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ON THE FRONT VIEW SHAPE OF REAL WING

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Abstract: The front view ideal wing (for model aeroplane, airplane) has a parabola shape. Its practical realisation is made using straight lines, resulting V, U, Uoptimum, W, and Woptimum wings. The Uoptimum wing is close to the parabola shape. Its deviation from the parabola is of maximum 0.08h (h – height increasing between tip and root wing) at 0.5t (2t - wingspan, 0 height is at the root wing), and to V wing (the most resistant to the lateral shocks) of only 0.25h, in the same point. The U wing, having the extremities starting from the half of the wings, presents a 0.25h deviation from parabola ideal wing and to V wing of 0.5h in the same point (0.5t), being less resistant to lateral shocks. The W wing presents less deviation to the parabola, but against straight line is of 0.33h at 0.66t. The Woptimum wing presents the smallest deviation to the parabola of 0.06h and to the straight line d1 of 0.25h at 0.5t, being most close to the parabola and straight line d1 (equivalent wing). **Key words:** ideal wing, U wing, Uoptimum wing, W wing, Woptimum wing.

1. INTRODUCTION

The maintaining in the air of a heavier than the air object is realized with the help of lifting surfaces. Lifting surfaces are wings of different shapes, having different airfoil profiles.

The flight started with the help of a lifting surface, based on observations, not throwing of

a stone in the air. The lifting surface may insure the lifting power, to maintain in the air, when moving in the air, as long it is a relative movement to the air flow.

It knows, the ideal shape of a wing is a parabola seen from the front [1], Figure 1, and an ellipse top view seen. The practical achieving in space of such surface having



Fig. 1. B-52 Stratofortress. An air-to-air front view of a B-52G Stratofortress aircraft [1]. It is seen the parabola shape of the wing.

parabola and ellipse (at 90^{0}) is difficult to make. Obtaining of such surface is not all, checking it is difficult, deviations are hard to see, and maintaining the shape is also difficult. Repairing, results of different situations, to bring the spatial surface to initial shape, is difficult, also. To excess the mentioned difficulties, there are produced approximations of surface by replacement of curves with straight lines.

The idea of ideal wing approximation issued in a paper by Arghir for V and U wing in Der Hangflieger, Germany, 1981 [2], but for Uoptimum wing was developed latter [3].

In the present paper, it is treated the ideal wing approximation by straight lines for model airplane, which approximation is valid for airplane also. Approximation by straight line of ideal wing is treated for front view only, resulting: V, U, Uoptimum, W, and Woptimum wings. There are presented calculations for proper dimensions, together with some observations, deviations to parabola, and to V wing – the most resistant to the lateral shocks.

2. IDEAL WING APPROXIMATION FOR DIFFERENT WINGS

<u>V wing</u> [2] is made by two straight lines, d1, passing (0, 0) and (t, h), and, respectively, (0, 0) (-t, h), Figure 2. Parabola, p, the ideal shape of the wing is passing through the same points (-t, h), (0, 0), and (t, h), and its axes on Oy and equation:



Fig. 2. V wing [2]. Parabola, p, approximated by two straight lines, d1 and its left symmetric, d2.

(5)

 $y = (h/t^2) x^2$. (1)

The straight line, d1, passing through (0, 0) and (t, h) has the equation:

y = (h/t) x, (2) where: 2t is the wingspan and h – height increasing between tip and root wing.

Deviation from parabola, p, (1) and straight line, d1 (2) is given by:

$$\Delta a 1 = (2) (x) - (1) (x) = (h/t) x - (h/t2) x2.$$

(h/t) x - (h/t²) x². (3) In order to find the maximum value of eq.

(3), it is derived to x and zeroed: 14 + 1 = 1/4 $2(1/4^2) = -0$

 $d\Delta a 1 = h/t - 2(h/t^2) x = 0,$

resulting:

$$x1 = t/2.$$
 (4)
Root (4) introduced in (3) gives:

 $\Delta a1 \max = h/4.$

Solution (4), x1 = t/2, shows at this position, the deviation of parabola, p, (1) to the straight line, d1, (2) is at maximum (5), $\Delta a1max = h/4$.

 $\Delta a 1 \max (5)$ is inside parabola, to the curvature. If V wing fails with straight line, d1 (1) vertical, then the bending moment, during impact to the Earth, on the structure of mechanical strength (leading edge, main longeron, secondary longeron, trailing edge) is nil, while at the parabolic wing it is proportional to (5), $\Delta a 1 \max = h/4$, in place of (4), x1 = t/2, at the half of a wing. The value of bending moment is proportional to h - height increasing between tip and root wing. V wing resists best to shocks, mechanical strength structure being subjected to compression and the parabola wing is subjected to bending.

<u>U wing</u> [2] is the wing approximated by three straight lines: one is tangent to parabola in central part in (0, 0) and the other one tangent to parabola at the wing tips at (-t, h) and, respectively, (t, h), d2, continuing until cut the horizontal line, d0, Figure 3.



Fig. 3. U wing [2]. Parabola, p, approximated by three straight lines, one tangent in central part, d0, and two tangent at tips, d2.

Tangent to parabola p (1) has the expression: $y' = 2(h/t^2)x$, (6)which in (t, h) has the value of $y'(t) = 2(h/t^2)t = 2(h/t).$ (7)Let us consider the straight line: y = mx + n, (8) with the slope given by (7): y = 2(h/t) x + n, (9) passing through (t, h): h = 2(h/t) t + n, (10)resulting: h = 2h + n, (11)and n = - h (12)and equation (8) imposed becomes, for conditions: y = 2(h/t) x - h, (13)which intersects to y = 0, (14)and gives

$$0 = 2(h/t) x 2 - h, (15)$$

resulting

$$x^2 = t/2.$$
 (16)

Maximum deviation to parabola p(1) of the intersecting point between of horizontal line d0 (14) and tangent of tip d2 (13) is:

$$\Delta a^2 = (h/t^2)(t/2)^2 = h/4, \qquad (17)$$

identical to $\Delta a 1 \max (5)$, but opposite to it, opposite to parabola curvature, being in the external part of parabola.

Considering:

 $\Delta a 1 \max + \Delta a 2 = h/4 + h/4 = h/2$, (18) the deviation of the point (t/2, 0) to the straight line d1 (2) results a double bending moment in the junction between the line d0 (14) and line d2 (2) for the strength structure. The double bending moment makes U wing to brake easier than parabolic wing, when it falls with the line d1 (2) vertical or close to the vertical.

<u>Uoptimum wing</u> [3], as in Figure 4, is approximated by three straight lines: one is



Fig. 4. Uoptimum wing [3]. Parabola, p, approximated by three straight lines: one tangent in the central part, d0, and two passing thru tips, d3, as close as parabola, $\Delta a3 = \Delta a4$.

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tangent in the central part in (0, 0), do, and the other two, d3, as that from the right d3 is passing thru (t, h) and has deviation $\Delta a3$ and $\Delta a4$ to parabola, equal and, respectively, minimum, and the left line is symmetrical. Parabola p (1) and line (8) passing true (t, h):

$$h = mt + n,$$

and

$$n = h - mt$$
, and

$$y = mx + h - mt.$$
 (20)

Straight line d3 (20) cuts

$$y = 0$$
 (21)
in x3, as
 $0 = mx3 + h - mt$, (22)

where

$$mx3 = mt - h$$

and

 $x_3 = (mt - h)/m.$ (24)

The deviation $\Delta a3$ is the distance between the intersection of straight line d3 (20) and d0 (21) to the parabola p (1), at x3 distance (24)

$$\Delta a3 = (h/t^2) x3^2 = (h/t^2) (mt - h)^2/m^2.$$

 $(h/t^2) (mt - h)^2/m^2$. (25) And the deviation $\Delta a4$ is between parabola p (1) and straight line d3 (20), at x4:

 $\Delta a4 = (mx4 + h - mt) - (h/t^2) x4^2$, (26) which has to be maximum, so derivative to x4 should be nil:

 $d\Delta a 4/dx4 = m - 2(h/t^2) x4 =$ $m - 2(h/t^2) x4 = 0,$ (27)

where

 $m = 2(h/t^2) x4,$ (28)

which is the line slope assuring $\Delta a4max$. From (28) results

$$x4 = (m/2) (t^2/h)$$
 (29)

and maximum value of
$$\Delta a4max$$
 for x4 (29) is

$$\Delta a4max = (m^2/4) (t^2/h) = h - mt.$$
(30)

It is imposed the condition $\Delta a3 = \Delta a4max$

and thus

resulting

$$m^{4} - 4(h/t) m^{3} + 8(t/h)^{3}m - 4(h/t)^{4} = 0.$$
(33)

$$p = h/t$$
(34)
resulting

$$m^4 - 4m^3p + 8mp^3 - 4p^4 = 0.$$
 (35)

To remember, as in equation (35) **m** is the the slope of the straight line d3 (20). For fourth order equation (35) solving was used a computer program, but initially entered numeric value:

$$p = 0.1,$$

resulting:
$$m^{4} - 0.1 m^{3} + 8 0.001 m - 4 0.0001 =$$
$$0.$$
(36)
Numerically solved equation gives roots:
$$m1 = -0.141381, (a)$$
(37)
$$m2 = 0.058621, (b)$$
$$m3 = 0.141422, (c)$$
$$m4 = 0.341412, (d)$$

which are real, and it is accepted the value m3 (37c), the line d3 (13) cuts Ox axis between 0 and t/2. It is accepted m3 – the straight line slope being between 0.1 (slope of straight line d1 (2)) and 0.2 (slope of straight line d3 (13)).

For m3 (37c) x3 (24) become:

$$x3 = t(1 - p/m) = 0.1(1 - 0.1/0.141422) = 0.293t.$$
 (38)

Here $\triangle a3$ becomes:

$$\Delta a3 = (h/t^{2})(t - h/m)^{2} = h(1 - p/m)^{2} =$$
0.0857h
(39)

and

(19)

(01)

(23)

(31)

$$\Delta a4max = (m^2/4)(t^2/h) + h - mt = 0.0857h.$$
(40)

Deviation to straight line d1 (2) is:

$$\Delta a5 = (h/t) \ 0.293t = 0.293h, \tag{41}$$

corresponding to x3. Deviation $\Delta a5$ (41) is quite great, much less than ($\Delta a1max + \Delta a2$ (18), being still a weak point in wing strength structure. The fact the $\Delta a5$ is at x3, close to the wing root, makes the wing stronger, static moments of inertia of the structural strength being bigger (as a solid of equally strength under bending) for x3 than for x2.

<u>**W wing</u>** is made by four straight lines: line d2 (13) and line d4, passing (0, 0) and (t/2, h/4) right and left, their symmetrical by reflex in Oy axis, Figure 5.</u>

Let us consider straight line d4, passing (0, 0) and (t/2, h/4):

$$y = (1/2)(h/t)x,$$
 (42)

cutting straight line d2 (13) in

$$x5 = (2/3)t$$
 (43)

and

$$y_5 = (1/3)h.$$
 (44)

Deviation $\Delta a6$ between the intersection of line d2 (13) and line d4 (42), and parabola p (1)is:

$$\Delta a6 = (1/3)h - (h/t)((2/3)t)^2 = -(1/9)h.$$
(45)

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(50)



Fig. 5. W wing. Parabola, p, approximated by four straight lines, two tangent to tips, d2, and the other two passing (0, 0) and (t/2, h/4), d4, on the right side and symmetrically on the left side.

The minus sign indicates that the deviation is outside the parabola. The deviation $\Delta a7$ between the intersection of d2 (13) and d4 (42), and straight line d1 (V wing) is:

 $\Delta a7 = (h/t)(2/3)t - (1/3)h.$ (46)This deviation, $\Delta a7$, is the biggest deviation between W wing and straight line d1, V wing, being the weakest point at shocks.

Deviation between straight line d4 and parabola p (1) for x6 is:

 $\Delta a 8 = (1/2)(h/t)x - (h/t^2)x^2,$ (47)which derivative is calculated in relation to x:

$$d\Delta a8/dx6 = (1/2)(h/t) - 2(h/t^2)x$$
(48)
equals zero, resulting x6:

x6 = (1/4)t, (49)which replaced in (47) leads to value $\Delta a8max$: $\Delta a8max = (1/16)h.$

Woptimum wing is approximated by four straight lines: two of which are straight line d4, passing (0, 0) and (t/2, t/4), and straight line d5, passing (t/2, h/4) and (t, h) right side and symmetrically by Oy reflex left side, Figure 6.



Fig. 6. Woptimum wing. Parabola, p, approximated by four straight lines, d4 passing (0,0) and (t/2, h/4), and d5 passing (t/2, h/4) and (t, h) in the right and symmetrically in the left.

Let's consider straight line d5, according to the mentioned conditions:

$$y = (3/2)(h/t)x - (1/2)h.$$
 (51) res

Deviation $\Delta a9$ is between straight line d5 (51) and parabola p(1) for x7:

$$\Delta a9 = (3/2)(h/t)x - (1/2)h - (h/t^2)x^2, \quad (52)$$

the derivative becomes zero for:

$$x7 = (3/4)t.$$
 (53)

esulting in a maximum of
$$\Delta a9max$$
:
 $\Delta a9max = (1/16)h.$ (54)

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Deviation $\Delta a8 \max (50)$ appears in this case at x = (3/4) as $\Delta a9 \max = (1/16)h (54)$.

Be noted that straight lines d4 (42) and d5 (51) considered for parabola p (1) approximation have maximum deviation of (1/16)h to parabola, placed x = (1/4)t (x6) and (3/4)t (x7), and their intersection is at (1/4)t away from the straight line d1 (V wing), the weak point of wing strength, located in x2 (16), x2 = (1/2)t.

3. CONCLUSIONS

Ideal wing - sustentation element - has a parabolic shape. Practically, its realization can be made by straight lines, resulting V, U, Uoptimum, W, and Woptimum. Uoptimum wing is close to the parabola shape. Its deviation from the parabola is a maximum 0.08h (h - height increasing between tip and root wing) at 0.5t (2t is the wingspan, 0 is at root wing), and to V wing (most resistant to the shocks) is only 0.25h, in the same point. U wing, having the extremities starting at the half of the wings, presents deviation to the ideal parabola wing of 0.25h and to the V wing of 0.5h in the same point (0.5t), being less resistant to the lateral shocks. The W wing presents less deviation to the parabola, but against straight line d1 (V wing) of 0.33h, at 0.66t. Woptimum presents the smallest deviation to the parabola, of 0.06h, but to the straight line, deviation is of 0.25h at 0.5t, being most close to parabola, and straight line d1 (equivalent V wing)..

4. REFERENCES

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ASUPRA VEDERII DIN FAȚĂ A ARIPII REALE

Rezumat: Vederea din față a aripii ideale (aeromodel, aeroplan, aerodină) are o formă parabolică. Realizarea sa practică se face prin utilizarea liniilor drepte rezultând aripă V, U, Uoptim, W si Woptim. Aripa Uoptim este apropiată formei parabolei. Abaterea sa de la parabolă este de maxim 0,08h (h – supraînălțarea extremității aripii) la 0,5t (2t – anvergura aripii, înălțimea 0 este la centrul aripii), iar la aripa V (cea mai rezistentă la șocurile laterale) este de 0,25h, în același punct. Aripa U are extremitățile începând de la jumătatea aripii, prezintă o abatere de 0,25t de la parabola aripii ideale și față de aripa V de 0,5h în același punct (0,5t), fiind mai puțin rezistentă la șocuri laterale. Aripa W prezintă abateri mai mici față de parabolă, insă față de dreaptă de 0,33h la 0,66t. Aripa Woptim prezintă cea mai mică abatere față de parabolă de numai 0,06h, insă față de linia dreaptă d1 de 0,25h la 0,5t, fiind foarte apropiată parabolei și liniei drepte d1 (corespunzătoare aripii V).

Cuvinte cheie: aripă ideală, aripă V, aripă U, aripă Uoptim, aripă W, aripă Woptim.

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