



CONTRIBUTION TO THE STUDY OF THE VIBRATORY CONVEYOR WITH AN ELASTIC BASE

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Abstract: In this paper is studied a vibratory conveyor that is placed on an elastic base. Using the Lagrange equation method it was determined the system that needs to be solved to obtain graphical representation for the generalized coordinates determining the position of the mechanical system elements. Developing a C program in which is used the Runge–Kutta and Gauss methods were obtained the numerical results of these coordinates.

Key words: lamellar spring, linear vibratory conveyor, vibration, elastic base, C program.

1. INTRODUCTION

The vibratory conveyors machines are used to transport different types of materials consuming reduced quantity of energy.

These machines have the following components: cradle, trough, elastic elements and the driven mechanism.

The free vibration movement of the mechanical system corresponding to the vibratory conveyor with an elastic bar will be studied.

In figure 1 is presented a vibratory conveyor actuated inertial that is placed on a base sustained with elastic elements that in our case are helical springs.

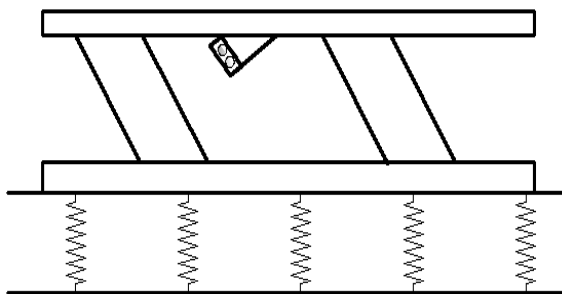


Fig.1

The trough is linked with the base with elastic beams.

In figure 2 is presented the mechanical system corresponding to the studied vibratory conveyor.

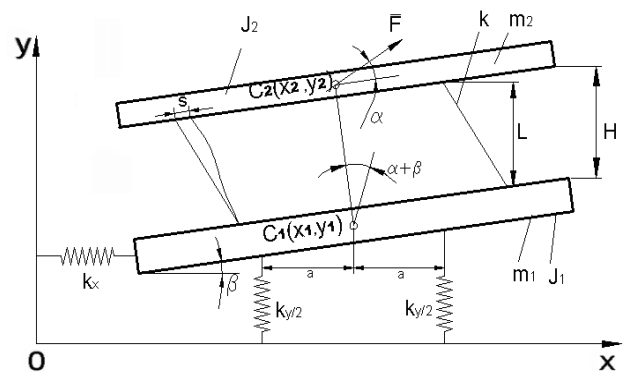


Fig.2

2. THEORETICAL DEVELOPMENT

The study of the vibratory conveyor placed on an elastic base was performed. As generalized coordinates the parameters x_1 , y_1 , β and s (figure 2) were considered.

The mass center coordinates x_2 and y_2 were calculated with respect of generalized coordinates.

To establish the simultaneous differential equations corresponding to mechanical system movements the Lagrange equation method will be used [1] [3] [4] [6] [9].

$$\begin{cases} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_1} \right) - \frac{\partial L}{\partial x_1} = 0 \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}_1} \right) - \frac{\partial L}{\partial y_1} = 0 \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\beta}} \right) - \frac{\partial L}{\partial \beta} = 0 \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{s}} \right) - \frac{\partial L}{\partial s} = 0 \end{cases} \quad (1)$$

were L is the Lagrange function, $L=E_c - E_p$.

The trough center mass coordinates are:

$$\begin{aligned} x_2 &= x_1 - L \sin(\alpha + \beta) - s \cos(\alpha + \beta) \\ y_2 &= y_1 + L \cos(\alpha + \beta) - s \sin(\alpha + \beta) \end{aligned} \quad (2)$$

and it's derivatives

$$\begin{aligned} \dot{x}_2 &= \dot{x}_1 - L\dot{\beta} \cos(\alpha + \beta) - \dot{s} \cos(\alpha + \beta) \\ &\quad + s\dot{\beta} \sin(\alpha + \beta) \\ \dot{y}_2 &= \dot{y}_1 - L\dot{\beta} \sin(\alpha + \beta) - \dot{s} \sin(\alpha + \beta) \\ &\quad - s\dot{\beta} \cos(\alpha + \beta) \end{aligned} \quad (3)$$

The kinetic and potential energy of mechanical system are:

$$\begin{aligned} E_c &= \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} J_1 \dot{\beta}^2 + \\ &\quad + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2} J_2 \dot{\beta}^2 \end{aligned} \quad (4)$$

$$\begin{aligned} E_p &= \frac{1}{2} k_x x_1^2 + \frac{1}{4} k_y (y_1^2 + x_1^2 + a^2 \beta^2) \\ &\quad + \frac{1}{2} k s^2 \end{aligned} \quad (5)$$

were:

- x_1, x_2, y_1, y_2 - are the center of mass coordinates related to the trough and to the cradle;

- β - angle of inclination of the trough;

- s - lamellar spring deformation;

- m_2 - trough mass;

- J_2 - moment of inertia of the trough;

- m_1 - cradle mass;

- J_1 - moment of inertia of the cradle;

- k - elasticity constant of the lamellar spring;

- k_x - elasticity constant of the helical spring on x axis;

- k_y - elasticity constant of the helical spring on y axis;

The Lagrange function has the following expression:

$$\begin{aligned} L=E_c - E_p &= \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} J_1 \dot{\beta}^2 + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) \\ &\quad + \frac{1}{2} J_2 \dot{\beta}^2 - \frac{1}{2} k_x x_1^2 - \frac{1}{4} k_y (y_1^2 + x_1^2 + a^2 \beta^2) - \frac{1}{2} k s^2 \end{aligned} \quad (6)$$

Introducing the values of x_2, \dot{x}_2, y_2 and \dot{y}_2 in Lagrange function expression, results:

$$\begin{aligned} L &= \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} J_1 \dot{\beta}^2 \\ &\quad + \frac{1}{2} m_2 (\dot{x}_1^2 + L\dot{\beta}^2 + \dot{s}^2 + s\dot{\beta}^2 - 2\dot{x}_1 \dot{\beta} L \cos(\alpha + \beta) \\ &\quad - 2\dot{x}_1 \dot{s} \cos(\alpha + \beta) + 2\dot{x}_1 \dot{\beta} s \sin(\alpha + \beta) \\ &\quad + \dot{y}_1^2 - 2\dot{y}_1 \dot{\beta} L \sin(\alpha + \beta) - 2\dot{y}_1 \dot{s} \sin(\alpha + \beta) - \\ &\quad 2\dot{y}_1 \dot{\beta} s \cos(\alpha + \beta) + 2\dot{\beta} \dot{s} L) + \frac{1}{2} J_2 \dot{\beta}^2 - \frac{1}{2} k_x x_1^2 \\ &\quad - \frac{1}{4} k_y (y_1^2 + x_1^2 + a^2 \beta^2) - \frac{1}{2} k s^2 \end{aligned} \quad (7)$$

To obtain the final form of the Lagrange equations the partial derivative with respect of generalized coordinates and generalized speeds are calculated followed by the derivatives with respect of time.

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_1} \right) &= m_1 \cdot \ddot{x}_1 + m_2 [\ddot{x}_1 - \ddot{\beta} \cdot L \cdot \cos(\alpha + \beta) + \\ &\quad \dot{\beta}^2 \cdot L \cdot \sin(\alpha + \beta) - \ddot{s} \cdot \cos(\alpha + \beta) + \dot{s} \cdot \dot{\beta} \cdot \sin(\alpha + \beta) + \\ &\quad \ddot{\beta} \cdot \dot{s} \cdot \sin(\alpha + \beta) + \dot{\beta}^2 \cdot s \cdot \cos(\alpha + \beta)] \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}_1} \right) &= m_1 \cdot \ddot{y}_1 + m_2 [\ddot{y}_1 - \ddot{\beta} L \sin(\alpha + \beta) - \\ &\quad \dot{\beta}^2 L \cos(\alpha + \beta) - \ddot{s} \sin(\alpha + \beta) - \dot{s} \dot{\beta} \cos(\alpha + \beta) - \\ &\quad \ddot{\beta} \dot{s} \cos(\alpha + \beta) + \dot{\beta}^2 s \sin(\alpha + \beta)] \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\beta}} \right) &= m_2 [L\ddot{\beta} + \dot{s}\dot{\beta} + s\ddot{\beta} - \ddot{x}_1 L \cos(\alpha + \beta) + \\ &\quad \dot{x}_1 \dot{\beta} L \sin(\alpha + \beta) + \ddot{x}_1 \dot{s} \sin(\alpha + \beta) + \dot{x}_1 s \dot{\beta} \cos(\alpha + \beta) \\ &\quad - \ddot{y}_1 L \sin(\alpha + \beta) - \dot{y}_1 \dot{\beta} L \cos(\alpha + \beta) - \dot{y}_1 \dot{s} \cos(\alpha + \\ &\quad + \beta) + \dot{y}_1 s \dot{\beta} \sin(\alpha + \beta) + \ddot{s} L] + (J_1 + J_2) \ddot{\beta} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{s}} \right) &= m_2 [\ddot{s} - \ddot{x}_1 \cos(\alpha + \beta) + \dot{x}_1 \dot{\beta} \sin(\alpha + \\ &\quad + \beta) - \ddot{y}_1 \sin(\alpha + \beta) - \dot{y}_1 \dot{\beta} \cos(\alpha + \beta) + \ddot{\beta} L] \end{aligned}$$

According to equation (1) the Lagrange differential equations are:

$$\begin{cases} A_1 \cdot \ddot{x}_1 + B_1 \cdot \ddot{y}_1 + C_1 \cdot \ddot{\beta} + D_1 \cdot \ddot{s} = E_1 \\ A_2 \cdot \ddot{x}_1 + B_2 \cdot \ddot{y}_1 + C_2 \cdot \ddot{\beta} + D_2 \cdot \ddot{s} = E_2 \\ A_3 \cdot \ddot{x}_1 + B_3 \cdot \ddot{y}_1 + C_3 \cdot \ddot{\beta} + D_3 \cdot \ddot{s} = E_3 \\ A_4 \cdot \ddot{x}_1 + B_4 \cdot \ddot{y}_1 + C_4 \cdot \ddot{\beta} + D_4 \cdot \ddot{s} = E_4 \end{cases} \quad (8)$$

were:

$$A_1 = m_1 + m_2$$

$$B_1 = 0$$

$$C_1 = -m_2 \cdot L \cos(\alpha + \beta) + m_2 \cdot \dot{s} \sin(\alpha + \beta)$$

$$D_1 = -m_2 \cdot \cos(\alpha + \beta)$$

$$E_1 = -m_2 \cdot \dot{\beta}^2 \cdot L \sin(\alpha + \beta) - m_2 \cdot \dot{s} \cdot \dot{\beta} \sin(\alpha + \beta) - m_2 \cdot \dot{\beta}^2 \cdot s \cos(\alpha + \beta) - k_x \cdot x_1 - k_y \cdot y_1$$

$$A_2 = 0$$

$$B_2 = m_1 + m_2$$

$$C_2 = -m_2 \cdot L \sin(\alpha + \beta) - m_2 \cdot \dot{s} \cos(\alpha + \beta)$$

$$D_2 = -m_2 \sin(\alpha + \beta)$$

$$E_2 = m_2 \cdot \dot{\beta}^2 \cdot L \cos(\alpha + \beta) + m_2 \cdot \dot{s} \cdot \dot{\beta} \cos(\alpha + \beta) - m_2 \cdot \dot{\beta}^2 \cdot s \sin(\alpha + \beta) - k_y \cdot y_1$$

$$A_3 = -m_2 \cdot L \cos(\alpha + \beta) + m_2 \cdot \dot{s} \sin(\alpha + \beta)$$

$$B_3 = -m_2 \cdot L \sin(\alpha + \beta) - m_2 \cdot \dot{s} \cos(\alpha + \beta)$$

$$C_3 = J_1 + J_2 + m_2 \cdot s^2 + m_2 \cdot L^2$$

$$D_3 = m_2 \cdot L^2$$

$$E_3 = -m_2 \cdot \dot{x}_1 \cdot \dot{\beta} \cdot L \sin(\alpha + \beta)$$

$$- m_2 \cdot \dot{x}_1 \cdot s \cdot \dot{\beta} \cos(\alpha + \beta) + m_2 \cdot \dot{y}_1 \cdot \dot{\beta} \cdot L \cos(\alpha + \beta)$$

$$- m_2 \cdot \dot{y}_1 \cdot s \cdot \dot{\beta} \sin(\alpha + \beta) + m_2 \cdot \dot{x}_1 \cdot \dot{\beta}^2 \cdot L \sin(\alpha + \beta)$$

$$+ m_2 \cdot \dot{x}_1 \cdot \dot{s} \cdot \dot{\beta} \sin(\alpha + \beta) + m_2 \cdot \dot{x}_1 \cdot \dot{\beta}^2 \cdot s \cos(\alpha + \beta)$$

$$- m_2 \cdot \dot{y}_1 \cdot \dot{\beta}^2 \cdot L \cos(\alpha + \beta) - m_2 \cdot \dot{y}_1 \cdot \dot{s} \cdot \dot{\beta} \cos(\alpha + \beta)$$

$$+ m_2 \cdot \dot{y}_1 \cdot \dot{\beta}^2 \cdot s \sin(\alpha + \beta) - \frac{k_y}{2} \cdot a^2 \cdot \beta$$

$$A_4 = -m_2 \cos(\alpha + \beta)$$

$$B_4 = -m_2 \sin(\alpha + \beta)$$

$$C_4 = m_2 \cdot L$$

$$D_4 = m_2$$

$$E_4 = +m_2 \cdot s \cdot \dot{\beta}^2 - k \cdot s$$

The system (8) composed of four differential equations of second order will be solved determining the expression of second order derivatives $\dot{x}_1, \dot{y}_1, \dot{\beta}$ and \dot{s} using the Gauss method for linear equation solving. As results were obtained four differential equations, each of them containing only one second order

derivatives of generalized coordinates, with respect of time

$$\begin{aligned} \ddot{x}_1 &= F_1(x_1, y_1, \beta, s, \dot{x}_1, \dot{y}_1, \dot{\beta}, \dot{s}) \\ \ddot{y}_1 &= F_2(x_1, y_1, \beta, s, \dot{x}_1, \dot{y}_1, \dot{\beta}, \dot{s}) \\ \ddot{\beta} &= F_3(x_1, y_1, \beta, s, \dot{x}_1, \dot{y}_1, \dot{\beta}, \dot{s}) \\ \ddot{s} &= F_4(x_1, y_1, \beta, s, \dot{x}_1, \dot{y}_1, \dot{\beta}, \dot{s}) \end{aligned} \quad (9)$$

Starting from differential equation (9) and noting $x_1 = z_1, \dot{x}_1 = z_2, y_1 = z_3, \dot{y}_1 = z_4, \beta = z_5, \dot{\beta} = z_6, s = z_7$ and $\dot{s} = z_8$ will result eight simultaneous differential equation each of them of first order. The obtained system of differential equation is solved step by step using the numerical method Runge-Kutta [5] [6].

3. NUMERICAL RESULTS

After the numerical solving of first order system of differential equations the displacements and speeds corresponding to the generalized coordinates were determined.

In the following figures 3, 4 and 5 some results are presented.

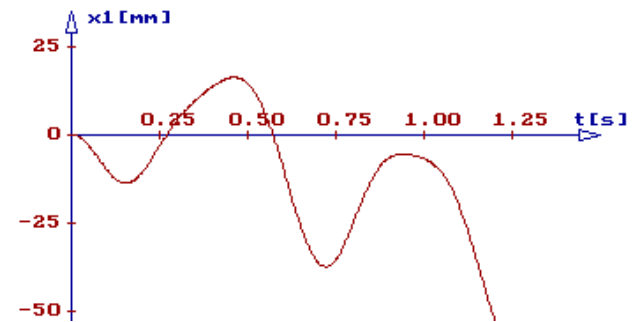


Fig.3 The variation of parameter x_1 depending on time t

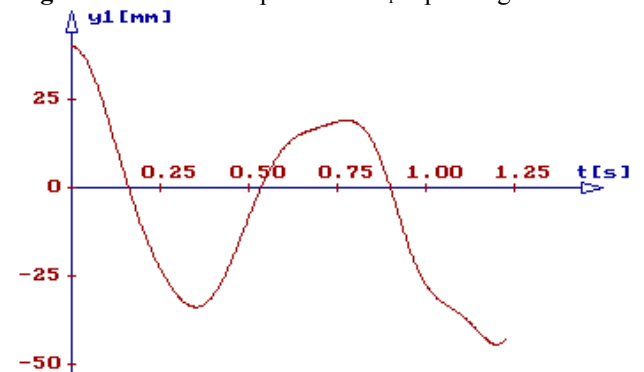


Fig.4 The variation of parameter y_1 depending on time t .

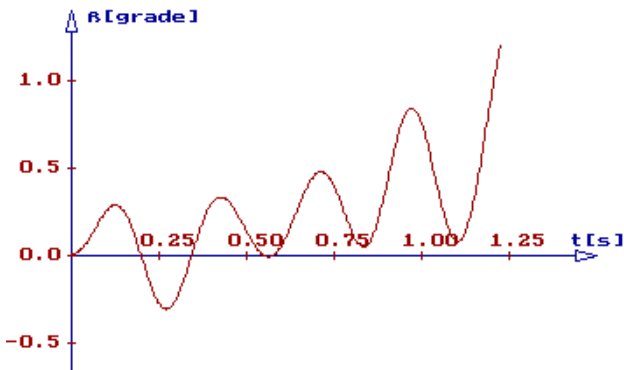


Fig.5 The variation of parameter β depending on time t .

4. CONCLUSIONS

Using Lagrange equations method was established the system of differential equations modeling the movements of the mechanical system.

The obtained differential system, after some transformation was numerically solved with Runge-Kutta method. The obtained results in case of free vibration were presented in figure 3, 4 and 5.

5. REFERENCES

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CONTRIBUȚII LA STUDIUL UNUI VIBROTRANSPORTOR AȘEZAT PE O FUNDATIE ELASTICĂ

Rezumat: În cadrul acestui articol s-a studiat un vibrotransportor situat pe o fundație elastică. Folosind metoda ecuațiilor lui Lagrange s-a determinat sistemul ce trebuie rezolvat. În urma realizării unui program în limbajul C, în care s-a folosit metoda lui Gauss și Runge-Kutta s-au obținut rezultatele numerice privind variația coordonatelor generalizate.

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