TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 54, Issue I, 2011

VARIANTS OF A NEW RECONFIGURABLE PARALLEL ROBOT WITH SIX, FIVE, FOUR,THREE AND TWO DEGREES OF FREEDOM

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Abstract: *The paper presents a new parallel reconfigurable robot with six degrees of freedom and three guiding kinematic chains of the platform. The robot uses six linear motors to perform movements in the workspace Depending on the area where it will be used ,the initial robot can be reconfigured easily and quickly in one with five, four, three and two degrees of freedom. The algorithms for the determination of the inverse and direct geometrical models for some structures are presented.* ***Key words:*** *reconfiguration, parallel robot, precision, repeatability, degrees of freedom, end-effector, linear active joint.*

1. INTRODUCTION

Robotic system have been developed in every field where a further progress was constricted due the human limitation in terms of speed, precision, fatigue, repeatability, strength,etc. Paralel robots have a series of advantages in comparation with the serial one, like: high stiffness, high accelerations and speeds, high precision and a modular simple construction. [1]

. Reconfigurable robots can be defined as a specific category of robots whose components: joints and links can be assembled in different configurations.

Coupling - uncoupling mechanisms are important elements in the field of reconfigurable robots. The structures will be simulated and analyzed using computers in order to coupling - uncupling elements and possible collisions between the robot elements. [2]

The first generation of parallel structures was used in amusement parks [3] and for welding robots, Gough uses a parallel mechanism with six points guidance platform for testing car tires [4] and Stewart presents parallel structures used nowadays for flight simulators [5]. In this book [6] J. P. Merlet realized one of the most complete studies regarding parallel robots. Studying the motion of the rigid body on curves and on fixed and mobile surfaces there are developed, as shown in [7]-[11], parallel mechanisms having three up to six degrees of mobility with guidance of the platform in three, four, five and six points.

A new approach for the analysis of a family of parallel reconfigurable robots is proposed by Gogu in [12]. The parallel structure, called Isogliden-TaRb, can have up to five degrees of freedom which are a combination of maximum 3 independent translations and maximum 2 rotations. The reconfiguration is obtained by blocking 1, 2, 3 or 4 actuators with no change in the architecture of the robot.

The paper is organized as follows: Section 2 is dedicated to the description and design of the parallel robot with six degrees of freedom and three guiding kinematic chains of the platform. The inverse and direct geometrical problems are the subject of the Section 3. In the last section the conclusions are presented.

2. DESIGN CONSIDERATION

Mobile platform (end-effector) in our case is a right angle triangle defined by A_1, A_2, A_3 related to the three kinematic chain structure via three spherical joints. The three kinematic chains using fork- slide joint type. Vertical displacement and rotation around Z axis is performed using six linear active joint mounted on three guides of the frame.

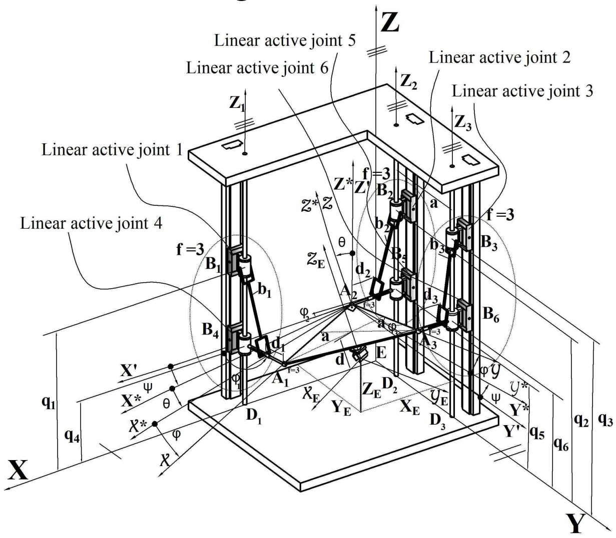

Fig. 1. Reconfigurable parallel robot with M=6 d.o.f and three guiding kinematik chains of the platform

In Figure 1 is presented a 6 d.o.f. parallel robot with three kinematic chains of the guiding platform. With the help of fasteners the robot can be reconfigured in one with five, four, three or two degrees of freedom[13].

The geometrical parameters of the robot are represented by $a, d, d_1, d_2, e_1, e_2, e_3, b_1, b_2, b_3$ and the coordinates of point $E(X_E, Y_E, Z_E)$.

3. THE GEOMETRICAL MODEL

In the paper [14] the direct geometrical model and the inverse geometrical model were calculated using Euler angles $Z'X^*z^*$. In order to deduct more simple relations for the geometrical models, for particular cases with five, four, three and two d.o.f will be used the Euler angles $Z'Y^*z^*$ as follows:

3.1 The inverse geometrical model

In this case there are given the generalized coordinates of the end effector $X_E, Y_E, Z_E, \psi, \theta, \varphi$. Required to be determined are the robot's generalized coordinates $q_1, q_2, ..., q_6$.

Based on Figure 1 we can write the following relation:

$$\begin{bmatrix} X_E - X_{A_i}\\ Y_E - Y_{A_i}\\ Z_E - Z_{A_i}\end{bmatrix} = \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha'''\\ c\beta' & c\beta'' & c\beta'''\\ c\gamma' & c\gamma'' & c\gamma'''\end{bmatrix}\begin{bmatrix} x_E - x_{A_i}\\ y_E - y_{A_i}\\ z_E - z_{A_i}\end{bmatrix}, i=1,2,3 \qquad (1)$$

From which are obtained the coordinates of the points $A_i, i=1,2,3$:

$$\begin{bmatrix} X_{A_i}\\ Y_{A_i}\\ Z_{A_i}\end{bmatrix} = \begin{bmatrix} X_E\\ Y_E\\ Z_E\end{bmatrix} - \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha'''\\ c\beta' & c\beta'' & c\beta'''\\ c\gamma' & c\gamma'' & c\gamma'''\end{bmatrix}\begin{bmatrix} x_{A_i} - x_E\\ y_{A_i} - y_E\\ z_{A_i} - z_E\end{bmatrix}, i=1,2,3 \qquad (2)$$

In relation (1) and (2) the angles between the axis are given below:

Correspondence of the coordinate systems

	ox	oy	oz
OX	α'	α''	α'''
OY	β'	β''	β'''
OZ	γ'	γ''	γ'''

(3)

In the case Euler angles $Z'Y^*z^*$,the rotation matrix elements have the expressions: (*)

$c\alpha' = c\psi c\theta c\varphi - s\psi s\varphi$	$c\alpha'' = -c\psi c\theta s\varphi - s\psi c\varphi$	$c\alpha''' = c\psi s\theta$
$c\beta' = s\psi c\theta c\varphi + c\psi s\varphi$	$c\beta'' = -s\psi c\theta s\varphi + c\psi c\varphi$	$c\beta''' = -s\psi s\theta$
$c\gamma' = -s\theta c\varphi$	$c\gamma'' = s\theta s\varphi$	$c\gamma''' = c\theta$

(4)

Using relations (4) and (2) it results:

$$\begin{bmatrix} X_{A_i}\\ Y_{A_i}\\ Z_{A_i}\end{bmatrix} = \begin{bmatrix} X_E\\ Y_E\\ Z_E\end{bmatrix} + \begin{bmatrix} c\psi c\theta c\varphi - s\psi s\varphi & -c\psi c\theta s\varphi - s\psi c\varphi & c\psi s\theta\\ s\psi c\theta c\varphi + c\psi s\varphi & -s\psi c\theta s\varphi + c\psi c\varphi & -s\psi s\theta\\ -s\theta c\varphi & s\theta s\varphi & c\theta\end{bmatrix}$$

$$\begin{bmatrix} x_{A_i} - x_E\\ y_{A_i} - y_E\\ z_{A_i} - z_E\end{bmatrix}, i=1,2,3 \qquad (5)$$

Relation from which the coordinates of the points $A_i, i=1,2,3$ are obtained according to the generalized coordinates of the end-effector, q_i and q_{i+3} (i=1,2,3) with obtaineed:

$$\begin{cases} X_{A_i} - X_{B_i} = \left[\sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i\right] c\varphi_i\\ Y_{A_i} - Y_{B_i} = \left[\sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i\right] s\varphi_i\\ Z_{A_i} = q_{i+3}\end{cases} \quad i=1,2,3 \qquad (6)$$

Using the equation (6) the angles can be determined (i = 1,2,3) obtained:

$$\left[\sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i\right]^2 = (X_{A_i} - X_{B_i})^2 + (Y_{A_i} - Y_{B_i})^2$$

$$Z_{A_i} = q_{i+3} \quad , i=1,2,3 \qquad (7)$$

In the first equation of the system (7):

$$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}, i=1,2,3 \qquad (8)$$

In the case of Figure 1:

*) Where s stands for sine, and c for cosine.

$$\begin{cases} X_{B_1} = X_{D_1} = e_1, Y_{B_1} = Y_{D_1} = 0 \\ X_{B_2} = X_{D_2} = 0, Y_{B_2} = Y_{D_2} = e_2 \\ X_{B_3} = 0, Y_{B_3} = e_3 \end{cases} \tag{9}$$

In the first relation of the system (7) we substitute relationship:

$$q_i^* = \sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i \tag{10}$$

Using (10) and (7)it results:

$$q_i^* = \sqrt{(X_{A_i} - X_{B_i})^2 + (Y_{A_i} - Y_{B_i})^2}, i = 1,2,3 \tag{11}$$

For his determination we use relation (10):

$$q_i - q_{i+3} = \sqrt{b_i^2 - (q_i^* - d_i)^2}$$

$$q_i = q_{i+3} + \sqrt{b_i^2 - (q_i^* - d_i)^2} \tag{12}$$

$$\begin{cases} Z_{A_i} = q_{i+3} \\ q_i = q_{i+3} + \sqrt{b_i^2 - (q_i^* - d_i)^2} \end{cases} \tag{13}$$

$$\begin{cases} Z_{A_i} = q_{i+3} \\ q_i = Z_{A_i} + \sqrt{b_i^2 - (q_i^* - d_i)^2} \end{cases} \tag{14}$$

From the equation (6) it results:

$$s\varphi_i = \frac{Y_{A_i} - Y_{B_i}}{\sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i}$$

$$c\varphi_i = \frac{X_{A_i} - X_{B_i}}{\sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i} \tag{15}$$

$$u_i = s\varphi_i = \frac{Y_{A_i} - Y_{B_i}}{q_i^*}, \quad w_i = c\varphi_i = \frac{X_{A_i} - X_{B_i}}{q_i^*} \tag{16}$$

$$\varphi_i = a\tan(u_i, w_i) \tag{17}$$

Table 1

The algorithm for the inverse geometrical model for the M=6 d.o.f and three guiding kinematic chains of the platform parallel robot.

Given: $X_E, Y_E, Z_E, \psi, \theta, \varphi$ **Unknown:** $q_1, q_2, q_3, q_4, q_5, q_6$	
Variables:	**Solving equation**
x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3	$x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0$ $x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0$ $x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0$
X_{D_i}, Y_{D_i}, Z_{D_i}, i=1,2,3	$X_{D_1} = e_1, Y_{D_1} = 0, Z_{D_1} = 0$ $X_{D_2} = 0, Y_{D_2} = e_2, Z_{D_2} = 0$ $X_{D_3} = 0, Y_{D_3} = e_2 + a = e_3, Z_{D_3} = 0$
x_E, y_E, z_E	$x_E = \frac{a}{3}$, $y_E = \frac{a}{3}$, $z_E = -d$
$X_{B_i}, Y_{B_i}; i = 1,2,3$	$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}; i = 1,2,3$
$c\alpha', c\beta', c\gamma'$, $c\alpha'', c\beta'', c\gamma''$, $c\alpha''', c\beta''', c\gamma'''$	$c\alpha' = c\psi c\theta c\varphi - s\psi s\varphi$; $c\alpha'' = -c\psi c\theta s\varphi - s\psi c\varphi$; $c\alpha''' = c\psi s\theta$ $c\beta' = s\psi c\theta c\varphi + c\psi s\varphi$; $c\beta'' = -s\psi c\theta s\varphi + c\psi c\varphi$; $c\beta''' = -s\psi s\theta$ $c\gamma' = -s\theta c\varphi$; $c\gamma'' = s\theta s\varphi$; $c\gamma''' = c\theta$
$X_{A_i}, Y_{A_i}, Z_{A_i}$, i=1,2,3	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} - \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \cdot \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$
q_i^*,i=1, 2, 3	$q_i^* = \sqrt{(X_{A_i} - X_{B_i})^2 + (Y_{A_i} - Y_{B_i})^2}, i = 1,2,3$
q_i, q_{i+3} i=1, 2, 3	$q_i = Z_{A_i} + \sqrt{b_i^2 - (q_i^* - d_i)^2}$, $q_{i+3} = Z_{A_i}$ i=1,2,3
u_i, w_i	$u_i = s\varphi_i = \frac{Y_{A_i} - Y_{B_i}}{q_i^*}$ $w_i = c\varphi_i = \frac{X_{A_i} - X_{B_i}}{q_i^*}$ i=1,2,3
φ_i	$\varphi_i = a\tan 2(u_i, w_i)$

3.2 The direct geometrical model

In the direct geometrical model of the robot the generalized coordinates are given q_i and q_{i+3} (i=1,2,3) generalized coordinates for the end effector are required: $X_E, Y_E, Z_E, \psi, \theta, \varphi$.

From the matrix relation (5) result the coordinates $X_{A_i}, Y_{A_i}, Z_{A_i}$, of the points A_i (i=1,2,3) belonging to the platform.

$$\begin{cases} X_{A_i} = X_E + (x_{A_i} - x_E)(c\psi c\theta c\varphi - s\psi s\varphi) + \\ (y_{A_i} - y_E)(-c\psi c\theta s\varphi - s\psi c\varphi) + (z_{A_i} - z_E)c\psi s\theta \\ Y_{A_i} = Y_E + (x_{A_i} - x_E)(s\psi c\theta c\varphi + c\psi s\varphi) + \\ (y_{A_i} - y_E)(-s\psi c\theta s\varphi + c\psi c\varphi) - (z_{A_i} - z_E)s\psi s\theta \\ Z_{A_i} = Z_E + (x_{A_i} - x_E)(-s\theta c\varphi) + (y_{A_i} - y_E)s\theta s\varphi + \\ (z_{A_i} - z_E)c\theta \end{cases} \tag{18}$$

Using relations (1) and (10), equations (7) become:

$$\begin{cases} F_i(q_i, X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv \left[X_{A_i}(X_E, \psi, \theta, \varphi) - X_{B_i}\right]^2 \\ + \left[Y_{A_i}(Y_E, \psi, \theta, \varphi) - Y_{B_i}\right]^2 - \left(q_i^*\right)^2 = 0 \\ F_{i+3}(q_{i+3}, Z_E, \theta, \varphi) = Z_{A_i}(Z_E, \theta, \varphi) - q_{i+3} = 0 \end{cases} \tag{19}$$

Using the (18) equations (19) can be written as:

$$\begin{cases} F_i(q_i, X_E, Y_E, \psi, \theta, \varphi) \equiv \\ \begin{bmatrix} X_E - (x_{A_i} - x_E)(c\psi c\theta c\varphi - s\psi s\varphi) + \\ (y_{A_i} - y_E)(-c\psi c\theta s\varphi - s\psi c\varphi) - (z_{A_i} - z_E)c\psi s\theta - X_{B_i} \end{bmatrix}^2 + \\ \begin{bmatrix} Y_E - (x_{A_i} - x_E)(s\psi c\theta c\varphi + c\psi s\varphi) - \\ (y_{A_i} - y_E)(-s\psi c\theta s\varphi + c\psi c\varphi) - (z_{A_i} - z_E)s\psi s\theta - Y_{B_i} \end{bmatrix}^2 - \\ -(q_i^*)^2 = 0 \\ F_{i+3}(q_{i+3}, Z_E, \theta, \varphi) \equiv Z_E - (x_{A_i} - x_E)(-s\theta c\varphi) - (y_{A_i} - y_E)\cdot \\ \cdot s\theta s\varphi - (z_{A_i} - z_E)c\theta = 0 \end{cases} \tag{20}$$

From the second equation of the system (20) a three equations system for i = 1,2,3 is deducted:

$$\begin{cases} i=1; F_4(q_4, Z_E, \theta, \varphi) \equiv Z_E - (x_{A_1} - x_E)(-s\theta c\varphi) - (y_{A_1} - y_E)s\theta s\varphi - \\ -(z_{A_1} - z_E)c\theta - q_4 = 0 \\ i=2; F_5(q_5, Z_E, \theta, \varphi) \equiv Z_E - (x_{A_2} - x_E)(-s\theta c\varphi) - (y_{A_2} - y_E)s\theta s\varphi - \\ -(z_{A_2} - z_E)c\theta - q_5 = 0 \\ i=3; F_6(q_6, Z_E, \theta, \varphi) \equiv Z_E - (x_{A_3} - x_E)(-s\theta c\varphi) - (y_{A_3} - y_E)s\theta s\varphi - \\ -(z_{A_3} - z_E)c\theta - q_6 = 0 \end{cases} \tag{21}$$

For:

$$\begin{aligned} & x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0 \\ & x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0 \\ & x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0 \\ & x_E = \frac{a}{3},\ y_E = \frac{a}{3},\ z_E = -d \end{aligned} \tag{22}$$

Using the (22) equation (21) becomes:

$$\begin{cases} F_4(q_4, Z_E, \theta, \varphi) \equiv Z_E - \frac{2a}{3}(-s\theta c\varphi) + \frac{a}{3}s\theta s\varphi - dc\theta - q_4 = 0 \\ F_5(q_5, Z_E, \theta, \varphi) \equiv Z_E + \frac{a}{3}(-s\theta c\varphi) + \frac{a}{3}s\theta s\varphi - dc\theta - q_5 = 0 \\ F_6(q_6, Z_E, \theta, \varphi) \equiv Z_E + \frac{a}{3}(-s\theta c\varphi) - \frac{2a}{3}s\theta s\varphi - dc\theta - q_6 = 0 \end{cases} \tag{23}$$

Substracting the 1rd equation from the 2st and the 3nd from the 2st, the following system is deducted:

$$\begin{cases} as\theta s\varphi + q_5 - q_4 = 0 \\ as\theta c\varphi + q_6 - q_4 = 0 \end{cases}$$

$$\begin{cases} s\theta s\varphi = \frac{q_5 - q_4}{a} \\ s\theta c\varphi = \frac{q_6 - q_4}{a} \end{cases} \tag{24}$$

By the squaring the equation (24) it results:

$$s^2\theta = \frac{1}{a^2}\left[(q_5 - q_4)^2 + (q_6 - q_4)^2\right]$$

Or

$$u_\theta = s\theta = \frac{1}{a}\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2} \tag{25}$$

Koming that $c\theta = \pm\sqrt{1 - s^2\theta}$ it results:

$$w_\theta = \pm\sqrt{1 - u_\theta^2} = \sqrt{1 - \frac{1}{a^2}\left[(q_5 - q_4)^2 + (q_6 - q_4)^2\right]}$$

$$w_\theta = c\theta = \pm\frac{1}{a}\sqrt{a^2 - (q_5 - q_4)^2 - (q_6 - q_4)^2} \tag{26}$$

Using relations (25) and (26) the angle θ will be:

$$\theta = a\tan 2(u_\theta, w_\theta) \tag{27}$$

From (24) the following equation result:

$$\begin{cases} u_\varphi = s\varphi = \frac{q_6 - q_4}{\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2}} \\ w_\varphi = c\varphi = \frac{q_5 - q_4}{\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2}} \end{cases} \tag{28}$$

Hence:

$$\varphi = a\tan 2(u_\varphi, w_\varphi) \tag{29}$$

From the last equation of the system (23) the Z_E coordinates can be determined:

$$Z_E = -u_\theta\left(\frac{2a}{3}w_\varphi + \frac{a}{3}u_\varphi\right) + dw_\theta + q_6 \tag{30}$$

Or

$$Z_E = -as\theta\left(\frac{2}{3}c\varphi + \frac{1}{3}s\varphi\right) + dc\theta + q_6 \tag{31}$$

Using relations (25) (26), (28) and (29) relations from the first system (20) becomes:

$$\begin{aligned} & F_i(q_i, X_E, Y_E, \psi) \equiv \\ & \begin{bmatrix} X_E - (x_{A_i} - x_E)(w_\theta w_\varphi c\psi - u_\varphi s\psi) + \\ (y_{A_i} - y_E)(-w_\theta u_\varphi c\psi + w_\varphi s\psi) - (z_{A_i} - z_E)u_\theta c\psi - X_{B_i} \end{bmatrix}^2 + \\ & \begin{bmatrix} Y_E - (x_{A_i} - x_E)(w_\theta w_\varphi s\psi + u_\varphi c\psi) - \\ (y_{A_i} - y_E)(-w_\theta u_\varphi s\psi + w_\theta c\psi) - (z_{A_i} - z_E)u_\theta s\psi - Y_{B_i} \end{bmatrix}^2 - \\ & -(q_i^*)^2 = 0 \end{aligned} \tag{32}$$

The three equations system with three unknowns X_E, Y_E, ψ can be solved using the Newton-Rapson method:

Denoting with:

$$X = [X_E, Y_E, \psi]^T$$

$$F(X) = [F_1(X_E, Y_E, \psi), F_2(X_E, Y_E, \psi), F_3(X_E, Y_E, \psi)]^T,$$

$$W(X)=\begin{bmatrix}\frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi}\\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi}\\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi}\end{bmatrix} \tag{33}$$

Using the equation (33) the three unknown X_E, Y_E, ψ are determined numerically by Newton-Raphson method with the formula:

$$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)})F(X^{(p)}), p = 0,1,2.... \tag{34}$$

Where p is the number of elevations.

Table 2

The algorithm for the direct geometrical model for the M=6 d.o.f and three guiding kinematic chains of the platform parallel robot.

Given: $q_1, q_2, q_3, q_4, q_5, q_6$ **Unknowns:** $X_E, Y_E, Z_E, \psi, \theta, \varphi$	
Variables	**Solving equations**
x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3	$x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0$ $x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0$ $x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0$
x_E, y_E, z_E	$x_E = \frac{a}{3}$, $y_E = \frac{a}{3}$, $z_E = -d$
X_{D_i}, Y_{D_i}, Z_{D_i}, i=1,2,3	$X_{D_1} = 0, Y_{D_1} = e_1, Z_{D_1} = 0$ $X_{D_2} = 0, Y_{D_2} = e_2 = e_1 + a, Z_{D_2} = 0$ $X_{D_3} = e_3, Y_{D_3} = 0, Z_{D_3} = 0$
X_{B_i}, Y_{B_i}; $i = 1,2,3$	$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}; i = 1,2,3$
u_θ, w_θ	$u_\theta = s\theta = \frac{1}{a}\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2}$ $w_\theta = c\theta = \pm\frac{1}{a}\sqrt{a^2 - (q_5 - q_4)^2 - (q_6 - q_4)^2}$
θ	$\theta = a\tan 2(u_\theta, w_\theta)$
u_φ, w_φ	$\begin{cases} u_\varphi = s\varphi = \dfrac{q_6 - q_4}{\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2}} \\ w_\varphi = c\varphi = \dfrac{q_5 - q_4}{\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2}} \end{cases}$
φ	$\varphi = a\tan 2(u_\varphi, w_\varphi)$
Z_E	$Z_E = -au_\theta(\frac{2}{3}w_\varphi + \frac{1}{3}u_\varphi) + dw_\theta + q_6$
q_i^*, $i = 1,2,3$	$q_i^* = \sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i, i = 1,2,3$
X_E, Y_E, ψ	$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)})F(X^{(p)}), p = 0,1,2...$ Where $X = \begin{bmatrix} X_E \\ Y_E \\ \psi \end{bmatrix}$, $F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \end{bmatrix}$, $W(X) = \begin{bmatrix}\frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi}\\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi}\\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi}\end{bmatrix}$, $F_i(q, X_E, Y_E, \psi) \equiv$ $\begin{bmatrix} X_E - (x_{A_i} - x_E)(w_\theta w_\varphi c\psi - u_\varphi s\psi) + \\ (y_{A_i} - y_E)(-w_\theta u_\varphi c\psi + w_\varphi s\psi) - (z_{A_i} - z_E)u_\theta c\psi - X_{B_i} \end{bmatrix}^2 +$ $\begin{bmatrix} Y_E - (x_{A_i} - x_E)(w_\theta w_\varphi s\psi + u_\varphi c\psi) - \\ (y_{A_i} - y_E)(-w_\theta u_\varphi s\psi + w_\theta c\psi) - (z_{A_i} - z_E)u_\theta s\psi - Y_{B_i} \end{bmatrix}^2 -$ $-(q_i^*)^2 = 0$ i=1,2,3

Parallel robot with M=5 d.o.f. Fig(2) ;

The drive coordinates are:

$$q_1, q_2, q_3, q_4, q_5 \tag{35}$$

The end-effector performs translations along OX, OY and OZ axes and two rotations (precession ψ and nutation θ angles). The coordinates of the end-effector are:

$$X_E, Y_E, Z_E, \psi, \theta, \varphi = 0 \tag{36}$$

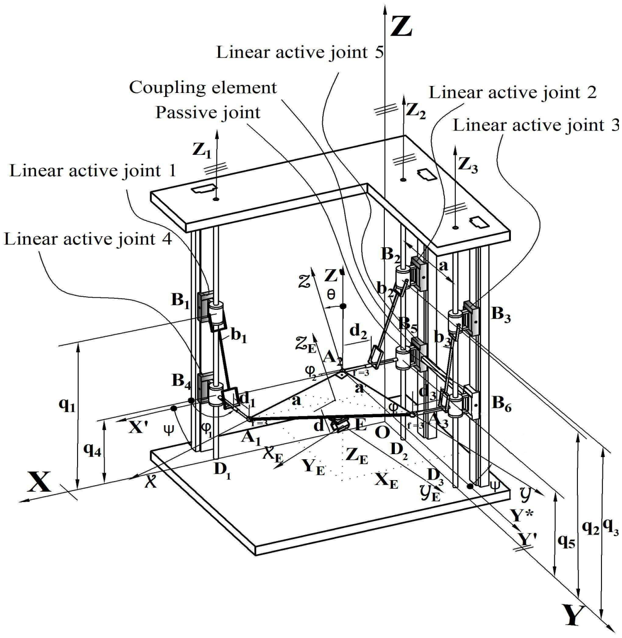

Fig. 2. Reconfigurable parallel robot with M=5 d.o.f and three guiding kinematik chains of the platform

Table 3

The algorithm for the inverse geometrical model for the M=5 d.o.f and three guiding kinematic chains of the platform parallel robot.

Given: $X_E, Y_E, Z_E, \psi, \theta$ **Unknown:** q_1, q_2, q_3, q_4, q_5	
Variables:	**Solving equation**
x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3	$x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0$ $x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0$ $x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0$
X_{D_i}, Y_{D_i}, Z_{D_i}, i=1,2,3	$X_{D_1} = e_1, Y_{D_1} = 0, Z_{D_1} = 0$ $X_{D_2} = 0, Y_{D_2} = e_2, Z_{D_2} = 0$ $X_{D_3} = 0, Y_{D_3} = e_2 + a = e_3, Z_{D_3} = 0$
x_E, y_E, z_E	$x_E = \frac{a}{3}$, $y_E = \frac{a}{3}$, $z_E = -d$
X_{B_i}, Y_{B_i}; i=1,2,3	$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}; i = 1,2,3$
$c\alpha', c\beta', c\gamma'$, $c\alpha'', c\beta'', c\gamma''$, $c\alpha''', c\beta''', c\gamma'''$	$c\alpha' = c\psi c\theta$; $c\alpha'' = -s\psi$; $c\alpha''' = c\psi s\theta$ $c\beta' = s\psi c\theta$; $c\beta'' = c\psi$; $c\beta''' = -s\psi s\theta$ $c\gamma' = -s\theta$; $c\gamma'' = 0$; $c\gamma''' = c\theta$
$X_{A_i}, Y_{A_i}, Z_{A_i}$, i=1,2,3	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} - \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \cdot \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$
q_i^*, i=1, 2, 3	$q_i^* = \sqrt{(X_{A_i} - X_{B_i})^2 + (Y_{A_i} - Y_{B_i})^2}, i = 1,2,3$
q_i, q_4, q_5 i=1, 2, 3	$q_i = Z_{Ai} + \sqrt{b_i^2 - (q_i^* - d_i)^2}$, i=1,2,3 $q_4 = Z_{A_1}, q_5 = Z_{A_2}$
u_i, w_i	$u_i = s\varphi_i = 0$ $w_i = c\varphi_i = 1$, i=1,2,3
φ_i	$\varphi_i = a\tan 2(u_i, w_i)$, i=1,2,3

Table 4

The algorithm for the direct geometrical model for the M=5 d.o.f and three guiding kinematic chains of the platform parallel robot.

Given: q_1, q_2, q_3, q_4, q_5 **Unknowns:** $X_E, Y_E, Z_E, \psi, \theta$	
Variables	**Solving equations**
x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3	$x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0$ $x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0$ $x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0$
x_E, y_E, z_E	$x_E = \frac{a}{3}$, $y_E = \frac{a}{3}$, $z_E = -d$
X_{D_i}, Y_{D_i}, Z_{D_i}, i=1,2,3	$X_{D_1} = 0, Y_{D_1} = e_1, Z_{D_1} = 0$ $X_{D_2} = 0, Y_{D_2} = e_2 = e_1 + a, Z_{D_2} = 0$ $X_{D_3} = e_3, Y_{D_3} = 0, Z_{D_3} = 0$
X_{B_i}, Y_{B_i}; i=1,2,3	$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}; i = 1,2,3$
u_θ, w_θ	$u_\theta = s\theta = \frac{1}{a}\sqrt{(q_5 - q_4)^2 + (q_6 - q_4)^2}$ $w_\theta = c\theta = \pm\frac{1}{a}\sqrt{a^2 - (q_5 - q_4)^2 - (q_6 - q_4)^2}$
θ	$\theta = a\tan 2(u_\theta, w_\theta)$
Z_E	$Z_E = -\frac{2}{3} a u_\theta + d w_\theta + q_5$
q_i^*, i = 1,2,3	$q_i^* = \sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i, i = 1,2,3$ $q_{i+3} = q_5$, for i=3
X_E, Y_E, ψ	$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)}) F(X^{(p)}), p = 0,1,2...$ Where $X = \begin{bmatrix} X_E \\ Y_E \\ \psi \end{bmatrix}$, $F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \end{bmatrix}$, $W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi} \end{bmatrix}$, $F_i(q_i, X_E, Y_E, \psi) \equiv$ $\begin{bmatrix} X_E - (x_{A_i} - x_E) w_\theta c\psi + \\ (y_{A_i} - y_E) s\psi) - (z_{A_i} - z_E) u_\theta c\psi - X_{B_i} \end{bmatrix}^2 +$ $\begin{bmatrix} Y_E - (x_{A_i} - x_E) w_\theta s\psi - \\ (y_{A_i} - y_E) w_\theta c\psi - (z_{A_i} - z_E) u_\theta s\psi - Y_{B_i} \end{bmatrix}^2 -$ $-(q_i^*)^2 = 0$ i=1,2,3

Parallel robot with M=4 d.o.f. Fig(3) ;

The drive coordinates are :

$$q_6 = q_5 = q_4, q_3, q_2, q_1 \qquad (37)$$

The end-effector performs translations along OX, OY, OZ axes and a rotation around OZ axis. The coordinates of the end-effector are:

$$X_E, Y_E, Z_E, \psi,\ \theta=0,\ \varphi=0 \tag{38}$$

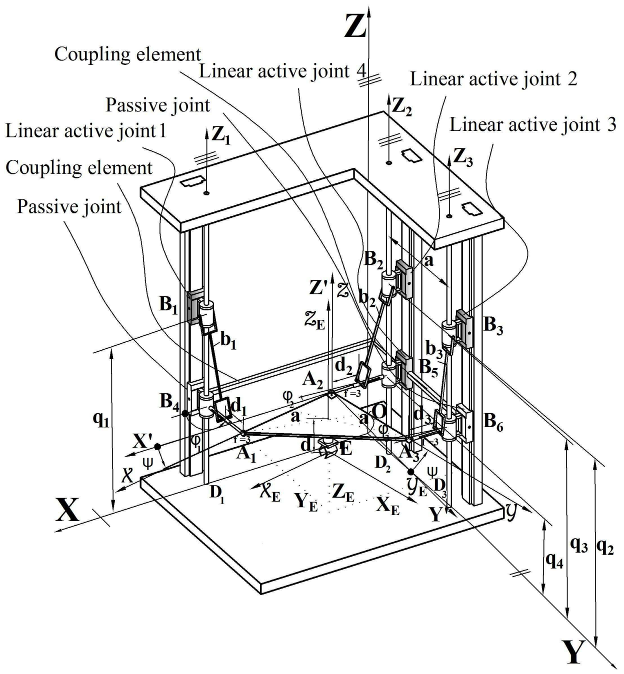

Fig. 3. Reconfigurable parallel robot with M=4 d.o.f and three guiding kinematik chains of the platform

Table 5

The algorithm for the inverse geometrical model for the M=4 d.o.f and three guiding kinematic chains of the platform parallel robot.

Given: X_E, Y_E, Z_E, ψ **Unknown:** q_1, q_2, q_3, q_4	
Variables:	**Solving equation**
x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3	$x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0$ $x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0$ $x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0$
X_{D_i}, Y_{D_i}, Z_{D_i}, i=1,2,3	$X_{D_1} = e_1, Y_{D_1} = 0, Z_{D_1} = 0$ $X_{D_2} = 0, Y_{D_2} = e_2, Z_{D_2} = 0$ $X_{D_3} = 0, Y_{D_3} = e_2 + a = e_3, Z_{D_3} = 0$
x_E, y_E, z_E	$x_E = \frac{a}{3}$, $y_E = \frac{a}{3}$, $z_E = -d$
X_{B_i}, Y_{B_i}; $i=1,2,3$	$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}; i = 1,2,3$
$c\alpha', c\beta', c\gamma'$, $c\alpha'', c\beta'', c\gamma''$, $c\alpha''', c\beta''', c\gamma'''$	$c\alpha' = c\psi$ ¦ $c\alpha'' = -s\psi$ ¦ $c\alpha''' = 0$ $c\beta' = s\psi$ ¦ $c\beta'' = c\psi$ ¦ $c\beta''' = 0$ $c\gamma' = 0$ ¦ $c\gamma'' = 0$ ¦ $c\gamma''' = 1$
$X_{A_i}, Y_{A_i}, Z_{A_i}$, i=1,2,3	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} - \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \cdot \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$
q_i^*, i=1, 2, 3	$q_i^* = \sqrt{(X_{A_i} - X_{B_i})^2 + (Y_{A_i} - Y_{B_i})^2}, i = 1,2,3$
q_i, q_{i+3} i=1, 2, 3	$q_i = Z_{A_i} + \sqrt{b_i^2 - (q_i^* - d_i)^2}$, i=1,2,3 $q_4 = Z_{A_1}$

Table 6

The algorithm for the direct geometrical model for the M=4 d.o.f and three guiding kinematic chains of the platform parallel robot.

Given: q_1, q_2, q_3, q_4 **Unknowns:** X_E, Y_E, Z_E, ψ	
Variables	**Solving equations**
x_{A_i}, y_{A_i}, z_{A_i}, i=1,2,3	$x_{A_1} = a, y_{A_1} = 0, z_{A_1} = 0$ $x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0$ $x_{A_3} = 0, y_{A_3} = a, z_{A_3} = 0$
x_E, y_E, z_E	$x_E = \frac{a}{3}$, $y_E = \frac{a}{3}$, $z_E = -d$
X_{D_i}, Y_{D_i}, Z_{D_i}, i=1,2,3	$X_{D_1} = 0, Y_{D_1} = e_1, Z_{D_1} = 0$ $X_{D_2} = 0, Y_{D_2} = e_2 = e_1 + a, Z_{D_2} = 0$ $X_{D_3} = e_3, Y_{D_3} = 0, Z_{D_3} = 0$
X_{B_i}, Y_{B_i}; $i=1,2,3$	$X_{B_i} = X_{D_i}, Y_{B_i} = Y_{D_i}; i = 1,2,3$
Z_E	$Z_E = d + q_4$
q_i^*, $i = 1,2,3$	$q_i^* = \sqrt{b_i^2 - (q_i - q_{i+3})^2} + d_i, i = 1,2,3$
X_E, Y_E, ψ	$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)})F(X^{(p)}), p = 0,1,2\ldots$ Where $X = \begin{bmatrix} X_E \\ Y_E \\ \psi \end{bmatrix}$, $F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \end{bmatrix}$, $W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi} \end{bmatrix}$,

	$F_i(q_i, X_E, Y_E, \psi) \equiv \begin{bmatrix} X_E - (x_{A_i} - x_E)c\psi + \\ (y_{A_i} - y_E)s\psi) - X_{B_i} \end{bmatrix}^2 + \begin{bmatrix} Y_E - (x_{A_i} - x_E)s\psi - \\ (y_{A_i} - y_E)c\psi - Y_{B_i} \end{bmatrix}^2 - (q_i^*)^2 = 0$ i=1,2,3

Parallel robot with M=3 d.o.f. spatial Fig(4);

The drive coordinates are:

$$q_1, q_2, q_3 \tag{39}$$

The end-effector performs only translations along OX, OY and OZ axes. The coordinates of the end-effector are:

$$X_E, Y_E, Z_E, \psi = 0,\ \theta = 0,\ \varphi = 0 \tag{40}$$

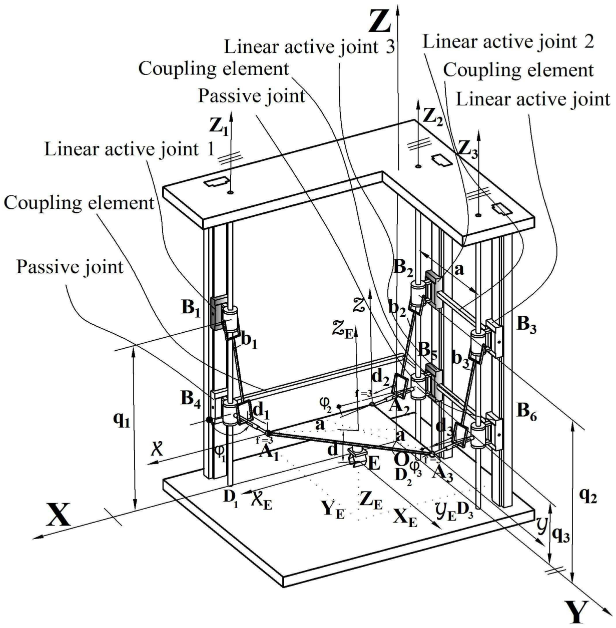

Fig. 4. Reconfigurable parallel robot with M=3 d.o.f and three guiding kinematik chains of the platform

Planar parallel robot with M=3d.o.f. Fig(5);

The drive coordinates are:

$$q_1, q_2, q_3, q_4 = q_5 = q_6 = ct \tag{41}$$

The end-effector performs translations along OX and OY and a rotation around OZ. The coordinates of the end-effector are:

$$X_E, Y_E, \psi, (Z_E = ct) \tag{42}$$

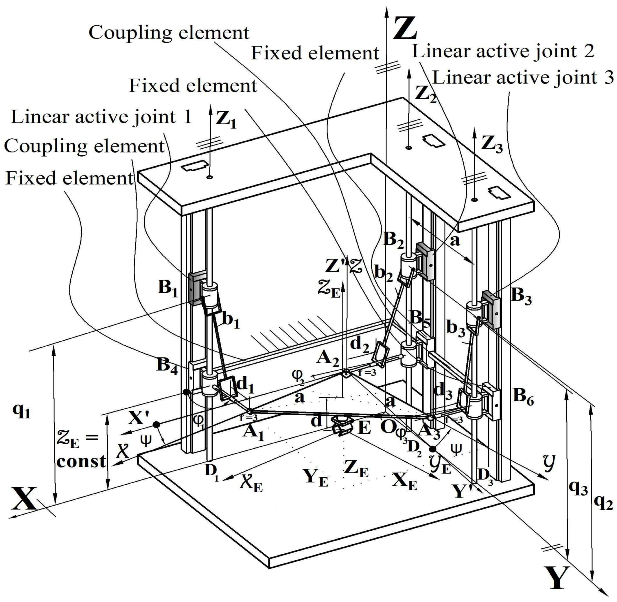

Fig. 5. Reconfigurable parallel robot with M=3 d.o.f and three guiding kinematik chains of the platform

Planar parallel robot with M=2 d.o.f. Fig(6);

The drive coordintes are:

$$q_1, q_2 \tag{43}$$

The end-effector performs translations along OX and OY axes. The coordinates of the end-effector are:

$$X_E, Y_E, Z_E = ct, \psi = 0,\ \theta = 0,\ \varphi = 0 \tag{44}$$

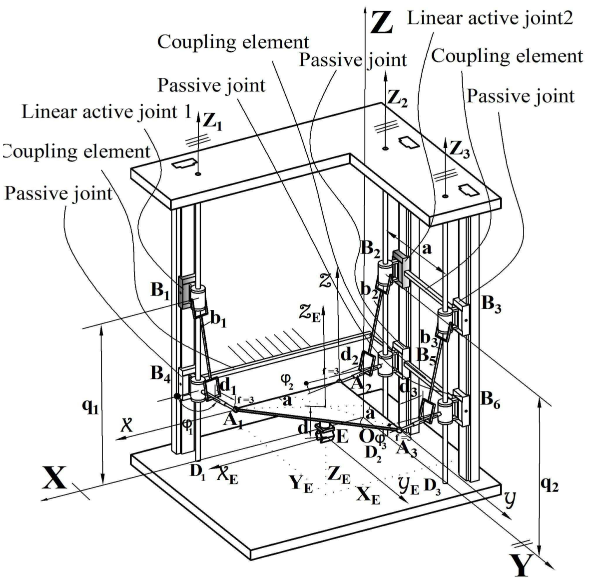

Fig. 6. Reconfigurable parallel robot with M=2 d.o.f and three guiding kinematik chains of the platform

4. CONCLUSION

This paper presents a new reconfigurable parallel robot with six degrees and three kinematic chains for guiding platform. The platform is driven by six linear active joint disposed on the three guides, having reduced weight and size allowing high speeds and accelerations .

The direct model and inverse geometrical models have been solved and further research will go into kinematics, dynamics and workspaceanalysis.

The robot is suitable for assembly and milling operations and can be used as a module in a minimally invasive surgical system.

5. ACKNOWLEDGEMENTS

This paper was supported by the project "Doctoral studies in engineering sciences for developing the knowledge based society-SIDOC”contract no. POSDRU/88/1.5/S/60078, project co-funded from European Social Fund through Sectorial Operational Program Human Resources 2007-2013.

6. REFERENCES

1. N.Plitea,J.Hesselbach „Modeling of parallel microrobots with two to six degrees of freedom” Acta Tehnica Napocensis 2007
2. Mândru Dan, Curs de Acționări în Mecanică Fină - 2004
3. J.E. Gwinnett, "Amusement Devices", US Patent No. 1, 789, 680, 1931.W.L. Pollard, "Position Controlling Apparatus", US Patent No. 2,286, 571, 1942.
4. V.E. Gough and S.GT. Whitehall, "Universal Tire Test Machine", Proc. 9'h Int. Technical Congress F.I.S.I.T.A.,vol. 117, pp. 117-135, May 1962.
5. D. Kohli, S.-H. Lee, K.-Y. Tsai, G.N., Sandor, "Manipulator configurations based on Rotary-Linear (R-L) actuators and their direct and inverse kinematics.", Mechanisms, Transmissions and Automation in Design, vol. 110, pp. 397-404, 1988.
6. D. Zlatanov, I.A. Bonev, C.M. Gosselin, "Constraint Singularities of Parallel Mechanisms", IEEE International Conference on Robotics and Automation (ICRA 2002), Washington D.C., 2002.
7. N. Plitea, "The study of rigid body motion guided on mobile surfaces with applications for the kinematic analysis and synthesis of spatial mechanisms with six degrees of mobility", The Scientific Bui. Of Polytechnic Institute Cluj, Series of Mechanics, vol. 14, pp. 47-52, 1971.
8. N. Plitea, "Study of guided rigid body motion upon moving and fixed surfaces applied to the kinematic analysis of spatial mechanisms with guidance", IFTOMM International Symposium on Linkages and Computer Design Methods, Bucharest, Romania, vol. B-36, pp. 477-487, 7-13 June, 1973.
9. N. Plitea, "Upon the rigid body motion with constraints with 3 degrees of freedom", The Scientific Bid. Of the Polytechnic Institute Timisoara, 1972.
10. N. Plitea, "Kinematiche Synthese von verzweigten raumlichen Getrieben mit Sechspunktfuhrung der Industrieroboter mit sechs Freiheitsgraden vom Typ I" The Fifth IFTOMM International Symposium on Linkages and Computer Aided Design Methods, Bucharest, Romania, pp. 495-503, 6-11 July, 1989..
11. N. Plitea, "Kinematiche Synthese von verzweigten raumlichen Getrieben mit Sechspunktfuhrung der Industrieroboter mit sechs Freiheitsgraden vom Typ III", The 10th Industrial Robots National Symposium, Bucharest, Romania, vol. 1, pp. 217-225, 18-20 April, 1991.
12. Gogu, George., Isogliden TaRb: a Family of up to Five Axes Reconfigurable and Maximally Regular Parallel Kinematic Machines, International Conference on Smart Machining Systems, (2007).
13. Plitea, N., Pisla, D., Vaida, C., Lese, D-B., Konya, B., Dadarlat, R., Scurtu, I., Sabou, C., *Familie de roboti paraleli cu sase grade de mobilitate*, Patent nr. A/10013/2011B.
14. Konya,C. Vaida,R. Dadarlat, N. Plitea : A new reconfigurable parallel robot with six degrees of freedom, International Conference on Quality and Innovation in Engineering and Management, (2011).

Variante ale unui nou robot reconfigurabil cu şase, cinci, patru, trei şi două grade de libertate

Lucrarea prezinta un nou robot parallel reconfigurabil cu sase grade de mobilitate si trei lanturi cinematice de ghidare a platformei. Robotul foloseste sase motoare liniare pentru a efectua miscarile in spatiu de lucru. In functie de domeniul unde va fi utilizat, robotul initial se poate reconfigura usor si repede intr-unul cu cinci, patru , trei sau doua grade de mobilitate. Algoritmii pentru modelul geometric direct si invers pentru unele struncturi sunt prezentati in lucrare.

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