot \eta_{c_4} i_{c_4}} (\pm \ddot{q}_2 + \ddot{q}_4). \qquad (12)$$

Forwards, for linear and angular velocities, according to [2], trapezoidal laws of variation depending on time were chosen (fig.2 and fig.3). Then, by numerical integration, according to [2], the laws of motion on the robot's axes were determined (fig.4 and fig.5). Also, by numerical differentiation, the laws of variation for the linear and angular accelerations were obtained (fig.6 and fig.7). By introducing this data and the values for the mechanical parameters of the robot in relations (9) - (12), the laws of variation, depending on time, for the expressions $\frac{P_{m_i}}{n_{m_1}}$, i=1 $\div$ 4 were obtained.

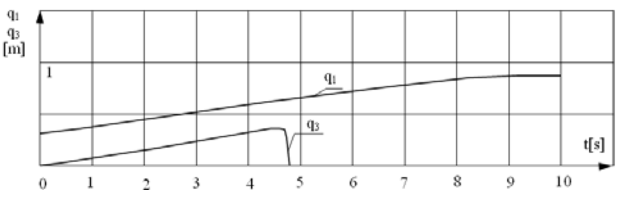

Fig.2. The variation of the generalized coordinates q_1 and q_3

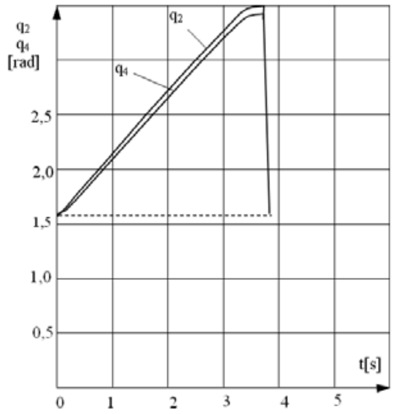

Fig. 3. The variation of the generalized coordinates q_2 and q_4

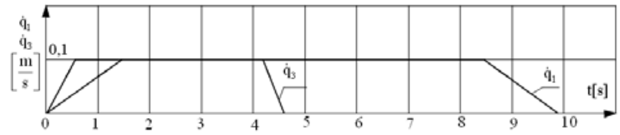

Fig. 4. The variation of the linear velocities $\dot{q}_1$ and $\dot{q}_3$

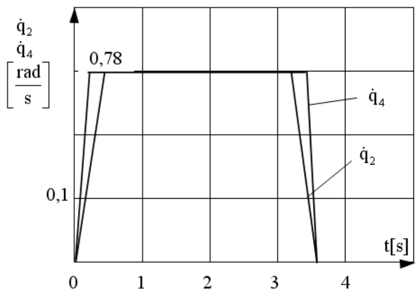

Fig. 5. The variation of the angular velocities $\dot{q}_2$ and $\dot{q}_4$

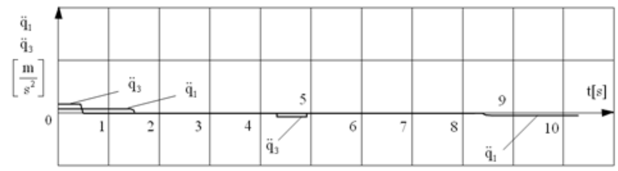

Fig. 6. The variation of the linear accelerations $\ddot{q}_1$ and $\ddot{q}_3$

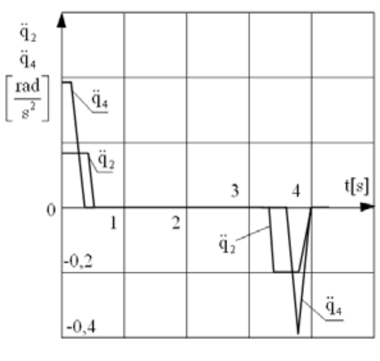

Fig. 7. The variation of the angular accelerations $\ddot{q}_2$ and $\ddot{q}_4$

The initial data for the generalized coordinates is:

$t_0=0$; $q_{10}=0,32[m]$; $q_{20}=\frac{\pi}{2}[rad]$; $q_{30}=0$;

$$q_{40}=\frac{\pi}{2}[rad]. \tag{13}$$

The data for the mechanical parameters, according to [2] and [4], is:

$m_1=43[kg]$; $m_2=41[kg]$; $m_3=30[kg]$; $m_4=15[kg]$; $G_1=421,8[N]$; $G_2=402,2[N]$; $G_3=294,3[N]$; $G_1=147,2[N]$; $l_4=0,04[m]$; $l_5=0,50[m]$; $J_{\Delta_2}^{(2)}=0,608[kgm^2]$; $J_{z_3}^{(3)}=0,12[kgm^2]$; $J_{z_4}^{(4)}=0,013[kgm^2]$; $d_{0_1}=0,0158[m]$; $d_{0_3}=0,0127[m]$; $p_1=0,00352[m]$; (14)
$p_3=0,00356[m]$; $s_1=s_3=0,008\cdot10^{-3}[m]$; $d_{b_1}=d_{b_3}=0,00198[m]$; $\theta_1=\theta_3=45°$; $\eta_{c_1}=\eta_{c_2}=\eta_{c_3}=\eta_{c_4}=0,97$; $\eta_{r_1}=\eta_{r_2}=\eta_{r_3}=0,995$; $i_{m_1}=i_{m_2}=i_{m_3}=30$; $\eta_{m_1}=\eta_{m_2}=\eta_{m_3}=0,78$; $i_{c_1}=i_{c_2}=i_{c_3}=i_{c_4}=1$.

By using the following relation:

$$M_{m_i}=9550\left(\frac{P_{m_i}}{n_{m_i}}\right)=[N\cdot m], i=1\div4, \tag{15}$$

the values for the driving moments were obtained (fig. $8\div13$).

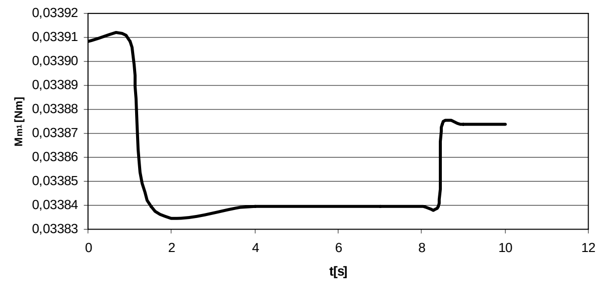

Fig. 8. The driving moment M_{m_1}

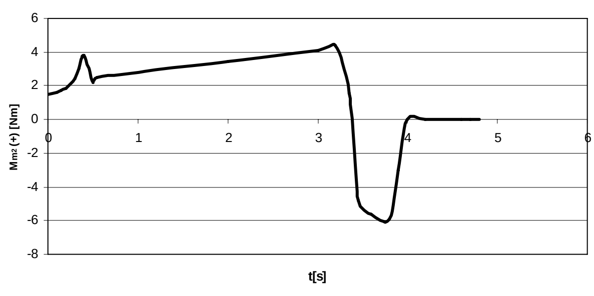

Fig. 9. The driving moment M_{m_2} (+)

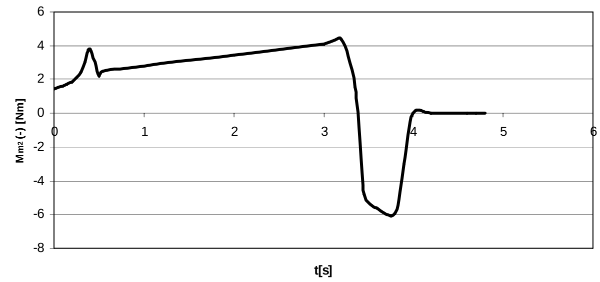

Fig. 10. The driving moment M_{m_2} (-)

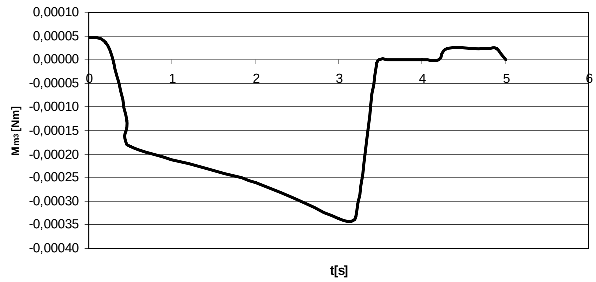

Fig. 11. The driving moment M_{m_3}

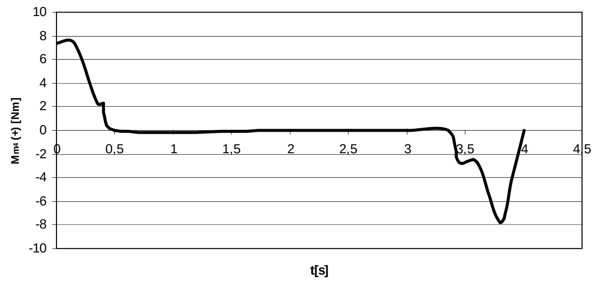

Fig. 12. The driving moment M_{m_4} (+)

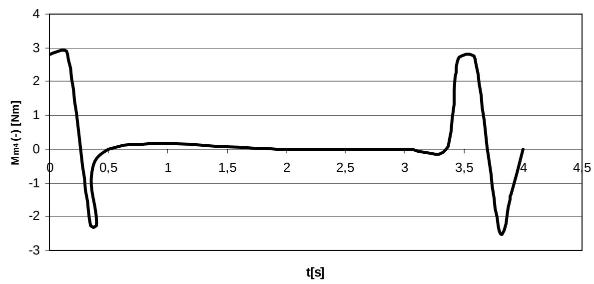

Fig. 13. The driving moment M_{m_4} (-)

From this charts the maximal values for the driving moments are chosen. Thereby:

$$M_{m_1} = 0{,}033908421\,[\text{Nm}];$$

$$M_{m_2}(+) = |-5{,}921|\,[\text{Nm}] = 5{,}921\,[\text{Nm}];$$

$$M_{m_2}(-) = |-5{,}91145|\,[\text{Nm}] = 5{,}91145\,[\text{Nm}];$$

$$M_{m_3} = |-0{,}000337019|\,[\text{Nm}] = 0{,}000337019\,[\text{Nm}]$$

;

$$M_{m_4}(+) = |-7{,}76415|\,[\text{Nm}] = 7{,}76415\,[\text{Nm}];$$

$$M_{m_4}(-) = 2{,}8077\,[\text{Nm}].$$

Forwards, these values are compared with the ones from the catalogue Kemmerich Elektromotoren [5]. After this comparison, the values immediately above are chosen. Doing so, the following engines were chosen:

- for the first module (MTV): A192/ACA 56 C-2 actuator (M_{STAS}=0,62 [Nm], n_{STAS}=2800 [U/min], P_{STAS}=180 [W]);
- for the second module (MRB): V47/LKM-K20R80G2 actuator (M_{STAS}=7,4 [Nm], n_{STAS}=2830 [U/min], P_{STAS}=2200 [W]);
- for the third module: A192/ACA 56 C-2 actuator (M_{STAS}=0,62 [Nm], n_{STAS}=2800[U/min], P_{STAS}=180 [W]);
- for the fourth module (MO): A192/ACA 90 LC-2 actuator (M_{STAS}=10,1[Nm], n_{STAS}=2890[U/min], P_{STAS}=3000[W]);

4. CONCLUSIONS

By combining the differential equations of movement with the relations established for the driving forces and moments at the exit of modules and considering the mechanical structure of each module, an original method of calculation was developed. Compared with other methods from literature, this method takes into consideration the dynamics of the entire robot and the actual construction of each module. In this way, by avoiding undersizing or oversizing, the most appropriate actuators were chosen.

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Calcularea motoarelor de acţionare pentru robotul TRTR1

În această lucrare autorii propun o metodă originală de calcul şi alegere a motoarelor de acţionare a modulelor robotului. Metoda constă în utilizarea ecuaţiilor dinamice ale robotului împreună cu relaţii stabilite pentru forţe şi momente motoare luând în considerare structura mecanică a fiecărui modul în parte. În acest sens, urmărind schema cinematică structurală a robotului serial TRTR1, se determină cu ajutorul ecuaţiilor lui Lagrange de speţa a II- a sistemul de ecuaţii diferenţiale de mişcare a robotului.

Adoptând, în continuare, diverse construcţii mecanice pentru modulele robotului, se scriu relaţii pentru forţele şi momentele motoare care conţin mărimi mecanice constructive ale modulelor, precum şi puterea, turaţia şi momentul motorului de acţionare. Prin introducerea acestor relaţii în ecuaţiile dinamice ale robotului, se obţin expresii pentru momentele motoarelor care acţionează modulele robotului.

Impunând legi de variaţie trapezoidale pe porţiunile de accelerare şi de frânare pentru vitezele liniare şi unghiulare şi precizând valori numerice pentru parametri mecanici constructivi, se obţin valorile maxime ale momentelor motoarelor de acţionare, necesare funcţionării robotului. Apoi, din cataloage de motoare specifice roboţilor se aleg motoarele care au valoarea momentului imediat superioară celei calculate.

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