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# APPROPRIATE CHOOSING THE BALL SPLINE FOR ROBOTIC APPLICATIONS

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***Abstract:*** The Ball spline is an innovative linear motion system in which balls accommodated in the spline nut transmit torque while linearly moving on precision-ground raceways on the spline shaft. The problem of choosing the appropriate type and size of the ball spline is very important to obtain maximum performances of the mechanical system driven by such mechanism. Based on the knowledge of specific operating conditions and data from the producer catalog, this paper provides a practical algorithm of choosing the correct size and verification of the ball spline.
**Key words:** ball spine, linear motion, robotic.

## 1. INTRODUCTION

Ball Spline demonstrates high performance in environments subject to vibrations and impact loads, locations where a high level of positioning accuracy is required or areas where high-speed kinetic performance is required. In addition, even when used as an alternative to a linear bushing, the Ball Spline achieves a rated load more than 10 times greater than the linear bushing with the same shaft diameter. Thus, the Ball Spline (see figure 1) provides a high degree of safety and long service life.

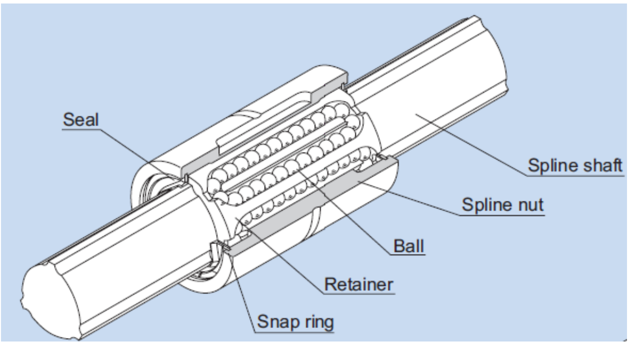
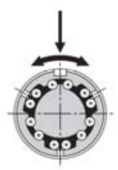
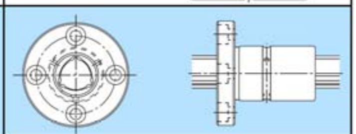
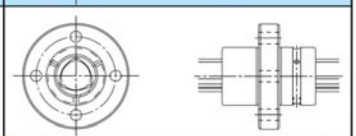
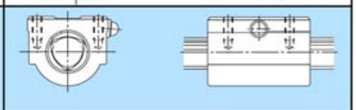
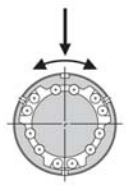
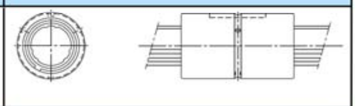
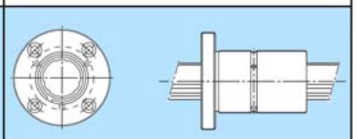
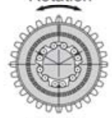
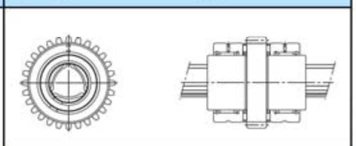
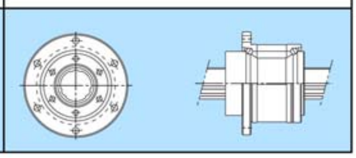



**Fig. 1.** The structure and the features of ball spline.

## 2. TECHNICAL DESCRIPTION

There are three types of the Ball Spline: *high torque* type, *medium torque* type and *rotary* type (table 1). In addition, wide arrays of spline nut shapes are available for each type, enabling the user to choose a desired shape according to the mounting or service requirements.

**Table 1. Ball spline classification** (by THK catalogue)

| Classification | | Type | Shape |
|---|---|---|---|
| High torque type |  | LBS<br>LBST |  |
| | | LBF |  |
| | | LBR |  |
| | | LBH |  |
| Medium torque type |  | LT |  |
| | | LF |  |
| Rotary type | Rotation<br> | LBG<br>LBGT |  |
| | Rotation<br> | LTR-A<br>LTR |  |

The following figure is a flowchart as a measuring stick for selecting a Ball Spline.

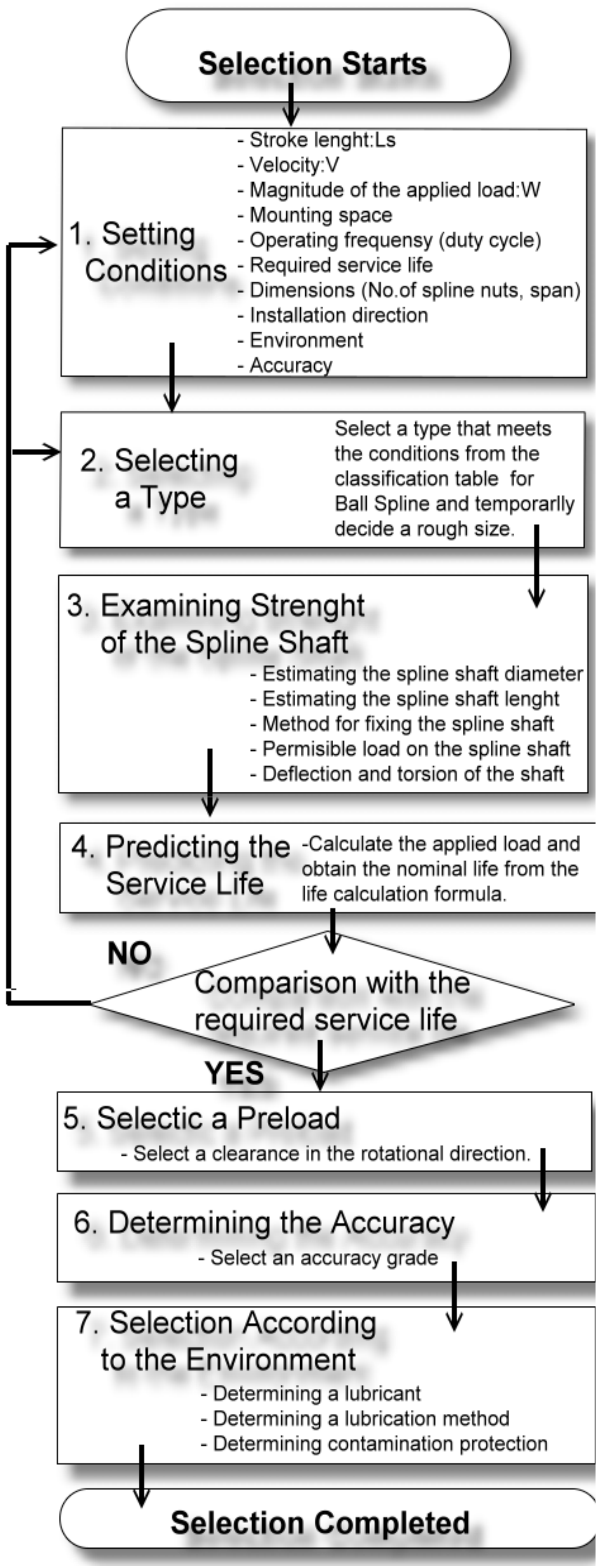



**Fig. 2.** The flowchart for selecting a Ball Spline.

The spline shaft of the Ball Spline is a compound shaft capable of receiving a radial load and torque. When the load and torque are large, the spline shaft strength must be taken into account.

**Studying the Spline Shaft Strength**

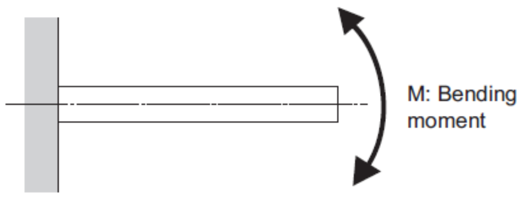



**Fig. 3.** The bending of the ball spline

When a bending load is applied to the spline shaft of a Ball Spline, obtain the spline shaft diameter using the equation (1).

$$M = \sigma \cdot Z \text{, or } Z = \frac{M}{\sigma} \text{, were:} \quad (1)$$

M : Maximum bending moment acting on the spline shaft (N-mm)

σ : Permissible bending stress of the spline shaft ($98N/mm^2$)

Z : Modulus section factor of the spline shaft (mm3)

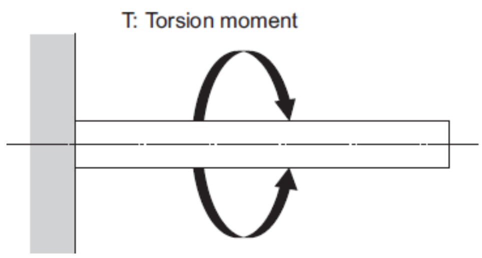



**Fig. 4.** The torsion of the ball spline

When a torsion load is applied on the spline shaft of a Ball Spline, obtain the spline shaft diameter using the equation (2).

$$T = \tau_a \cdot Z_p \text{, or } Z_p = \frac{T}{\tau_a} \text{, were} \quad (2)$$

T : Maximum torsion moment (N-mm)

τa : Permissible torsion stress of the spline shaft ($49\ N/mm^2$)

Zp : Polar modulus of section of the spline nut ($mm^3$)

When the spline shaft of a Ball Spline receives a bending load and a torsion load simultaneously, calculate two separate spline shaft diameters: one for the equivalent bending moment (Me) and the other for the equivalent torsion moment (Te). Then, use the greater value as the spline shaft diameter.

Equivalent bending moment

$$M_e = \frac{M + \sqrt{M^2 + T^2}}{2} = \frac{M}{2}\left\{1 + \sqrt{1 + \left(\frac{T}{M}\right)^2}\right\} \quad (3)$$

with the condition: $M_e \le \sigma \cdot Z_p$ (4)

Equivalent torsion moment

$$Te = \sqrt{M^2 + T^2} = M \cdot \sqrt{1 + \left(\frac{T}{M}\right)^2}$$

with the condition: $T_e \le \tau_a \cdot Z_p$ (5)

Rigidity of the Spline Shaft

The rigidity of the spline shaft is expressed as a torsion angle per meter of shaft length. Its value should be limited within 1°/4.

Rigidity of the shaft: $\theta u = \frac{\theta}{L} < \frac{1^o}{4}$ (6)

$$\theta = 57.3 \cdot \frac{T \cdot L}{G \cdot I_p}$$ , were: (7)

θ : Torsion angle (°)
L : Spline shaft length (mm)
G : Transverse elastic modulus ($7.9 \times 10^4$ N/mm$^2$)
l : Unit length (1000 mm)
$I_p$ : Polar moment of inertia (mm$^4$)

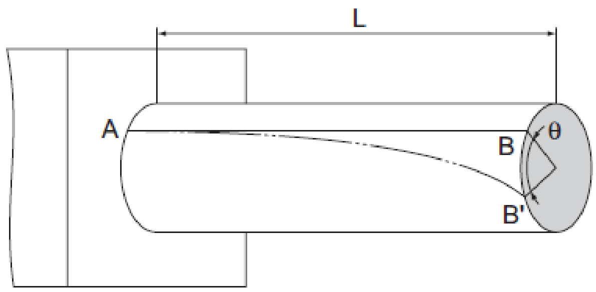



**Fig. 5.** The deflection ungle of an ball spline

The deflection and deflection angle of the Ball Spline shaft (see figure 5) need to be calculated using equations that meet the relevant onditions. Table2 represent these conditions and the corresponding equations.

**Dangerous Speed of the Spline Shaft**

When a Ball Spline shaft is used to transmit power while rotating, as the rotational speed of the shaft increases, the rotation cycle nears the natural frequency of the spline shaft. It may cause resonance and eventually result in inability to move. Therefore, the maximum shaft speed must be limited to a level that does not cause resonance. If the shaft's rotation cycle exceeds or nears the resonance point during operation, it is necessary to reconsider the spline shaft diameter.

The critical speed of the spline shaft is obtained using the equation (8) below, in which the value is multiplied by a safety factor of 0.8.

Critical Speed

$$N_c = \frac{60 \cdot \lambda^2}{2\pi \cdot l_b^2} \cdot \sqrt{\frac{E \cdot 10^3 \cdot I}{\gamma \cdot A}} \cdot 0.8$$ , were: (8)

Nc : Dangerous speed (min-1)
$l_b$: Distance between mounting surfaces (mm)
E : Young's modulus ($2.06 \times 10^5$ N/mm$^2$)
I : Minimum geometrical moment of inertia of the shaft (mm$^4$)

$$I = \frac{\pi}{64} \cdot d^4$$ , d -Minor diameter (mm)

γ: Density (specific gravity) ($7.85 \times 10^{-6}$ kg/mm$^3$)

$$A = \frac{\pi \cdot d^2}{4}$$

λ : Factor according to the mounting method (see figure 5)
(1) Fixed - free λ=1.875
(2) Supported - supported λ=3.142
(3) Fixed - supported λ=3.927
(4) Fixed - fixed λ=4.73

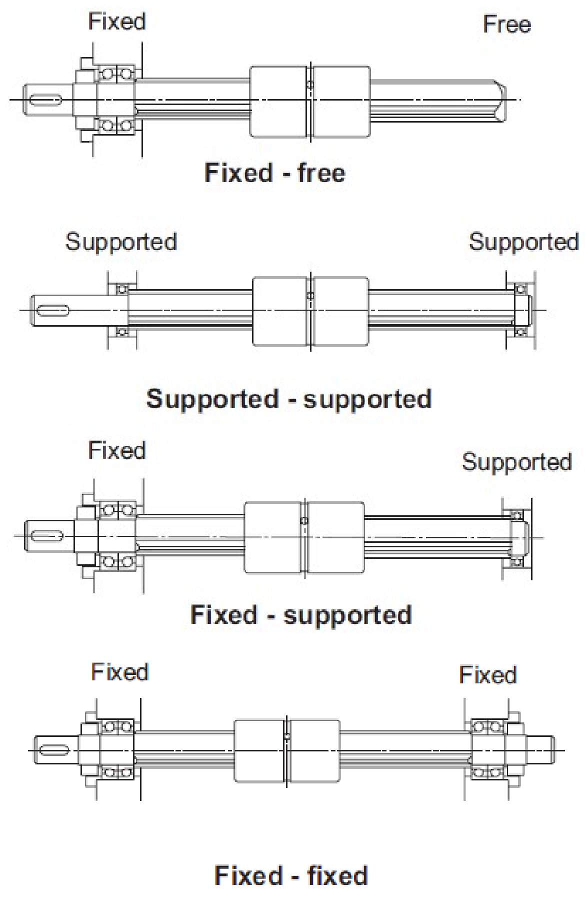
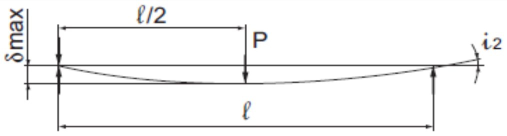
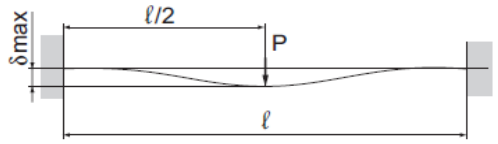
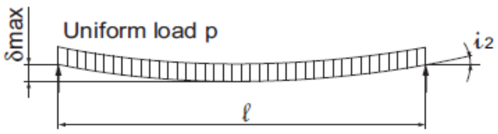
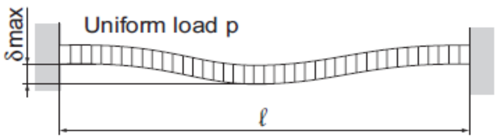
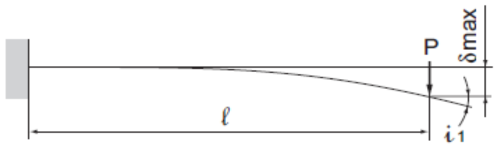
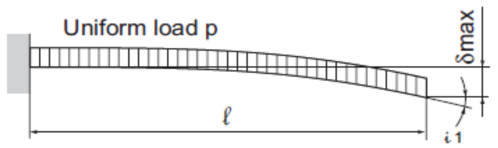
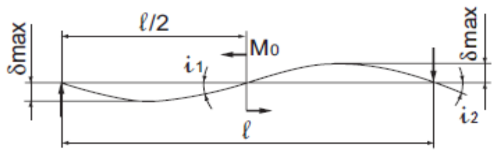
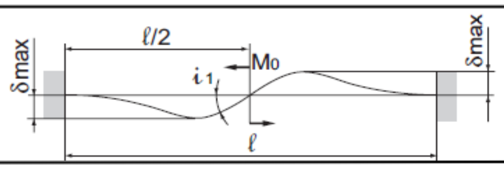



**Fig. 5.** The mounting methods of an ball spline

**Table 2. Deflection and Deflection Angle Equations**

| Support method | Condition | Deflection equation | Deflection angle equation |
|---|---|---|---|
| Both ends free |  | $\delta_{max} = \frac{P\ell^3}{48EI}$ | $i_1 = 0$ <br> $i_2 = \frac{P\ell^2}{16EI}$ |
| Both ends fas-tened |  | $\delta_{max} = \frac{P\ell^3}{192EI}$ | $i_1 = 0$ <br> $i_2 = 0$ |
| Both ends free |  | $\delta_{max} = \frac{5p\ell^4}{384EI}$ | $i_2 = \frac{p\ell^3}{24EI}$ |
| Both ends fas-tened |  | $\delta_{max} = \frac{p\ell^4}{384EI}$ | $i_2 = 0$ |
| One end fas-tened |  | $\delta_{max} = \frac{P\ell^3}{3EI}$ | $i_1 = \frac{P\ell^2}{2EI}$ <br> $i_2 = 0$ |
| One end fas-tened |  | $\delta_{max} = \frac{P\ell^4}{8EI}$ | $i_1 = \frac{P\ell^3}{6EI}$ <br> $i_2 = 0$ |
| Both ends free |  | $\delta_{max} = \frac{\sqrt{3}M_0\ell^2}{216EI}$ | $i_1 = \frac{M_0\ell}{12EI}$ <br> $i_2 = \frac{M_0\ell}{24EI}$ |
| Both ends fas-tened |  | $\delta_{max} = \frac{M_0\ell^2}{216EI}$ | $i_1 = \frac{M_0\ell}{16EI}$ <br> $i_2 = 0$ |

δmax: Maximum deflection (mm)
$M_0$: Moment (N-mm)
l: Span (mm)
I:Geometrical moment of inertia ($mm^4$)
$i_1$: Deflection angle at loading point
$i_2$: Deflection angle at supporting point
P: Concentrated load (N)
p: Uniform load (N/mm)
E: Modulus of longitudinal elasticity
$2.06 \times 10^5$ (N/mm2)

**Predicting the Service Life**

The service life of a Ball Spline varies from unit to unit even if they are manufactured through the same process and used in the same operating conditions. Therefore, the nominal life defined below is normally used as a guidepost for obtaining the service life of a Ball Spline.

Nominal life is the total travel distance that 90% of a group of identical ball splines independently operating under the same conditions can achieve without showing flaking (scale-like pieces on a metal surface).

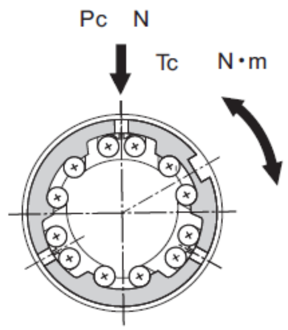



**Fig. 5.** Torque and radial load applied of an ball spline.

### Calculating the Nominal Life

The nominal life of a Ball Spline varies with types of loads applied during operation: torque load, radial load and moment load. The corresponding nominal life values are obtained using the equations (9) to (11) below.

*Note: The basic load ratings in these loading directions are indicated in the manufacturer catalogue for the corresponding model number.*

When a Torque Load is applied

$$L = \left( \frac{f_T \cdot f_C}{f_W} \cdot \frac{C_T}{T_C} \right)^3 \cdot 50 \qquad (9)$$

When a Radial Load is applied

$$L = \left( \frac{f_T \cdot f_C}{f_W} \cdot \frac{C}{P} \right)^3 \cdot 50 \qquad (10)$$

When a torque load and a radial load are simultaneously applied, calculate the nominal life by obtaining the equivalent radial load using the equation (11) below.

$$P_E = P_C + \frac{4 \cdot T_C \cdot 10^3}{i \cdot d_p \cdot \cos\alpha} \qquad (11)$$

$C_T$ : Basic dynamic torque rating (N-m)
$T_C$ : Calculated torque applied (N-m)
C : Basic dynamic load rating (N)
$P_C$ : Calculated radial load (N)
$f_T$ : Temperature factor (see Figure 6)
$f_C$ : Contact factor (see Table 3)
$f_W$ : Load factor (see Table 4)
$P_E$ : Equivalent radial load (N)
cosα : Contact angle i=number of rows of balls under a load

(Type LBSα=45° i=2(LBS10 or smaller)
i=3(LBS15 or greater)
Type LTα=70° i=2(LT13 or smaller)
i=3(LT16 or greater))
dp : Ball center-to-center diameter (mm)

When a Moment Load is applied to a a single nut or two nuts in close contact with each other, obtain the equivalent radial load using the equation (12) below.

$$P_U = K \cdot M \qquad (12)$$

Pu : Equivalent radial load (N) (with a moment applied)
K : Equivalent Factors *(see manufacturer technical sheets/catalogue)*
M : Applied moment (N-mm). However, M should be within the range of the static permissible moment.

When a Moment Load and a Radial Load are Simultaneously Applied calculate the nominal life from the sum of the radial load and the equivalent radial load.

### Calculating the Service Life Time

When the nominal life (L) has been obtained in the equation above, if the stroke length and the number of reciprocations per minute are constant, the service life time is obtained using the equation (13) below.

$$L_h = \frac{L \cdot 10^3}{2 \cdot l_s \cdot n_1 \cdot 60} \quad \text{(h)}, \qquad (13)$$

were:
$L_h$ : Service life time (h)
$l_S$ : Stroke length (m)
$n_1$ : Number of reciprocations per minute (opm)

### $f_T$:Temperature Factor

If the temperature of the environment surrounding the operating Ball Spline exceeds 100°C , take into account the adverse effect of the high temperature and multiply the basic load ratings by the temperature factor indicated in Figure 6. In addition, the Ball Spline must be of a high temperature type. *Note: If the environment temperature exceeds 80°C, high temperature types of seal and retainer are required.*

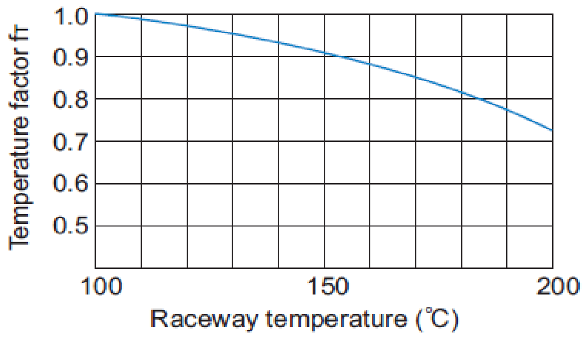



**Fig. 6.** Temperature factor.

**$f_C$: Contact Factor**

When multiple spline nuts are used in close contact with each other, their linear motion is affected by moments and mounting accuracy, making it difficult to achieve uniform load distribution. In such applications, multiply the basic load rating (C) and ($C_0$) by the corresponding contact factor in Table 3.

**Table 3. Contact factor for the ball spline**

| Number of spline nuts in close contact with each other | Contact factor $f_c$ |
|---|---|
| 2 | 0.81 |
| 3 | 0.72 |
| 4 | 0.66 |
| 5 | 0.61 |
| Normal use | 1 |

**$f_W$: Load Factor**

In general, reciprocating machines tend to involve vibrations or impact during operation. It is extremely difficult to accurately determine vibrations generated during high-speed operation and impact during frequent start and stop. When loads applied on a Ball Spline cannot be measured, or when speed and impact have a significant influence, divide the basic load rating (C or $C_0$), by the corresponding load factor in the table of empirically obtained data on Table 4.

**Table 4. Load factor for the ball spline**

| Vibrations/ impact | Speed(V) | $f_w$ |
|---|---|---|
| Faint | Very low $V \leqq 0.25$m/s | 1 to 1.2 |
| Weak | Slow $0.25 < V \leqq 1$m/s | 1.2 to 1.5 |
| Medium | Medium $1 < V \leqq 2$m/s | 1.5 to 2 |
| Strong | High $V > 2$m/s | 2 to 3.5 |

## 3. CONCLUSIONS

The proposed calculation method and algorithm for choosing the appropriate size of the ball spline and checking of its life time is an important tool for designers of industrial robots and complex installations driven by sucs mechanisms.

The calculation and validation methods are based on manufacturer's catalog data and takes into account all static and dynamic loading conditions of the ball spline.

## 4. REFERENCES

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[3]. THK - General Catalogue; Linear Motion Systems

[4] www.thk.com/us

**Alegerea adecvata a transisiei canelate cu bile pentru aplicatii robotice.**

**Rezumat**: Lucrarea prezintă premisele teoretice necesare si algoritmul alegerii optime a tipului şi mărimii transmisiilor canelate cu bile ce intră în structura mecanica a sistemelor de acționare de mare precizie, caz specific acționarii roboților industriali şi altor instalații complexe. Lucrarea se bazeaza pe condițiile reale de încărcare statică şi dinamică ale unui arbore canelat cu bile, precum si pe datele de catalog ale unor firme de prestigiu (ex.THK-SUA).

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