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MATHEMATICAL MODEL AND PROGRAM FOR ANALYZING THE THERMOREFRACTORY INSULATION OF FURNACES

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Abstract: *The paper presents a mathematical model and calculating program for determining the thermal field of some multilayer walls. It represents a useful tool for the efficient design of thermo-refractory insulations of industrial furnaces. The authors have successfully emplayed this program with the energetic audit of metallurgical furnaces.*

Key words: *thermal field, block diagram, thermo-refractory insulation, industrial furnaces*

1. INTRODUCTION

The temperature field represents the total instantaneons values of temperature in the investigated area.

In the case of metallurgical furnaces of classic masonry, after about 16 – 20 hours from the start the temperature field inside the walls becomes stationary.

Theoretical and practical elements presented in this work relates to a wall, parallel, with little thickness in that report with other dimensions, so marginal effects in the process of thermal conducted can be neglected, [1],[2].

2. DESIGNATIONS

a, b – coefficients of the function the thermal conductivity varies with temperature.
C_o – radiation coefficient of black body, $[W/m^2 \cdot K^4]$;
c – coefficient dependent on surface orientation;
i – the layer index;
j – the index of the median plane of the layer;
t_a – ambient temperature, [°C];
t_c – furnace temperature, [°C];
t_{pe} – wall outer surface temperature ,[°C];
t_0 – wall inner surface temperature, [°C];
α_{int} – heat transfer coefficient at the inner surface of the wall, $[W/m^2K]$;
α_{ext} – heat transfer coefficient at the outer surface of the wall to the environment, $[W/m^2K]$;
δ_i – the thickness “i” of wall,[m];
λ_i – thermal conductivity of layer material “i”,[W/m· K];
ε_{pe} – emissivity of wall outer surface;
.

3. MATHEMATICAL MODEL

We consider a wall consisting of “n” homogeneons layers, being perfect contact with one another (fig.1). Density of the global thermal flux q transmithed from the working area to the environment, in steady state regime, is given by the relation:

$$q=\frac{t_c-t_a}{\frac{1}{\alpha_{\text{int}}}+\sum_{i=1}^{n}\frac{\delta_i}{\lambda_i}+\frac{1}{\alpha_{ext}}}\quad\left[\frac{W}{m^2}\right]\qquad(1)$$

In steady state, the regim staționar, density of heat flow is constant and equal for each layer, being equal to the density of heat flow transmitted from the outer wall to the environment:

$$q=\frac{t_c-t_1}{\frac{\delta_1}{\lambda_1}}=\ldots=\frac{t_{i-1}-t_i}{\frac{\delta_i}{\lambda_i}}=\ldots=\frac{t_n-t_a}{\frac{1}{\alpha_{ext}}}\quad\left[\frac{W}{m^2}\right]\qquad(2)$$

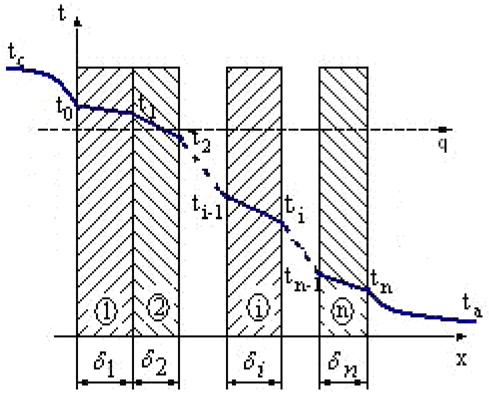

Fig.1. Variation of temperature in the cross section of a plane wall composed of "n" layers.

The superficial coefficient of heat transfer from the outer surface of the wall to the environment is calculated with the relation:

$$\alpha_{ext}=c\cdot\sqrt[4]{t_{pe}-t_a}+\frac{\varepsilon_{pe}\cdot C_c}{t_{pe}-t_a}\cdot\ldots$$
$$\ldots\cdot\left[\left(\frac{t_{pe}+273}{100}\right)^4-\left(\frac{t_a+273}{100}\right)^4\right]\quad\left[\frac{W}{m^2\cdot K}\right]\qquad(3)$$

The value of temperature on one the component layer is obtained on the basis of relatin:

$$t_j=t_c-q\cdot\sum_{i=1}^{j}\frac{\delta_i}{\lambda_i}\quad[°C]\qquad(4)$$

Applying the simplifying assumption of a consyant thermal conductivity and neglecting possible discontinuities at the level of layer interfaces, the temperature curve in the wall thickness is in the form of a broken line (fig.1), whese straight segments correspond to the linear variation of temperature over the thickness of each homogeneons layer.

For determining the temperature pattern from the wall of a furnace the fallawing steps are to be taken, [3]:

- The temperature of the inner surface of the wall to is measured, or estimated fuction of working area temperature t_c:

$$t_0=t_c-\Delta t\quad[°C]\qquad(5)$$

where Δt = 50 … 150 °C;

- The temperature of the environment t_a is measured;
- The onter surface temperature of the wall t_{pe} is measured or imposed.
- The mean temperature of each layer is aproximately determined;
- The value of α_{ext} coefficient is determined;
- The value of thermal conductivity of layers λ_i, as function of mean temperature of each layer medii;

The value of heat flow density $\dot{q}$ is Se calculated with the relation (1) ;

The mean temperatures on each layer are recalculated with the relation:

$$\overline{t_j}=t_c-q\cdot\sum_{i=1}^{j}\left(\frac{\delta_i}{\lambda_i}-\frac{\delta_j}{2\cdot\lambda_j}\right)\quad[°C]\qquad(6)$$

The temperature on the outer surface of the wall is recalculated with the relation:

$$t'_{pe}=t_0-q\cdot\sum_{i=1}^{n}\frac{\delta_i}{\lambda_i}\quad[°C]\qquad(7)$$

The mean temperatures of each layer is compared with the mean temperatures obtained with the first approximation and we compare the temperature on the outer surface t'_{pe} with the initially calculated temperature t_{pe};

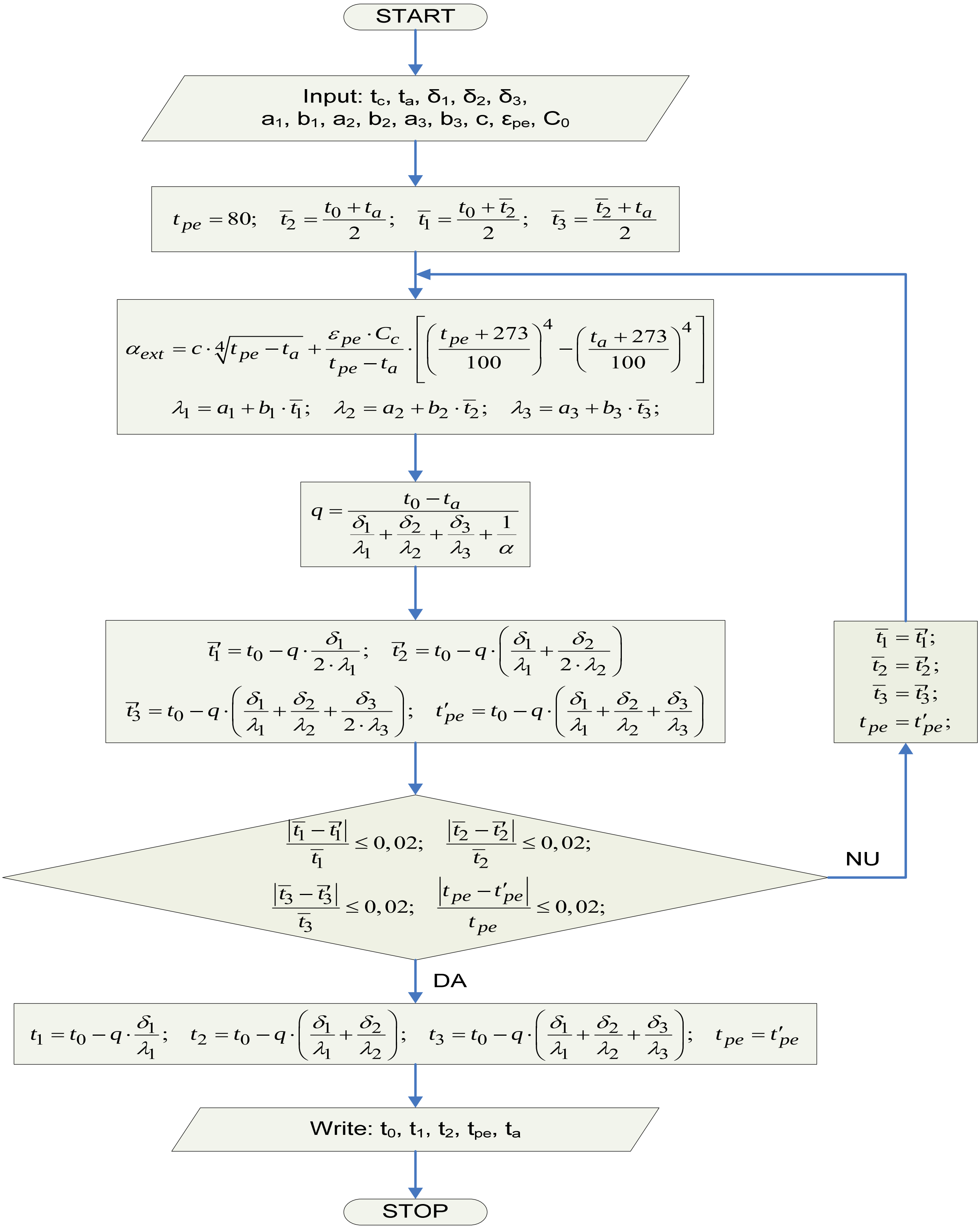

Fig. 2. Black diagram.

- If as a result of comparisons effected in the previons stage, differences greater than a previonsly established threshold result, we return to step/stage five.
- The temperatures on separation surfaces between layers are determined.

4. CASE STUDY

As one can deduce from the block diagram previonsly shown, the calculation of temperatures (t_1, t_2, ... , t_n) on the separation surfaces between layers, can not be directly obtained, it can be effected only through an iteration calculation method due to the dependence of thermal conductivity of the layer materials on the mean temperatures of each layer.

The authors have designed a program enabling them to calculate temperatures based on the algorithm previonsly presented. The program is written in language C , in BorlandC programming medium 3.1, [6],[7].

The program allows calculation of temperatures on the interface of wall component layers, and the analysis of some influencing parameters on heat flow variation, as well.

An example of calculus effected whith the aid of this program is shown as it follows.

The input data are given in table 1.

Table 1.

Denotation	Symbol	Value
Number of layers	n	3
Furnace temperature [°C]	t_c	1200
Ambient temperature[°C]	t_a	23
Layer thickness 1	δ_1	0.064
Layer thickness 2	δ_2	0.065
Layer thickness 3	δ_3	0.124
Layer thermal conductivity 1	λ_1	$0{,}9+ 0{,}7\cdot10^{-3}\cdot t$
Layer thermal conductivity 2	λ_2	$0{,}139+0{,}23\cdot10^{-3}\cdot t$
Layer thermal conductivity 3	λ_3	$0{,}059+0{,}186\cdot10^{-3}\cdot t$
Coefficient dependent on surface orientation (vertical wall)	c	2,56
Radiation coefficient of black body [W/m^2K^4]	C_o	5,775
Temperature of wall outer surface [°C];	t_{pe}	80
Emissivity of wall anter surface	ε_{pe}	0,8

The block diagram afferent o the algoritm presented is shown in figure 2.

Results from rolling programme with dates of entry are presented in table 2:

Table 2

Denotation	Symbol	Value
Furnace temperature [°C]	t_c	1200
Ambiant temperature [°C]	t_a	23
Layer 1 mean temperature [°C]	t_{1m}	1072,60
Layer 2 mean temperature [°C]	t_{2m}	948,16
Wall outer surface temperature [°C];	t_{pe}	77,01

5. CONCLUSIONS

Table 3

Denotation		**Real Balance**	**Optimized balance**	**Variation [%]**
Thermal efficiency, [%]		29,57	41,47	+40
Specific fuel consumption	$[\frac{m^3_{N\ natural\,gas}}{t_{gorgings}}]$	285,93	209,11	-26,87
	$[\frac{t_{ep}}{t_{gorgings}}]$	0,245	0,179	-26,87

The calculation program designed on the mathematical model presented above, allows, firstly, to determine the stationary temperature pattern, required for the calculation of heat lost in an industrial furnace.It may be used both with furnaces working in stationary thermal regime if one knows the temperature variation diagram inside the furnace, tc. Likewise the program may be used by designers, in view of optimizing dimensions of furnace thermo-refractory linings.

The results obtained in optimizing thermorefractory insulations resulted in major energy savings, see table 3.. [4],[5].

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MODEL MATEMATIC ŞI PROGRAM PENTRU ANALIZA IZOLAȚIILOR TERMOREFRACTARE ALE CUPTOARELOR

Rezumat:
Lucrarea prezintă un model matematic şi programul de calcul aferent pentru determinarea câmpului termic al unor pereți multistrat.Ea reprezintă un instrument util pentru analiza energetică sau pentru proiectarea eficientă, a izolațiilor termorefractare ale cuptoarelor industriale.Autorii au folosit cu secces acest program la auditul energetic al unui cuptor metalurgic

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