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PARALLEL RECONFIGURABLE ROBOT WITH SIX DEGREES OF FREEDOM AND TWO GUIDING KINEMATIC CHAINS OF THE PLATFORM AND ITS VARIANTS WITH FIVE, FOUR, THREE AND TWO DEGREES OF FREEDOM

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Abstract: *The paper presents a proposed structure for a new parallel robot with six degrees of freedom and two guiding kinematic chains of the platform actuated by four linear and two rotational motors. The robot can be reconfigured in parallel structures with five, four, three and two degrees of freedom actuated from the base. These structures, along with the inverse and direct geometrical models for each of them are presented. The robot can be used as module in minimally invasive procedures as well as for complex assembly and laser cutting operations.*
Key words: *reconfigurable parallel robot, multiple DoF, design, geometrical models.*

1. INTRODUCTION

The need for robotic structures comes into focus in all areas where progress is constricted due to the limitation imposed by the human body in terms of speed, stamina, strength, precision, repeatability [17].

Industrial robotics has, in recent times, welcomed the development of a new generation of robots based on closed-loop kinematic chain mechanisms. Due to the fact that these structures are commonly symmetrical and have more than one kinematic chain linking the base plate to the end-effector, they have been named parallel robots [4].

Reconfigurable robots are structures that can change configuration in order to respond to the requirements of the human operator or the robot for which it operates [20]. The structures will be simulated and analyzed using computers in order to determine possible collisions between the robot elements.

Compared to the serial robot, which has only one kinematic chain (or arm), parallel structures have at least two kinematic chains linking the base to the end-effector. Another major difference between these two types of robots can be found in the way they function: the serial robot has a motor (or actuator) for each joint, where the parallel robot has most of the actuators situated on the base plate, employing passive joints and mechanical restraints in order to function properly [4].

Some of the advantages parallel structures have over serial ones are: higher stiffness, higher accelerations and speeds, higher precision, simple construction, smaller positioning errors, much better weight to load ratio. However, the workspace for the parallel robots is usually much smaller than for the serial ones [9].

These parallel structures are currently being used for complex operations in areas like biotechnology, microsurgery, micro assembly, electronics circuit testing and also for less complex ones such as packing. The latter requires less degrees of freedom (DoF) with more speed and repeatability, where the more complex operations require more DoF [6]-[14].

References [1]-[3], [5]-[16] and [18]-[19] illustrate, through the cooperation between the

Technical Universities of Cluj-Napoca and Braunschweig, the development of a series of new parallel robots and microrobots with multiple applications, like assembly, micro-assembly, industry of automobile components manufacturing.

Based on an existing structure already built and working (also a parallel robot) [16], a new, innovative parallel structure was proposed, with two kinematic chains linking the base to the end-effector. Using four linear actuators (electric motors) and two revolute actuators (also electric motors) and only passive, class five joints, the end-effector can achieve the six degrees of freedom imposed.

Using the backbone of the initial structure, a series of parallel robots with fewer degrees of freedom can be obtained (with five, four, three or two degrees of freedom respectively), some of them only with minor modifications to the actuator system and/or the passive joints. The passive joints normally used for such a parallel structure are either class five joints (revolute joints), class four joints (cardanic joints) or class three joints (spherical joints).

The development of parallel robots has been brought on also by product miniaturization trend and by the need for modular design [17].

A general view over the structural synthesis (inverse and direct model) of the proposed structures with fewer degrees of freedom (DoF) is presented in this paper.

2. STRUCTURAL OVERWIEV OF THE PARALLEL RECONFIGURABLE ROBOT

Starting with two parallel robots mounted on the same base, each having three degrees of freedom, a new parallel structure was proposed using only class five revolute joints [23]. The three degrees of freedom are achieved through three electric motors, two linear motors and a rotary one, giving each robot freedom of movement. All this, combined with the mechanical restrictions imposed by each kinematic link working with the chosen passive joints used, yield a six degree of freedom parallel structure. Figure 1 shows the basic structure of the robot, along with the coordinate reference systems and the annotations used in the theoretical calculations [21].

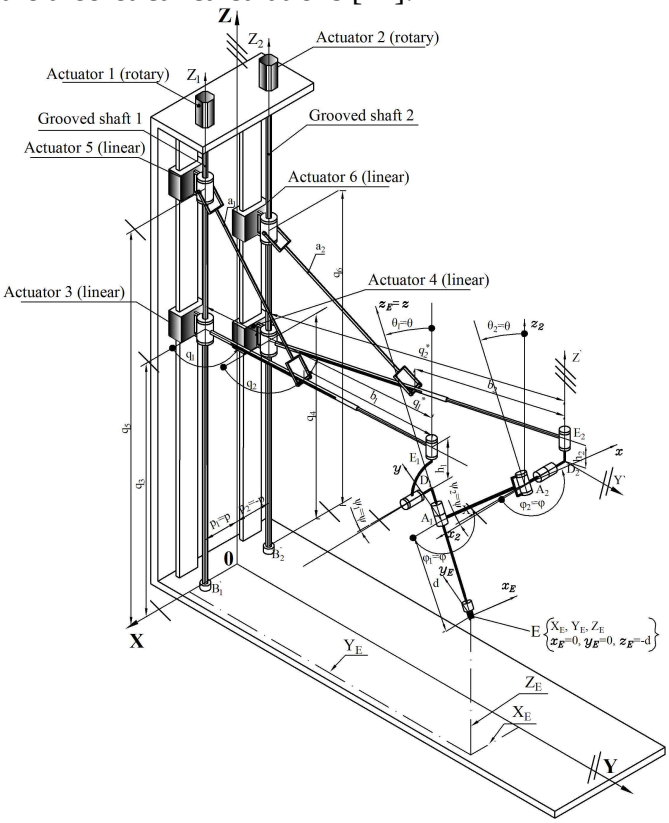


Fig. 1. Structural overview and reference coordinate systems for the six DoF parallel robot

The annotations that are used throughout the paper are as follows: M – mobility degree of the mechanism; F – mechanism family; N – number of mobile elements in the structure; C_i – number of "i" class joints.

By applying particular restrictions to the M=6 DoF parallel robot (Fig. 1) the other variants with M=5, M=4, M=3 and three versions of the M=2 DoF are obtained.

2.1 The inverse geometric model for the six DoF structure with two kinematic chains of the platform [21] *)

Table 1

Algorithm for the inverse geometrical model of the parallel mechanism with 6 DoF.

Given:	$X_E, Y_E, Z_E, \Psi, \theta, \varphi$ $a_1, a_2, b_1, b_2,$ $h_1, h_2, d, d_2,$ p, e_1, e_2
Unknown:	$q_i, i = 1,2,...,6$
Variables:	Solving equations:

x_E, y_E, z_E	$x_E = 0, y_E = 0, z_E = -d$
$x_{A_1}, y_{A_1}, z_{A_1}$	$x_{A_1} = 0, y_{A_1} = 0, z_{A_1} = 0$
$x_{A_2}, y_{A_2}, z_{A_2}$	$x_{A_2} = d_2, y_{A_2} = 0, z_{A_2} = 0$
$x_{D_1}, y_{D_1}, z_{D_1}$	$x_{D_1} = 0, y_{D_1} = 0, z_{D_1} = e_1$
p_1, p_2	$p_1 = p, p_2 = -p$
$c\alpha', ..., c\gamma'''$	$c\alpha' = c\psi c\varphi - s\psi c\theta s\varphi$; $c\alpha'' = -c\psi s\varphi - s\psi c\theta c\varphi$; $c\alpha''' = s\psi s\theta$ $c\beta' = s\psi c\varphi + c\psi c\theta s\varphi$; $c\beta'' = -s\psi s\varphi + c\psi c\theta c\varphi$; $c\beta''' = -c\psi s\theta$ $c\gamma' = s\theta s\varphi$; $c\gamma'' = s\theta c\varphi$; $c\gamma''' = c\theta$
$X_{A_1}, Y_{A_1}, Z_{A_1}$	$X_{A_1} = X_E + dc\alpha'''$ $Y_{A_1} = Y_E + dc\beta'''$ $Z_{A_1} = Z_E + dc\gamma'''$
$X_{A_2}, Y_{A_2}, Z_{A_2}$	$X_{A_2} = X_E + d_2 c\alpha' + dc\alpha'''$ $Y_{A_2} = Y_E + d_2 c\beta' + dc\beta'''$ $Z_{A_2} = Z_E + d_2 c\gamma' + dc\gamma'''$
$X_{D_1}, Y_{D_1}, Z_{D_1}$	$X_{D_1} = X_{A_1} + e_1 c\alpha'''$ $Y_{D_1} = Y_{A_1} + e_1 c\beta'''$ $Z_{D_1} = Z_E + e_1 c\gamma'''$
u_1, w_1	$u_1 = sq_1 = \frac{Y_{D_1}}{q_1^*}, w_1 = cq_1 = \frac{X_{D_1}}{q_1^*}$
q_1	$q_1 = \text{atan2}(u_1, w_1)$
q_3	$q_3 = Z_{D_1} + h_1$
q_1^*	$q_1^* = \sqrt{(X_{D_1} - p_1)^2 + Y_{D_1}^2}$
q_5	$q_5 = q_3 + \sqrt{a_1^2 - (q_1^* - b_1)^2}$
$\psi_2, \theta_2, \varphi_2$	$\psi_2 = \psi, \theta_2 = \theta, \varphi_2 = \varphi$
$X_{D_2}, Y_{D_2}, Z_{D_2}$	$X_{D_2} = X_{A_2} - e_2 c\psi$ $Y_{D_2} = Y_{A_2} - e_2 s\psi_2$ $Z_{D_2} = Z_{A_2}$
q_2^*	$q_2^* = \sqrt{(X_{D_2} - p_2)^2 + Y_{D_2}^2}$
u_2, w_2	$u_2 = sq_2 = \frac{Y_{D_2}}{q_2^*}, w_2 = cq_2 = \frac{X_{D_2} - p_2}{q_2^*}$
q_2	$q_2 = \text{atan2}(Y_{D_2}, X_{D_2} - p_2)$ or $q_2 = \text{atan2}(u_2, w_2)$
q_4	$q_4 = Z_{D_2} + h_2$
q_6	$q_6 = q_4 + \sqrt{a_2^2 - (q_2^* - b_2)^2}$

*) – please refer to the letter "c" as cosine and to the letter "s" as sine.

2.2 The direct geometric model for the six DoF structure with two kinematic chains of the platform [21]

Table 2

Algorithm for the inverse geometrical model of the parallel mechanism with 6 DoF.

Given:	$q_i, i = 1,2,...,6$ $a_1, a_2, b_1, b_2, h_1, h_2, d, d_2, p, e_1, e_2$
Unknown:	$X_E, Y_E, Z_E, \Psi, \theta, \varphi$
Variables:	Solving equations:
x_E, y_E, z_E	$x_E = 0, y_E = 0, z_E = -d$
$x_{A_1}, y_{A_1}, z_{A_1}$	$x_{A_1} = 0, y_{A_1} = 0, z_{A_1} = 0$
$x_{A_2}, y_{A_2}, z_{A_2}$	$x_{A_2} = d_2, y_{A_2} = 0, z_{A_2} = 0$
$x_{D_1}, y_{D_1}, z_{D_1}$	$x_{D_1} = 0, y_{D_1} = 0, z_{D_1} = e_1$
p_1, p_2	$p_1 = p, p_2 = -p$
$X_{D_i}, Y_{D_i}, Z_{D_i}$ $i = 1,2$	$\begin{cases} X_{D_i} = \\ = p_i + \left[b_i + \sqrt{a_i^2 - (q_{i+4} - q_{i+2})^2} \right] cq_i \\ Y_{D_i} = \\ = \left[b_i + \sqrt{a_i^2 - (q_{i+4} - q_{i+2})^2} \right] sq_i \\ Z_{D_i} = \\ = q_{i+2} - h_i \end{cases}$ $i = 1,2$
ψ, θ, φ	$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)})F(X^{(p)}),$ $p = 0,1,2,...$ where $X = \begin{bmatrix} \psi \\ \theta \\ \varphi \end{bmatrix}, F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \end{bmatrix},$ $W = \begin{bmatrix} \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \gamma} \\ \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \gamma} \\ \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \gamma} \end{bmatrix}$ $F(X) =$ $= \begin{bmatrix} F_1(\psi, \theta, \varphi) \equiv \\ d_2(c\psi c\varphi - s\psi c\theta s\varphi) + ds\psi s\theta + \\ e_2 c\psi - X_{D_2} - (e_1 + d)s\psi s\theta + X_{D_1} \\ \hline F_2(\psi, \theta, \varphi) \equiv \\ d_2(s\psi c\varphi + c\psi c\theta s\varphi) - dc\psi s\theta - \\ e_2 s\psi - Y_{D_2} + (e_1 + d)c\psi s\theta + Y_{D_1} \\ \hline F_3(\psi, \theta, \varphi) \equiv \\ d_2 s\theta s\varphi + dc\theta - Z_{D_2} - \\ (e_1 + d)c\theta + Z_{D_1} \end{bmatrix}$

X_E, Y_E, Z_E	$\begin{cases} X_E = X_{D_1} - (e_1 + d)s\psi s\theta \\ Y_E = Y_{D_1} + (e_1 + d)c\psi s\theta \\ Z_E = Z_{D_1} - (e_1 + d)c\theta \end{cases}$

2.3 Parallel robot with M=5 DoF (Fig. 2)

For this particular case only the kinematic scheme and some info regarding the algorithm tables for the inverse and direct geometrical models are presented. Reference [22] further elaborates on the matter.

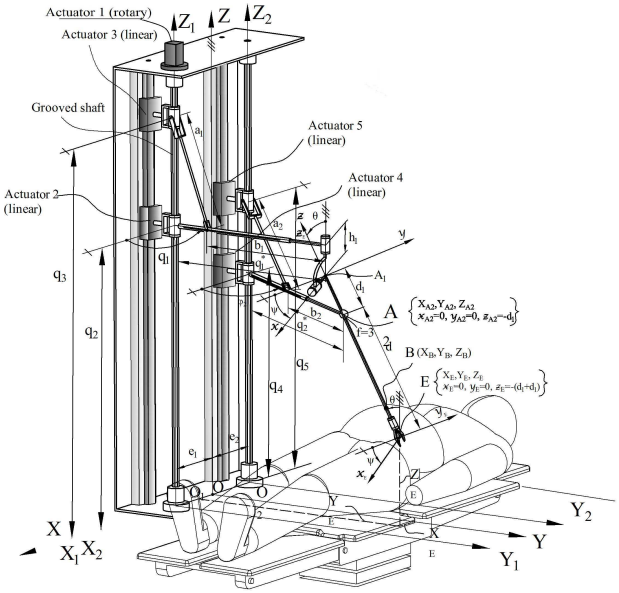


Fig. 2. Structural overview and reference coordinate systems for the five DoF parallel robot variant

2.3.1 Inverse geometric algorithm

For the inverse geometric problem of the parallel mechanism, the position and orientation coordinates X_E, Y_E, Z_E, ψ, θ are given and the coordinates of the active joints q_1, q_2, q_3, q_4, q_5 are unknown. In this case the problem consists in solving a system of five equations with five unknowns [22].

Table 3

Algorithm for the inverse geometrical model of the parallel mechanism with 5 DoF.

Given:	
X_E, Y_E, Z_E, ψ, θ, φ=0;	
h_1 [m]	d [m] e_2 [m]
b_1 [m]	b_2 [m] d_1 [m]
e_1 [m]	a_1 [m] a_2 [m]
Unknown:	
q_1, q_2, q_3, q_4, q_5	
Variables	Solving equations
e_1, e_2	$e_1 = e$, $e_2 = -e$
x_{A_1}, y_{A_1}, z_{A_1}	$x_{A_1} = 0$, $y_{A_1} = 0$, $z_{A_1} = 0$
x_{A_2}, y_{A_2}, z_{A_2}	$x_{A_2} = 0$, $y_{A_2} = 0$, $z_{A_2} = 0$
$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$C\alpha' = C\psi$ | $C\alpha'' = -S\psi C\theta$ | $C\alpha''' = S\psi S\theta$ $C\beta' = S\psi$ | $C\beta'' = C\psi S\theta$ | $C\beta''' = -C\psi S\theta$ $C\gamma' = 0$ | $C\gamma'' = S\theta$ | $C\gamma''' = C\theta$
q_1^*	$q_1^* = +\sqrt{(X_E - z_E \cdot C\alpha''' - e_1)^2 + (Y_E - z_E \cdot C\beta$
u_1, w_1	$u_1 = Sq_1 = \dfrac{Y_E - z_E \cdot C\beta'''}{q_1^*}$ $w_1 = Cq_1 = \dfrac{X_E - z_E \cdot C\alpha''' - e_1}{q_1^*}$
q_1	$q_1 = \text{atan2}(Y_E - z_E \cdot C\beta''', X_E - z_E \cdot C\alpha''' - e_1)$
q_2	$q_2 = h_1 + Z_E - z_E \cdot C\gamma'''$
q_3	$q_3 = q_2 + \sqrt{a_1^2 - (q_1^* - b_1)^2}$
x_{A_2}, Y_{A_2}, Z_{A_2}	$X_{A_2} = X_E + d \cdot C\alpha'''$ $Y_{A_2} = Y_E + d \cdot C\beta'''$ $Z_{A_2} = Z_E + d \cdot C\gamma'''$
q_4	$q_4 = Z_E + d \cdot C\theta$
q_2^*	$q_2^* = +\sqrt{(X_{A_2} - e_2)^2 + Y_{A_2}}$
q_5	$q_5 = q_4 + \sqrt{a_2^2 - \left[-b_2 + \sqrt{(X_{A_2} - e_2)^2 + Y_{A_2}}\right.}$

2.3.2 Direct geometric algorithm

The objective of the direct geometric model is to define a mapping from the known set of the actuated joints coordinates to the unknown position of the laparoscope. For the forward geometric problem of the parallel mechanism, coordinates of the active joints q_1, q_2, q_3, q_4, q_5 are known, and the position coordinates and orientation angles X_E, Y_E, Z_E, ψ, θ of the end-effector are unknown [22].

Table 4

Algorithm for the direct geometrical model of the parallel mechanism with 5 DoF.

Given:	
q_1, q_2, q_3, q_4, q_5	
h_1 [m]	d [m] e_2 [m]
b_1 [m]	b_2 [m] d_1 [m]
e_1 [m]	a_1 [m] a_2 [m]
Unknown:	
X_E, Y_E, Z_E, ψ, θ, φ=0;	

Variables	Solving equations
$e_1,\ e_2$	$e_1 = e,\ e_2 = -e$
$x_{A_1},\ y_{A_1},\ z_{A_1}$	$x_{A_1} = 0,\ y_{A_1} = 0,\ z_{A_1} = 0$
$x_{A_2},\ y_{A_2},\ z_{A_2}$	$x_{A_2} = 0,\ y_{A_2} = 0,\ z_{A_2} = -d_1$
$x_E,\ y_E,\ z_E$	$x_E = 0,\ y_E = 0,\ z_E = (d_1 + d)$
q_1^*	$q_1^* = b_1 + \sqrt{a_1^2 - (q_3 - q_2)^2}$
$X_{A_1},\ Y_{A_1},\ Z_{A_1}$	$X_{A_1} = e_1 + q_1^* \cdot Cq_1$ $Y_{A_1} = q_1^* \cdot Sq_1$ $Z_{A_1} = q_2 - h_1$
u_1, w_1	$u_1 = S\theta = \dfrac{\sqrt{d_1^2 - (Z_{A_1} - q_4)^2}}{d_1}$ $w_1 = C\theta = \dfrac{Z_{A_1} - q_4}{d_1}$
θ	$\theta = a\tan 2(u_1, w_1)$
Z_E	$Z_E = q_4 - d \cdot C\theta$
a, b, c	$a = 2d_1 \cdot u \cdot Y_{A_1}$ $b = -2d_1 \cdot u\left(X_{A_1} - e_2\right)$ $c = \left(q_2^*\right)^2 - \left(X_{A_1} - e_2\right)^2 - Y_{A_1}^2 - d_1^2 \cdot u^2$
ψ	$\psi = \text{atan2}\left(c,\ \pm\sqrt{a^2 + b^2 - c^2}\right) - \text{atan2}(a, b)$
$X_E,\ Y_E$	$X_E = X_{A_1} - (d_1 + d) \cdot u \cdot S\psi$ $Y_E = Y_{A_1} + (d_1 + d) \cdot u \cdot C\psi$
$u_2,\ w_2$	$u_2 = S\varphi_2 = \dfrac{Y_{A_2}}{q_2^*} \quad w_2 = C\varphi_2 = \dfrac{X_{A_2} - e_2}{q_2^*}$
φ_2	$\varphi_2 = a\tan 2\left(Y_{A_2},\ X_{A_2} - e_2\right)$

2.4 Parallel robot with M=4 DoF with the end-effector that moves in a planar translation (Fig. 3)

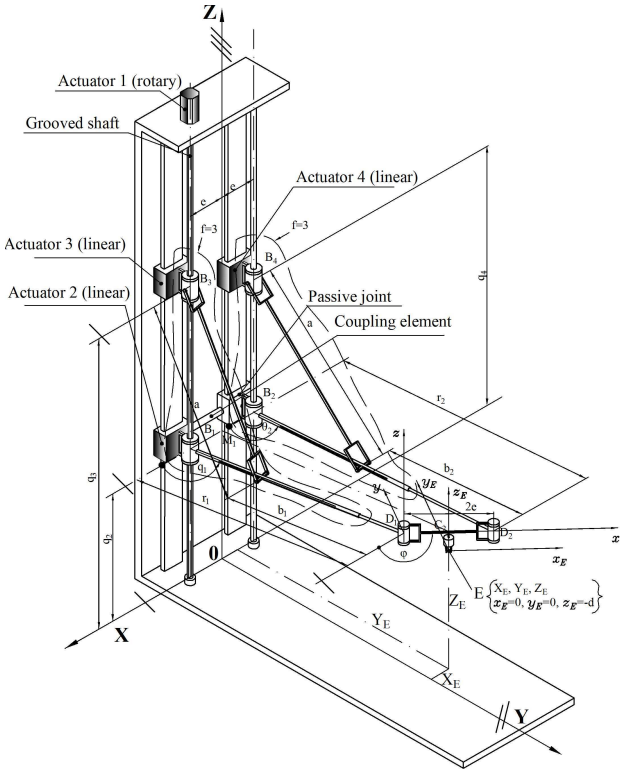


Fig. 3. Structural overview and reference coordinate systems for the four DoF parallel robot variant

For this particular case the inverse geometrical model is presented below.

2.4.1 The inverse geometric model for the four DoF variant

Using the given structure shown in figure 3 and the theoretical coordinates X_E, Y_E, Z_E, φ of the end-effector, a generalized expression of the coordinates for $q_i (i = 1,2,...,4)$ is to be established.

$$\begin{cases} X_E = e + r_1 cq_1 - ec\varphi \\ Y_E = r_1 sq_1 - es\varphi \\ Z_E = q_2 - d \end{cases} \tag{1}$$

where

$$r_1 = b + \sqrt{a - (q_3 - q_2)^2} \tag{2}$$

The X, Y and Z coordinates of the end-effector can also be written as follows:

$$\begin{cases} X_E = -e + r_2 c\theta_2 + ec\varphi \\ Y_E = r_2 s\theta_2 - es\varphi \\ Z_E = q_2 - d \end{cases} \tag{3}$$

where

$$r_2 = b + \sqrt{a - (q_4 - q_2)^2} \tag{4}$$

Using relations (1) through (4), the q_2 value (or parameter) can be obtained:

$$\begin{cases} r_1 c q_1 = X_E - e + ec\varphi \\ r_1 s q_1 = Y_E + es\varphi \\ q_2 = Z_E + d \end{cases} \quad (5)$$

$$q_2 = Z_E + d \quad (6)$$

Using the first two relations from equation (5), the r_1 parameter is obtained:

$$r_1 = \sqrt{(X_E - e + ec\varphi)^2 + (Y_E - es\varphi)^2} \quad (7)$$

$$sq_1 = \frac{Y_E + es\varphi}{r_1}, cq_1 = \frac{X_E - e + ec\varphi}{r_1} \quad (8)$$

$$q_1 = \arctan 2(Y_E + es\varphi, X_E - e - ec\varphi) \quad (9)$$

Using the relations in system (3) the following relations are deducted:

$$\begin{cases} r_2 c\theta_2 = X_E + e - ec\varphi \\ r_2 s\theta_2 = Y_E + es\varphi \\ q_2 = Z_E + d \end{cases} \quad (10)$$

The solving equation for r_2 is obtained from the first two relations of equation (10):

$$r_2 = \sqrt{(X_E + e - ec\varphi)^2 + (Y_E + es\varphi)^2} \quad (11)$$

$$s\theta_2 = \frac{Y_E + es\varphi}{r_2}, c\theta_2 = \frac{X_E + e - ec\varphi}{r_2} \quad (12)$$

$$\theta_2 = \arctan 2(Y_E + es\varphi, X_E + e - ec\varphi) \quad (13)$$

Using relation (2) the q_3 parameter is deducted:

$$\begin{aligned} (r_2 - b)^2 &= a^2 - (q_4 - q_2)^2 \\ (q_4 - q_2)^2 &= a^2 - (r_2 - b)^2 \\ q_4 &= q_2 + \sqrt{a^2 - (r_2 - b)^2} \end{aligned} \quad (14)$$

Based on the relations (1) through (14) the algorithm for the inverse geometrical model of the parallel mechanism with 4 DoF can be established.

Table 5

Algorithm for the inverse geometrical model of the parallel mechanism with 4 DoF.

Given:	X_E, Y_E, Z_E, φ a, b, e, d
Unknown:	$q_i, i = 1,2,...,4$
Variables:	Solving equations:
r_1, r_2	$r_1 = \sqrt{(X_E - e + ec\varphi)^2 + (Y_E - es\varphi)^2}$ $r_2 = \sqrt{(X_E + e - ec\varphi)^2 + (Y_E + es\varphi)^2}$
q_1	$q_1 = \arctan 2(Y_E + es\varphi, X_E - e - ec\varphi)$
θ_2	$\theta_2 = \arctan 2(Y_E + es\varphi, X_E + e - ec\varphi)$
q_2	$q_2 = Z_E + d$
q_3	$q_3 = q_2 + \sqrt{a^2 - (r_1 - b)^2}$
q_4	$q_4 = q_2 + \sqrt{a^2 - (r_2 - b)^2}$

2.5 Parallel robot with M=3 DoF and continuous orientation of the end-effector (Fig. 4)

Based on Fig. 4 both the inverse and direct geometric models can be deducted.

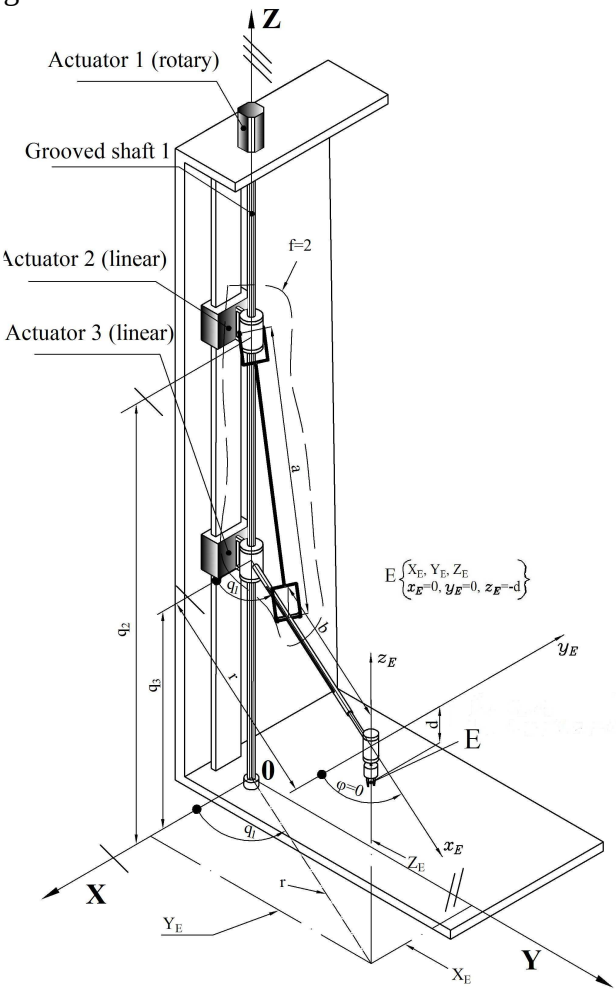


Fig. 4. Structural overview and reference coordinate systems for the three DoF parallel robot variant

2.5.1 The direct geometric model for the three DoF variant

The objective of the direct geometric model is to define a mapping from the known set of the actuated joints coordinates to the unknown position of the laparoscope. For the forward geometric problem of the parallel mechanism, coordinates of the active joints q_1, q_2, q_3 along with the a, b and d values are known, and the

position coordinates and orientation angles X_E, Y_E, Z_E of the end-effector are unknown.

$$X_E = rcq_1, Y_E = rsq_1, Z_E = q_3 - d \quad (15)$$

where

$$r = b + \sqrt{a^2 - (q_2 - q_3)^2} \quad (16)$$

Table 6

Algorithm for the direct geometrical model of the parallel mechanism with 3 DoF.

Given:	$q_i, i = 1,2,3$ a, b, d
Unknown:	X_E, Y_E, Z_E
Variables:	Solving equations:
r	$r = b + \sqrt{a^2 - (q_2 - q_3)^2}$
X_E, Y_E, Z_E	$X_E = rcq_1, Y_E = rsq_1, Z_E = q_3 - d$

2.5.2 The inverse geometric model for the three DoF variant

Given: X_E, Y_E, Z_E, a, b, d

Unknown: q_1, q_2, q_3

Using the first two equations from relation (15), the value for r is deducted:

$$r = \sqrt{X_E^2 + Y_E^2} \quad (17)$$

$$sq_1 = \frac{Y_E}{r}, cq_1 = \frac{X_E}{r}, q_3 = Z_E + d \quad (18)$$

$$q_1 = \arctan 2(Y_E, X_E) \quad (19)$$

$$q_2 = q_3 + \sqrt{a^2 - (r - b)^2} \quad (20)$$

Table 7

Algorithm for the inverse geometrical model of the parallel mechanism with 3 DoF.

Given:	X_E, Y_E, Z_E a, b, d
Unknown:	$q_i, i = 1,2,3$
Variables:	Solving equations:
r	$r = \sqrt{X_E^2 + Y_E^2}$
q_1	$q_1 = \arctan 2(Y_E, X_E)$
q_3	$q_3 = Z_E + d$
q_2	$q_2 = q_3 + \sqrt{a^2 - (r - b)^2}$

2.6 Parallel robot with M=2 DoF and variable orientation of the end-effector – ver. 1 (Fig. 5)

The variant for the parallel robot with M=2 DoF is presented in Fig. 5. It has only 2 linear actuators.

The proposed mechanism has N=2 mobile elements, one class 5 passive joint $(C_5 = 1)$ of the f=1 family, two class 4 joints $(C_4 = 2)$ highlighted in Fig. 5 with the dotted line of the f=2 family.

The whole mechanism is considered to be of the F=3 family, since none of its elements can move along the OZ axis or revolve around the OX or OY axis.

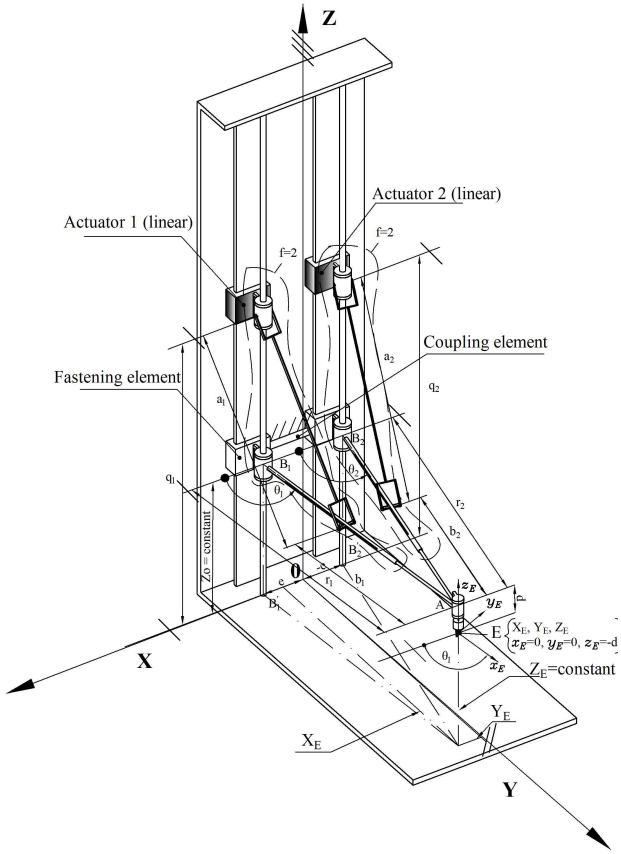


Fig. 5. Structural overview and reference coordinate systems for the two DoF parallel robot variant – ver. 1

For

$$F = 3, N = 2, C_5 = 1, C_4 = 2 \quad (21)$$

$$M = 3N - 2C_5 - C_4 = \\ = 3 \times 2 - 2 \times 1 - 2 = 2 \quad (22)$$

2.6.1 The direct geometric model for the two DoF variant – ver. 1

In the case of the direct geometric model the generalized coordinates q_1, q_2 are given and the generalized coordinates X_E, Y_E of the end-effector are unknown.

Based on Fig. 5 the following equations can be deducted:

$$\begin{cases} r_1 = b + \sqrt{a^2 - (q_1 - Z_0)^2} \\ r_2 = b + \sqrt{a^2 - (q_2 - Z_0)^2} \end{cases} \quad (23)$$

$$\begin{cases} X_E - X_{B_1} = r_1 c\theta_1 \\ Y_E - Y_{B_1} = r_1 s\theta_1 \end{cases} \quad (24)$$

$$\begin{cases} X_E - X_{B_2} = r_2 c\theta_2 \\ Y_E - Y_{B_2} = r_2 s\theta_2 \end{cases} \quad (25)$$

Using relations (24) and (25) following equations are obtained:

$$\begin{cases} (X_E - X_{B_1})^2 + (Y_E - Y_{B_1})^2 = r_1^2 \\ (X_E - X_{B_2})^2 + (Y_E - Y_{B_2})^2 = r_2^2 \end{cases} \quad (26)$$

With

$$\begin{cases} X_{B_1} = e, Y_{B_1} = 0 \\ X_{B_2} = e, Y_{B_2} = 0 \end{cases} \quad (27)$$

relations (26) become:

$$\begin{cases} (X_E - e)^2 + Y_E^2 = r_1^2 \\ (X_E + e)^2 + Y_E^2 = r_2^2 \end{cases} \quad (28)$$

$$\begin{cases} X_E^2 - 2eX_E + e^2 + Y_E^2 = r_1^2 \\ X_E^2 + 2eX_E + e^2 + Y_E^2 = r_2^2 \end{cases} \quad (29)$$

Deducting the two relations, equation (30) is obtained:

$$4eX_E = r_2^2 - r_1^2 \quad (30)$$

$$X_E = \frac{r_2^2 - r_1^2}{4e} \quad (31)$$

Using the first relation from system (29), coordinate Y_E:

$$Y_E = \sqrt{r_1^2 - (X_E - e)^2} \quad (32)$$

Coordinate Z_E for the E point based on Fig. 5 will be:

$$Z_E = Z_0 - d \text{ (constant)} \quad (33)$$

Table 8

Algorithm for the direct geometrical model of the parallel mechanism with 2 DoF – ver. 1.

Given:	$q_i, i = 1,2$ a, b, d, e $Z_E = Z_0 (ct)$
Unknown:	X_E, Y_E
Variables:	Solving equations:
r_1, r_2	$\begin{cases} r_1 = b + \sqrt{a^2 - (q_1 - Z_0)^2} \\ r_2 = b + \sqrt{a^2 - (q_2 - Z_0)^2} \end{cases}$
X_E, Y_E	$X_E = \frac{r_2^2 - r_1^2}{4e}$ $Y_E = \sqrt{r_1^2 - (X_E - e)^2}$

2.6.2 The inverse geometric model for the two DoF variant – ver. 1

In the case of the inverse geometric model we are given the coordinates of the end-effector and the generalized coordinates of the robot are unknown.

The problem is solved by determining the values of r_1, r_2 using relations (28):

$$\begin{cases} r_1 = \sqrt{(X_E - e)^2 + Y_E^2} \\ r_2 = \sqrt{(X_E + e)^2 + Y_E^2} \end{cases} \quad (34)$$

$$\begin{cases} a^2 - (q_1 - Z_0)^2 = (r_1 - b)^2 \\ a^2 - (q_2 - Z_0)^2 = (r_2 - b)^2 \end{cases}$$

$$\begin{cases} (q_1 - Z_0)^2 = a^2 - (r_1 - b)^2 \\ (q_2 - Z_0)^2 = a^2 - (r_2 - b)^2 \end{cases} \quad (35)$$

$$q_1 = Z_0 + \sqrt{a^2 - (r_1 - b)^2}$$

$$q_2 = Z_0 + \sqrt{a^2 - (r_2 - b)^2}$$

Table 9

Algorithm for the inverse geometrical model of the parallel mechanism with 2 DoF – ver. 1.

Given:	X_E, Y_E a, b, d, e $Z_E = Z_0 (ct)$
Unknown:	$q_i, i = 1,2$
Variables:	Solving equations:
r_1, r_2	$\begin{cases} r_1 = \sqrt{(X_E - e)^2 + Y_E^2} \\ r_2 = \sqrt{(X_E + e)^2 + Y_E^2} \end{cases}$
q_1	$q_1 = Z_0 + \sqrt{a^2 - (r_1 - b)^2}$
q_2	$q_2 = Z_0 + \sqrt{a^2 - (r_2 - b)^2}$

2.7 Parallel robot with M=2 DoF – ver. 2 – with the end-effector that moves only in an horizontal plane (Fig. 6)

The M=2 DoF mechanism is obtained by imposing two other restrictions to the M=4 DoF mechanism (Fig. 3):

$$q_3 = q_4$$
$$q_2 = Z_0 = const \quad (36)$$

In this particular case, the mechanism plane is fixed at the Z_0 = constant height.

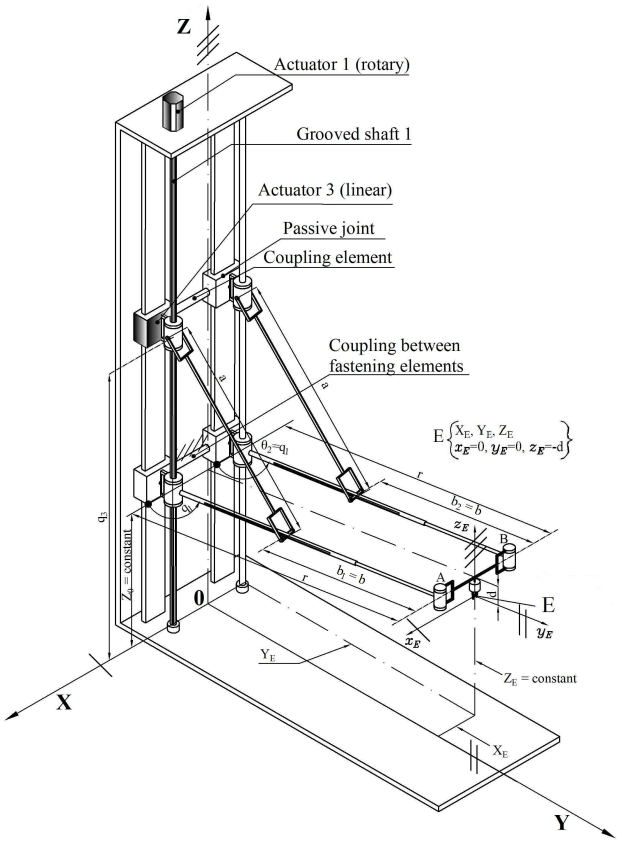


Fig. 6. Structural overview and reference coordinate systems for the two DoF parallel robot – ver. 2

2.7.1 The direct geometric model for the 2 DoF variant – ver. 2

Table 10

Algorithm for the direct geometrical model of the parallel mechanism with 2 DoF – ver. 2.

Given:	$q_i, i = 1,2$ a, b, d, e $Z_0(ct)$
Unknown:	X_E, Y_E
Variables:	Solving equations:
r	$r = b + \sqrt{a^2 - (q_2 - Z_0)^2}$
X_E, Y_E, Z_E	$X_E = rcq_1$ $Y_E = rsq_1$ $Z_E = Z_0 - d$ (constant)

2.7.2 The inverse geometric model for the 2 DoF variant – ver. 2

Using relations of the six DoF parallel structure geometric model, following equations are deducted:

$$r = \sqrt{X_E^2 + Y_E^2} \quad (37)$$
$$sq_1 = \frac{Y_E}{r}, cq_1 = \frac{X_E}{r} \quad (38)$$
$$q_1 = \arctan 2(Y_E, X_E) \quad (39)$$
$$q_2 = Z_0 + \sqrt{a^2 - (r-b)^2} \quad (40)$$

Based on relations (37) through (40), the algorithm for the inverse geometric model can be established.

Table 11

Algorithm for the inverse geometrical model of the parallel mechanism with 2 DoF – ver. 2.

Given:	$q_i, i = 1,2$ a, b, d, e $Z_E = Z_0 - d(ct)$ X_E, Y_E
Unknown:	$q_i, i = 1,2$
Variables:	Solving equations:
r	$r = \sqrt{X_E^2 + Y_E^2}$
q_1	$q_1 = \arctan 2(Y_E, X_E)$
q_2	$q_2 = Z_0 + \sqrt{a^2 - (r-b)^2}$

2.8 Parallel robot with M=2 DoF – ver. 3 – with variable orientation of the end-effector (Fig. 7)

The mobility degree of the structure is:

$$M = 3N - C_4 = 3 \times 1 - 1 = 2 \quad (41)$$

Based on Fig. 7 the geometric models can be established.

2.8.1 The direct geometric model for the 2 DoF variant – ver. 3

$$\begin{cases} X_E = r_A cq_1 \\ Y_E = r_A sq_1 \\ Z_E = Z_0 - d \end{cases} \quad (42)$$

where

$$r_A = b + \sqrt{a^2 - (q_2 - Z_0)^2} \quad (43)$$
$$\varphi = q_1 \quad (44)$$

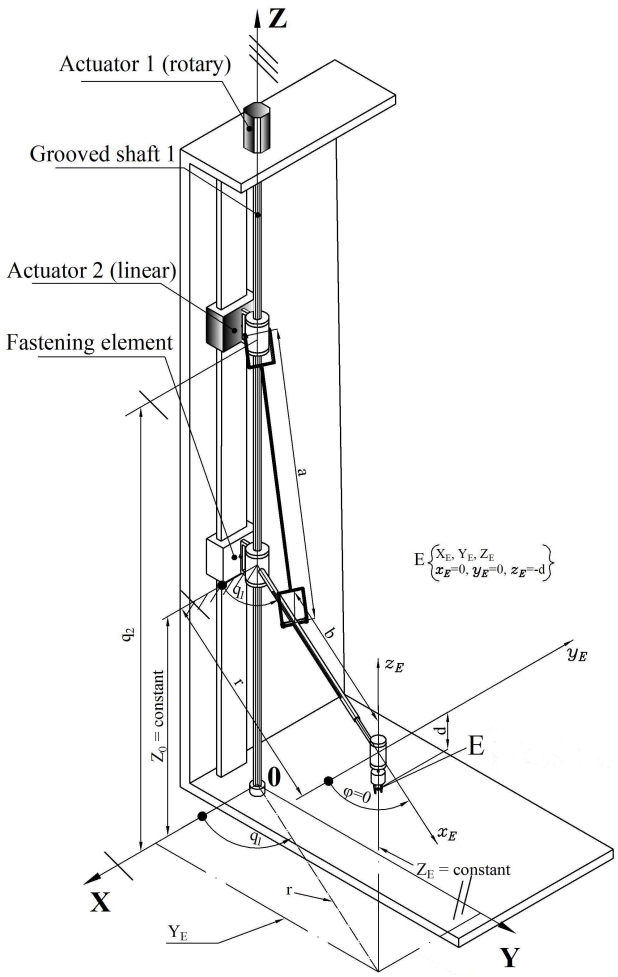


Fig. 7. Structural overview and reference coordinate systems for the two DoF parallel robot – ver. 3

Table 12

Algorithm for the direct geometrical model of the parallel mechanism with 2 DoF – ver. 3.

Given:	$q_i, i=1,2$ a,b,d,e $Z_0(ct)$
Unknown:	X_E, Y_E, Z_E
Variables:	Solving equations:
r_A	$r_A = b + \sqrt{a^2 - (q_2 - Z_0)^2}$
X_E, Y_E, Z_E	$\begin{cases} X_E = r_A c q_1 \\ Y_E = r_A s q_1 \\ Z_E = Z_0 - d \end{cases}, \varphi = q_1$

2.8.2 The inverse geometric model for the 2 DoF variant – ver. 3

Table 13

Algorithm for the inverse geometric model of the parallel mechanism with 2 DoF – ver. 3.

Given:	$X_E, Y_E, Z_0 = ct$ a,b,d,e
Unknown:	$q_i, i=1,2, \varphi$
Variables:	Solving equations:
r, Z_E	$r = \sqrt{X_E^2 + Y_E^2}$, $Z_E = Z_0 - d$
q_1	$q_1 = \arctan 2(Y_E, X_E), \varphi = q_1$
q_2	$q_2 = Z_0 + \sqrt{a^2 - (r-b)^2}$

3. CONCLUSIONS

This paper presents the variants of a six degree of freedom reconfigurable parallel robot structure that can be used as a module in minimal invasive surgery or for industrial purposes such as welding or assembling work [11].

An overview of the inverse and direct geometrical model for each of them is also presented, along with the complete algorithm.

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ROBOT PARALEL RECONFIGURABIL CU ŞASE GRADE DE LIBERTATE ŞI DOUĂ LANȚURI CINEMATICE DE GHIDARE A PLATFORMEI ŞI VARIANTELE SALE CU CINCI, PATRU, TREI ŞI DOUĂ GRADE DE LIBERTATE

Abstract: *Lucrarea prezintă o structură cinematică propusă pentru un nou robot paralel cu șase grade de libertate şi două lanțuri cinematice de ghidare a platformei acționate de patru motoare liniare şi două rotative. Robotul poate fi reconfigurat în structuri paralele cu cinci, patru, trei şi două grade de libertate acționate de la bază. Aceste structuri, împreună cu modelele geometrice corespunzătoare fiecăreia, sunt prezentate în lucrare. Robotul poate fi folosit ca şi modul în chirurgia minim invazivă, precum şi pentru operațiuni industriale precum asamblări complexe şi tăiere cu laser.*

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