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MODELING THE SOUND FIELD INSIDE AN ENCLOSURE USING "RAY–TRACING" METHOD

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Abstract: *In this paper, a program developed in C applying the "ray-tracing" method was used to study the distribution of sound energy inside an enclosure. The geometric problem was solved, which includes: defining the boundaries of the enclosure, setting the positions of source and receiver. The followings were calculated: positions of the sound rays leaving the source, the points of intersection with the obstacles, directions of the reflected rays. This process continues until every ray travels a way of a predefined length. Another step in this study is to determine the absorption due to obstacles and attenuation of sound ray in the air, the maximum length of the path covered by the sound ray - direct and reflected - defining the level of sound pressure.*

Key words: *ray-tracing method, sound field modeling, acoustics, sound power*

1. INTRODUCTION

One of the most important topics in acoustics is the study and modeling of sound field [1], [2], [3], [4], [5], [6]. In the literature there are presented several methods for the study of sound waves propagation. Among them, we may mention the Helmholtz's partial differential equation method that allows the accurate determination of analytical expressions of the solutions. Another category of methods is based on domain discretization, using finite element method, boundary element method and finite difference method.

A third method is based on the parabolic equations that approximate the partial differential equation.

The fourth category includes geometrical acoustics methods, based on approximations of propagation paths: "ray-tracing" method, "image source" method and others.

The "ray-tracing" method is a direct application of geometrical acoustics. This method can be used to study sound waves propagation in an enclosure (classroom, concert hall, church), in industrial areas, on both sides of a highway or railway, inside the area protected by noise barriers, in water, also near airports.

When studying the distribution of the sound energy into an enclosure, due to a source placed inside it, the use of the "ray-tracing" method involves multiple mathematical and programming aspects.

A rectangular enclosure is presented in figure 1, with a sound source S placed inside it. Its walls (sound barriers) are defined by known coordinates of vertexes.

The paths traveled by the sound rays should to be accurately determined, considering a given number of reflections and a maximum distance that can be traveled before the sound energy falls below a certain limit.

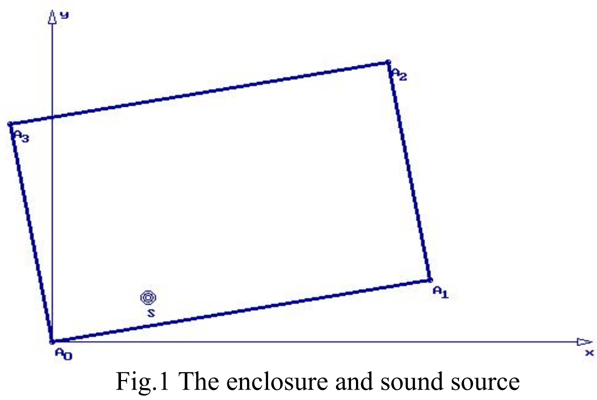

Fig.1 The enclosure and sound source

Two different problems need to be solved:

1. A sound ray that starts from the source S, in a direction that makes an angle φ_1 with the

horizontal direction, will intersect the rectangle's walls in a point with the coordinates X_I and Y_I. The values of these two coordinates must be determined.

2. The sound ray is reflected, according to Snell's law, from the previously determined point (X_I ,Y_I), in a new direction, defined by an angle φ_2, measured counter-clockwise from horizontal.

According to this method, real sound waves are replaced with incident sound rays equal angular spaced (thousands of rays) starting from a sound source and emitted in all directions. Sound rays lose their energy when propagate in air on straight directions and due to collisions with the walls, followed by reflections.

2. THEORETICAL CONSIDERATIONS

The sound source S is placed into an enclosure that is assumed to be convex (figure 2) [7], [8], [9], [10], [11], [12], [13].

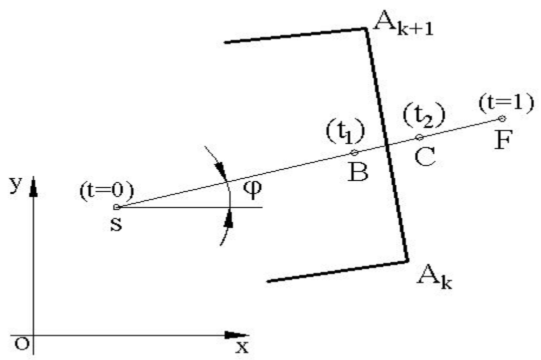

Fig.2 Sound source S, sound ray and a portion of the rectangular enclosure

The line passing through the point S and making an angle φ to the axis Ox has the equation (*m* being its slope):

$$y - y_s = m\left(x - x_s\right), \quad m = \tan\varphi \qquad (1)$$

The equations of straight lines passing through two successive points from the contour of the rectangle are:

$$\frac{x - x_k}{x_{k+1} - x_k} = \frac{y - y_k}{y_{k+1} - y_k}, \; k = 0,1,2,3. \qquad (2)$$

If equation (2) is written as:

$$E(x,y) = (x - x_k)\left(y_{k+1} - y_k\right) - \\ -\left(y - y_k\right)\left(x_{k+1} - x_k\right) = 0 \qquad (3)$$

and two points B and C are considered in the plane, placed on different sides of the line passing through the points A_k and A_{k+1}, then the following relation stands:

$$E(x_B, y_B)E\left(x_C, y_C\right) < 0 \qquad (4)$$

Condition (4) will be considered to establish the coordinates of the intersection point between SF and the right side of the rectangular wall. Point F is chosen on the xOy plane, outside the rectangular boundaries, on the line that passes through point S, of known slope.

Determination of intersection points between sound rays and the rectangular contour lines can be made by consecutively solving the two equations (1) and one equation from (2). A number of intersection points equal to the number of rectangle's lines will be obtained (unless the slope m is equal to one or more slopes of the rectangle's lines). Still, the intersection point must be properly chosen, by checking a number of conditions.

A simple and efficient way is to work with the parametric equations of the line SF:

$$x = x_S + \left(x_F - x_S\right)t$$

$$y = y_S + \left(y_F - y_S\right)t \qquad (5)$$

and to give values from 0 to 1, with a step Δt, for the parameter t.

If for two successive values of the parameter t, t_1 and $t_2 = t_1 + \Delta t$, the following expression:

$$E\left(x_t\,,\, y_t\right)\, E\left(x_{t+\Delta t}\,,\, y_{t+\Delta t}\right) \qquad (6)$$

is negative, it means that the parameter which defines the position of the intersection point of the two lines (SF and a contour line of the rectangle) is inside the interval $[t_1, t_1 + \Delta t]$.

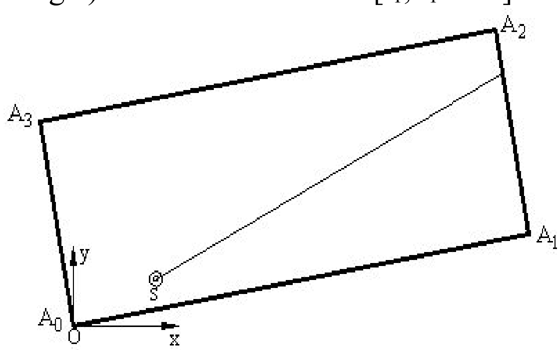

Fig.3 Enclosure, sound source and the first sound ray

This calculation must be performed considering all the contour lines of the rectangle: A_0A_1, A_1A_2, A_2A_3, A_3A_0, and the intersected rectangular line is the one corresponding to the case when the lowest value of the parameter t is obtained.

In this way we can obtain the coordinates x_I and y_I of the point of incidence I of the sound ray with the rectangle's contour line.

For the given problem, the coordinates of the point where the incident sound ray reaches one of the lines of the rectangle have been found. Once determined this point, if the obstacle's surface (wall, sound barrier) reflects the sound ray, in order to draw it, the angle of the reflected ray has to be calculated.

In figure 4 is presented the case when the sound ray SI reaches the wall and it is reflected. The angles noted φ_1 and φ_2, are between the horizontal axis and the two sound rays - direct and reflected. The coordinates of points S, A_{k+1} and I being known, the vertices of S A_{k+1} I triangle, the lengths of its sides can be calculated:

$$a=\sqrt{\left(x_S-x_{k+1}\right)^2+\left(y_S-y_{k+1}\right)^2} \qquad (7)$$

$$b=\sqrt{\left(x_S-x_I\right)^2+\left(y_S-y_I\right)^2} \qquad (8)$$

$$c=\sqrt{\left(x_I-x_{k+1}\right)^2+\left(y_I-y_{k+1}\right)^2} \qquad (9)$$

Then, using the cosinus formula, the value of angle ψ will be:

$$\psi=a\cos\left(\frac{b^2+c^2-a^2}{2bc}\right) \qquad (10)$$

In figure 4 the angle between A_kA_{k+1} segment and the horizontal direction is considered in the interval $[0, \pi]$. Because the angle between the reflected ray and the segment A_kA_{k+1} is $\pi-\psi$, the expression for angle φ_2 is as follows:

$$\varphi_2=u_{k,k+1}+\pi-\psi \qquad (11)$$

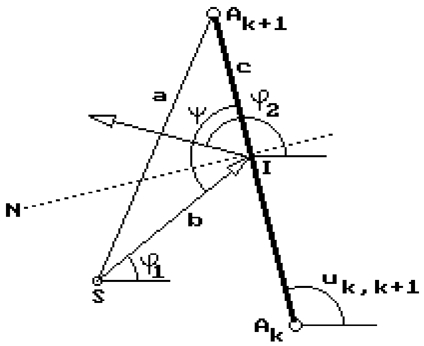

Fig.4. Direct and reflected sound rays

The same formula (11) is obtained even if the point S (sound source) is placed above the normal NI or the angle $u_{k,\,k+1}$ is less than $\pi / 2$.

Knowing the coordinates of point I and the angle between the horizontal and the reflected sound ray, it will be possible to draw it, following to calculate the next point of intersection between this ray and one of the contour lines of the rectangle. In a similar manner, the positions of all reflected rays can be successively established.

3. NUMERICAL RESULTS

The calculation method described above has been programmed using the C language, results being presented in graphical form.

The sound ray presented in figure 5, starting from source S, has a given initial angle with the horizontal direction and performs five reflections during its way inside the enclosure.

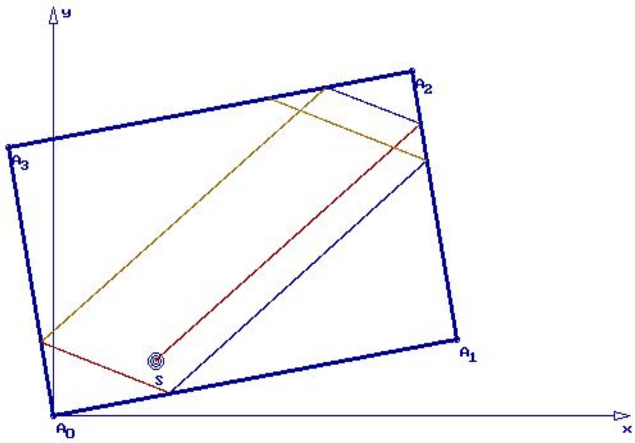

Fig.5. Enclosure, sound source, direct ray and five reflected rays

In the "ray-tracing" method it is considered that from the point S (the source) starts any number of rays, which, after reaching the enclosure's walls, perform one or more reflections.

The procedure used for one ray can be applied to each ray of the beam emitted by source S. In the figure 6 there are 12 such initial rays, presented together with their reflected rays.

The propagation of sound waves in a fluid is influenced by several factors, occurring phenomena whose study should be performed.

If it is studied the sound field generated by a source placed inside an enclosure or the length of the propagation path of sound rays is small, the atenuation phenomenon can be neglected.

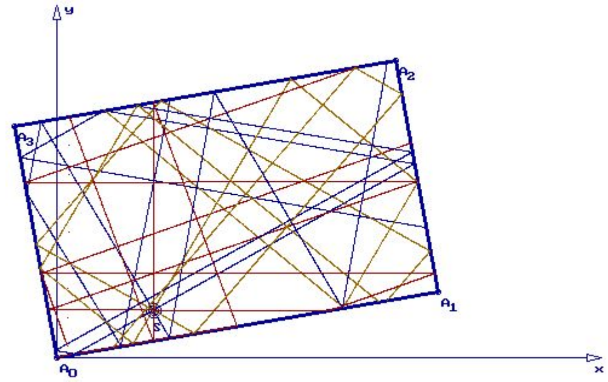

Fig. 6 Enclosure, sound source S, direct ray and reflected rays

During the sound wave traveling in space its power decreases with the distance x, according to an exponential law:

$$P = P_{init} \; e^{-m\,x} \qquad (12)$$

where m is the attenuation coefficient due to the environment (air), which depends on: temperature, atmospheric pressure, humidity and frequency of the sound.

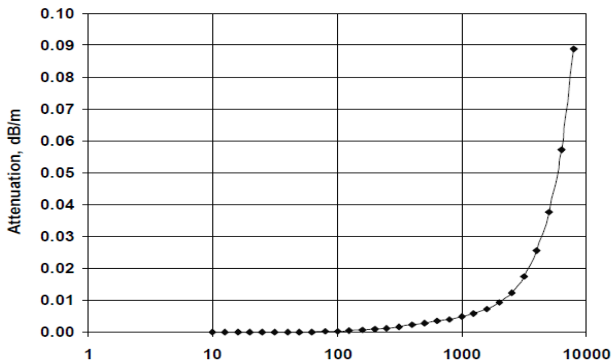

Fig.7. Sound attenuation in air, in dB/m, depending on frequency

A part of the sound wave energy is absorbed by the barrier, due to the properties of the materials it is built, the initial power of the sound wave decreasing by law:

$$P = P_{init}\left(1-\alpha\right) \qquad (13)$$

where α is an absorption coefficient.

The sound rays that "leave" from source S loose their energy due to the air attenuation, according to the exponential law (12) which depends on the attenuation coefficient m [m^{-1}], and also due to the contact with the enclosure's walls, the coefficient being noted with α. The following numerical values of the two coefficients are considered:

m = 0.001151 [m^{-1}], α = 0.025

Value of m was calculated using the formula m = 0.230258A, A is the attenuation and has the value A = 0.0050 [dB/m], and the value of α corresponds to concrete walls or obstacles.

For each sound ray, the length of the trajectory was establish to 42.0 [m], between two successive reflections the sound ray energy decreases according to the exponential law (12), and after each reflection there is a loss of energy calculated with formula (13).

If the source power is noted with P_{source} and the number of sound rays with *n*, then for each sound ray will correspond a power $P_{init} = P_{source} / n$. The power of the sound ray will decrease with the traveled distance, according to (12):

$$P\left(x\right) = P_{init}\, e^{-m\,x} = \frac{P_{source}}{n}\, e^{-mx} \qquad (14)$$

until the sound ray reaches the first obstacle.

When reaching an obstacle (wall, sound barrier, window, etc.) the power of the sound ray will decrease because of absorption, according to (13):

$$P_1^{(obstacle)} = P_1^{(path)}\left(1-\alpha\right) = \left(1-\alpha\right) e^{-m\,L_1} P_{init} \qquad (15)$$

After reaching an obstacle, the contact point between the incident sound ray and the obstacle can be considered as a new source, whose power is P_1 (obstacle). The process is repeated several times until the completion of the imposed travel distance of a sound ray, either until the sound power is reduced below a certain limit.

The final formula for calculating the power of the sound ray, after traveling *k* paths and after encountering obstacles is:

$$P_k^{(obstacle)} = \left(1-\alpha\right)^k e^{-m\,L} P_{init} \quad L = \sum_{i=1}^{k} L_i \qquad (16)$$

Figure 8 presents the path of a sound ray inside the enclosure, during which the ray has five reflections. At the bottom of the figure, the variation of sound power is given by a diagram.

Its sudden variations correspond to the moments of reflections of the sound ray, due to the phenomenon of walls absorption.

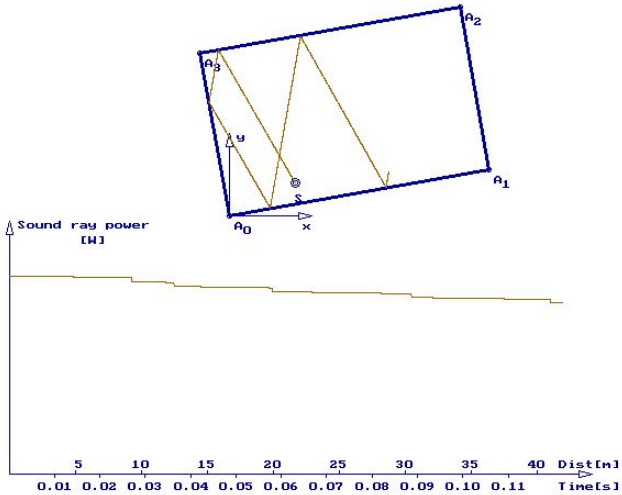

Fig.8. The path of the direct and reflected sound ray and the reduction of its sound power due to walls and attenuation in the air

The "ray-tracing" method enables the modeling of the sound field distribution inside the enclosure and also in a restricted area in the proximity of a receiver.

Coordinates of the four corners of the enclosure are: A_0 [0.0, 0.0]; A_1 [15.8, 2.8]; A_2 [14.0, 12.6]; A_3 [-1.7, 9.8] and the Cartesian coordinates of the two points inside the enclosure which define the positions for sound source and receiver are: xS [m] = 4.00; yS [m] = 2.00; xR[m] = 10.00; yR [m] = 8.00.

It has been considered a circular area around the point R defining the position of the receiver, and the power of each sound ray that passes through this area. The contribution of each sound ray, in terms of energy, during the crossing of the circular area is defined by:

$$E = P_{average}\left(t_{fin} - t_{init}\right) \qquad (17)$$

where $P_{average}$ is the average power of the sound ray during its travel through the circular surface, $t_{fin} - t_{init}$ is the time interval in which the sound ray crosses the surface.

Figure 9 presents the positions of 12 sound rays after they have traveled a distance of 20 [m] inside the enclosure, some of them making several reflections on its walls.

In figure 10 is presented the variation of the sound power of the rays which pass near the receiver (point R) and intersect the circle of radius r, defined around the receiver. The calculation was made by considering a number of 12 sound rays and a time interval enough for each sound ray to travel 20 [m].

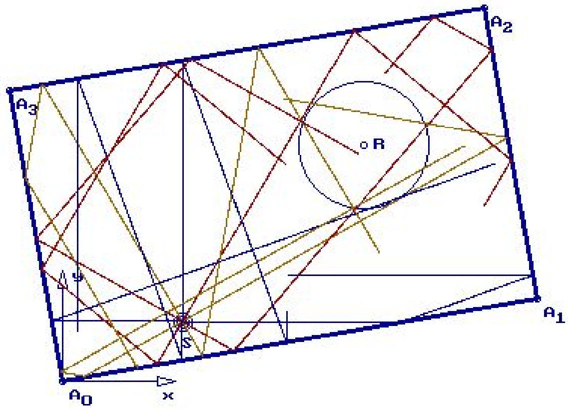

Fig.9. Enclosure, sound source and 12 sound ray after 20 [m] covered on the trajectories

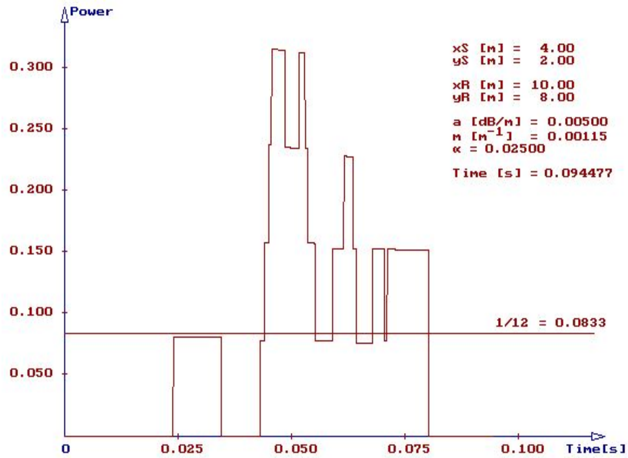

Fig.10. Sound power variation in the receiver's vicinity

4. CONCLUSIONS

A program written in C language was developed to implement the method described above. It was used to model the trajectories of the sound rays starting from a point sound source, placed inside a rectangular enclosure.

The algorithm takes into consideration the reduction of sound rays energy through absorption and attenuation, enabling the estimation of sound pressure inside the enclosure and also in a restricted area in the proximity of the receiver.

5. ACKNOWLEDGEMENTS

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Modelarea câmpului sonor într-o incintă folosind metoda geometrică "ray-tracing"

Rezumat: *În această lucrarea s-a folosit un program realizat în limbajul C utilizând metoda "ray–tracing" pentru un studiu al distribuţiei energiei sonore într-o incintă şi rezolvarea geometrică a problemei care cuprinde: definirea marginilor domeniului, fixarea poziţie sursei şi a receptorului. Au fost determinate poziţiile razelor sonore care pleacă din sursă, coordonatele punctelor de intersecţie cu obstacolele, direcţiile acestor raze după ce sunt reflectate, procesul fiind continuat până la parcurgerea de către fiecare rază a unui drum de o lungime predefinită. O altă etapă în acest studiu este determinarea absorbţiei datorate obstacolelor şi atenuării energiei undei sonore în mediu, lungimea maximă parcursă de raza sonoră – directă şi reflectată - determinând nivelul de presiune acustică.*

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