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CAR VERTICAL DYNAMICS 3D SIMULATOR USING A 7 DOF MODEL

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Abstract: This paper presents a 3D car simulator, concerning only the vertical dynamics, described using a 7 DOF car model. This simulator is realized in Matlab/Simulink and will be used for further studies on wheel-road adherence, passenger comfort and the use of appropriate suspension shock absorbers to improve both adherence and comfort. This simulator will also be used to validate a road response simulator, designed as a reconfigurable haptic interface that includes real-time wheel/road dynamic contact simulation in its control system.
Key words: *7DOF car model, vertical dynamics, 3D simulator, Matlab/Simulink, dynamic contact, wheel-road adherence, passenger comfort.*

1. INTRODUCTION

In a previous study concerning a new self-adjustable suspension shock absorber for improving both wheel-road adherence and passenger comfort, Niculescu, Dumitriu et al. [1] have firstly used a quarter-car model, in order to simulate the vertical interaction between a rear car wheel and the road. But such a quarter-car model has only 2 DOF (degrees of freedom), neglecting the roll and the pitch motion of the car.

Thus, the next step was to build a half-car 2D model simulator for the vertical interaction between car and road, considering the pitch motion of the car but neglecting its roll motion [2]. This half-car 2D model has 4 degrees of freedom, i.e., the vertical displacement of the front of the car body (front bounce), the vertical displacement of the back of the car body (rear bounce), the vertical displacement of the front wheel center (front wheel hop) and the vertical displacement of the rear wheel center (rear wheel hop). The same half-car 2D model can be used to study the roll motion of the car while neglecting its pitch motion.

In order to better approach the real 3D environment, by considering both the roll and the pitch motion of the car, this paper presents a classical 3D car simulator. This simulator concerns only the vertical dynamics of the car, neglecting the small motions in the horizontal plane (longitudinal and transversal oscillations). The car vertical dynamics equations below are those proposed in the literature [3,4]. Some suggestions concerning the road conditions to be used for simulations are provided as well.

This car simulator is realized in Simulink, the dynamic equations being implemented as C MEX S-functions from specifications and source code written in Ansi-C compatible.

2. 3D CAR VERTICAL DYNAMICS MODEL

Fig. 1 shows the 3D dynamic model [3,4] used to study the vertical interaction between car and road, considering both the roll and the pitch motion of the car. The case of independent suspensions both for the front and rear part of the car is considered here.

The 3D vertical model has 7 degrees of freedom, i.e., the vertical displacement z_{GC} of the gravity center GC of the car body (sprung mass), the roll angle φ of the car body (around the x axis passing through the gravity center GC), the pitch angle θ of the car body (around the y axis passing through GC), the vertical displacement z_1 of the front right wheel center

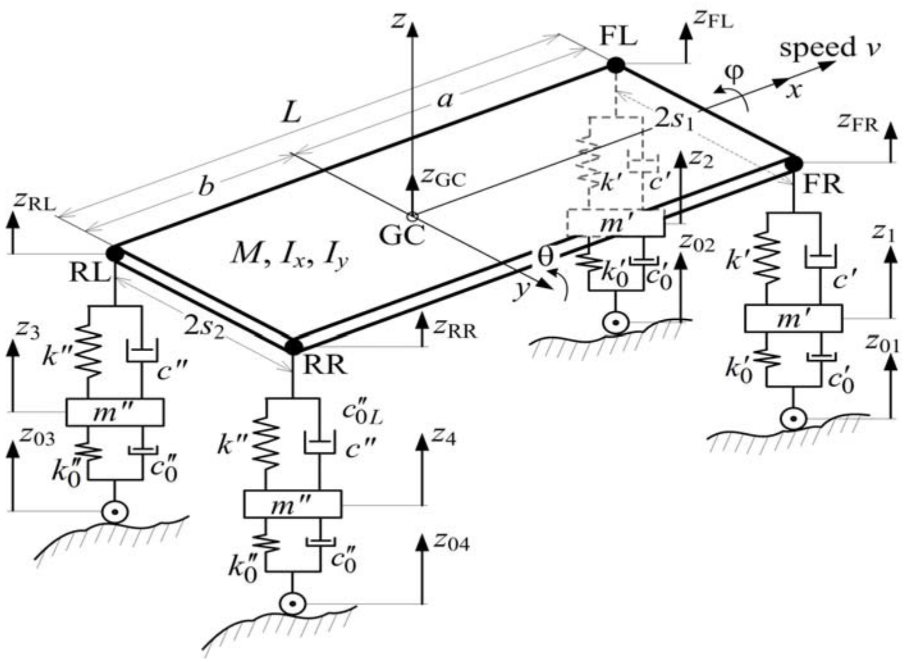

Fig. 1. Car vertical dynamics 3D model.

(front right wheel hop), the vertical displacement z_2 of the front left wheel center (front left wheel hop), the vertical displacement z_3 of the rear left wheel center (rear left wheel hop) and the vertical displacement z_4 of the rear right wheel center (rear right wheel hop). The vertical displacements z_i (i=1,2,3,4) are obviously calculated with respect to the equilibrium position.

At time t, the vertical profile of the road (road roughness) corresponding to the front right wheel is denoted by $x_{01}(t)$, the road roughness corresponding to the front left wheel is denoted by $x_{02}(t)$, the road roughness corresponding to the rear left wheel is denoted by $x_{03}(t)$, finally the road roughness corresponding to the rear right wheel is denoted by $x_{04}(t)$.

The model contains two levels of elastic and damping elements: one level between the wheels and the road, characterized by the stiffness coefficients k_0' and k_0'' of the tires and the damping coefficients c_0' and c_0'' of the tires; the second level between the wheels and the body (vehicle suspension), characterized by the spring rates of suspension k' and k'' and the damping coefficients c' and c'' of the shock absorbers. Here $'$ corresponds to front wheels, while $''$ corresponds to rear wheels.

The 3D car/road vertical interaction model in Fig. 1 implies the following geometrical and inertial characteristics:
- the distance a between the mass center GC of the sprung mass M and the front axle (passing through points FR and FL);
- the distance b between the mass center GC of the sprung mass M and rear axle (passing through points RR and RL);
- the car wheelbase L (obviously $L=a+b$);
- the front wheel track $2s_1$ (distance measured across the front axle from the center line of the front left tyre tread to the center line of the opposite front right tyre tread);
- the rear wheel track $2s_2$ (similarly, distance between left and right rear wheels);
- the front left wheel unsprung mass m';
- the front right wheel unsprung mass m';
- the rear left wheel unsprung mass m'';
- the rear right wheel unsprung mass m'';
- the mass M of the car body (sprung mass M), which normally takes values between M_{empty} (unloaded car case, including only seat+driver and fuel masses) and M_{full} (maximum admissible car loading case);
- the moment of inertia I_x of the sprung mass M with respect to the longitudinal x axis passing through the mass center GC of the sprung mass;

- the moment of inertia I_y of the sprung mass M with respect to the transversal y axis passing through the mass center GC of the sprung mass.

3. CAR BODY AND WHEELS VERTICAL DYNAMICS EQUATIONS

The vertical dynamics equations are obtained using the Newton-Euler formulation, as provided in the literature [3,4,5]. The aerodynamic forces are neglected here. There are 3 scalar dynamic equations of the car body (sprung mass): one in terms of vertical forces (z direction), one in terms of moments with respect to the longitudinal x axis and the third one in terms of moments with respect to the transversal y axis. Since the roll angle φ and the pitch angle θ are small, it can be considered that $\sin\varphi \approx 0$ and $\sin\theta \approx 0$, in this case the vertical dynamic equations of the car body can be simplified as follows [4]:

$$\begin{cases} M\ddot{z} = -(F_{\mathrm{FL}} + F_{\mathrm{FR}} + F_{\mathrm{RL}} + F_{\mathrm{RR}})\cos\varphi\cos\theta \\ I_x\ddot{\varphi} = (F_{\mathrm{FL}} - F_{\mathrm{FR}})s_1\cos\varphi\cos\theta + \\ \qquad + (F_{\mathrm{RL}} - F_{\mathrm{RR}})s_2\cos\varphi\cos\theta \\ I_y\ddot{\theta} = (F_{\mathrm{FL}} + F_{\mathrm{FR}})a\cos\theta - (F_{\mathrm{RL}} + F_{\mathrm{RR}})b\cos\theta \end{cases} \tag{1}$$

where the resulting forces at the points where the sprung mass is attached to the suspension system, i.e., FL, FR, RL and RR, are given by:

$$\begin{aligned} F_{\mathrm{FR}} &= F_{c,\mathrm{FR}} + k'(x_{\mathrm{FR}} - x_1) - F_{e,\mathrm{bumper,FR}}, \\ F_{\mathrm{FL}} &= F_{c,\mathrm{FL}} + k'(x_{\mathrm{FL}} - x_2) - F_{e,\mathrm{bumper,FL}}, \\ F_{\mathrm{RL}} &= F_{c,\mathrm{RL}} + k''(x_{\mathrm{RL}} - x_3) - F_{e,\mathrm{bumper,RL}}, \\ F_{\mathrm{RR}} &= F_{c,\mathrm{RR}} + k''(x_{\mathrm{RR}} - x_4) - F_{e,\mathrm{bumper,RR}}. \end{aligned} \tag{2}$$

Here $F_{c,\mathrm{FR}}$ stands for the damping force given by the front right shock absorber. As for $F_{e,\mathrm{bumber,FR}}$, it designates the elastic striking force when the piston hits either the rebound bumper ($F_{e,\mathrm{bumber,FR}} < 0$ case) or the compression bumper ($F_{e,\mathrm{bumber,FR}} > 0$ case) of the front right suspension. Obviously, similar notations are used for the front left (FL), rear right (RR) and rear left (RL) suspensions.

The dynamic equations of the front and rear wheels are as follows [1-5]:

$$\begin{cases} m'\ddot{x}_1 = F_{\mathrm{FR}} - c_0'(\dot{x}_1 - \dot{x}_{01}) - k_0'(x_1 - x_{01}) \\ m'\ddot{x}_2 = F_{\mathrm{FL}} - c_0'(\dot{x}_2 - \dot{x}_{02}) - k_0'(x_2 - x_{02}) \\ m''\ddot{x}_3 = F_{\mathrm{RL}} - c_0'(\dot{x}_3 - \dot{x}_{03}) - k_0'(x_3 - x_{03}) \\ m''\ddot{x}_4 = F_{\mathrm{RR}} - c_0'(\dot{x}_4 - \dot{x}_{04}) - k_0'(x_4 - x_{04}) \end{cases} \tag{3}$$

The second order differential equations of motion (1) and (3) can be easily transformed in a system of 14 first order explicit ordinary differential equations, ready to be numerically integrated by usual methods, e.g., the Runge-Kutta method.

4. ROAD CONDITIONS

In what concerns the road conditions, the best choice is to consider a real road roughness profile (such as the Californian road profile shown in Fig. 2) or randomly generated road profiles with specified spectral density [6]. Obviously, the road profile for the left wheels can be different compared with the one for the right wheels.

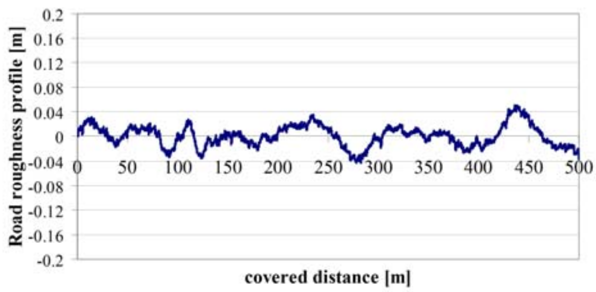

Fig. 2. Sample of real Californian road profile.

For testing purposes, a simple harmonic function $x_0 = a\sin(2\pi f + \phi)$ can be used to simulate the road profile [1,2], where a, f and ϕ stand for the amplitude, the frequency and the phase of the harmonic function.

Another choice is to simulate the road profile as the sum of three harmonic functions:

$$\begin{aligned} x_0 = a_1\sin(2\pi f_1 + \phi_1) + a_2\sin(2\pi f_2 + \phi_2) + \\ + a_3\sin(2\pi f_3 + \phi_3) \end{aligned} \tag{4}$$

The amplitudes, frequencies and phases of the harmonic functions above can be chosen so that to correspond to extreme, resonant road profile excitations (interesting for testing purposes).

5. CONCLUSION

A 3D Matlab/Simulink car simulator has been built based of a simplified 7 DOF model for car vertical dynamics. The dynamic equations are implemented in Simulink as C MEX S-functions from specifications and source code written in Ansi-C compatible.

This 3D car simulator will be used for further studies on wheel-road adherence, passenger comfort and the use of appropriate suspension shock absorbers to improve both adherence and comfort. This simulator will also be used to validate a road response simulator, designed as a reconfigurable haptic interface that includes real-time wheel/road dynamic contact simulation in its control system.

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Simulator 3D al dinamicii autovehiculelor, folosind un model cu 7 grade de libertate al mişcării în plan vertical

Rezumat: Lucrarea prezintă elementele ce stau la baza unui simulator 3D al dinamicii autovehiculelor, folosind un model cu 7 grade de libertate al mişcării în plan vertical. Simulatorul este realizat în Matlab/Simulink şi va fi folosit pentru studii privind folosirea unor suspensii adecvate pentru a îmbunătăți simultan atât aderența roată-drum cât şi confortul pasagerilor. De asemenea, simulatorul va fi folosit pentru validarea unui alt simulator, dedicat răspunsului drumului, proiectat ca o interfață haptică reconfigurabilă ce include simularea în timp real a contactului dinamic roată-drum în cadrul sistemului său de control.

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