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KINEMATICS, WORKSPACE AND SINGULARITY ANALYSIS OF A NEW RECONFIGURABLE PARALLEL ROBOT

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Abstract: *Reconfigurable robots are actuated mechanisms capable of achieving different industrial applications, using the same equipment. Different configurations of these robots involve different degrees of freedom and different workspaces. This paper starts by presenting some achievements in this field and continues by presenting Recrob – the robot under study, the kinematics, workspace and singularity analysis of this robot. The mathematical models presented here are made for the general case in witch the robot has 6 degrees of freedom. Some configuration possibilities are also presented.*
Key words: *Parallel robot, kinematics, workspace, singularity, reconfiguration.*

1. INTRODUCTION

A parallel reconfigurable robot has multiple links and joints that can be assembled into different robot geometries. Compared to a conventional industrial robot with fixed geometry, such a system can provide great flexibility to the user, enabling him to accomplish a variety of tasks through proper selection and reconfiguration of the parallel robot.

A parallel reconfigurable robot can be rapidly modified by changing leg positions, joint types, link lengths or by adding different constraint elements.

This allows for flexibility, variety in use, rapid changeover and ease of maintenance.

Reconfigurability is, as mentioned in [1], a change of the characteristics of the robot during operation. Stechert proposes in his paper a classification of reconfiguration: static and dynamic reconfiguration.

Static reconfiguration assumes a manual reconstruction of the robot. For example the orientation of actuators can be changed, thus the workspace is changed.

There are two types of dynamic reconfiguration: The first type uses the transition of singularities of type I and II in order to create an additional area of the workspace. The second uses a change of kinematic characteristics in operation. In order to do this, variable length links and multiple degree joints that can block one degree of freedom are used.

Reconfigurable robot prototypes have been developed in research institutes worldwide, the most important are presented below.

Chen and Dash presents in [2] and [3] a modular reconfigurable robot that can be assembled into a serial or a parallel structure depending on the desire of the user.

Among the most well known achievements in modular robotics area with reconfiguration possibilities are M-Tran II (Distributed Systems Design Research Group- Tsukuba, Japan), Polybot (Xerox-Palo Alto Research Center, USA), Telecube (Xeroc-PARC), CONRO (University of Southern California-Los Angeles), Crystalline and Molecule (Dartmouth Robotics Lab., SUA), I-Cube (Advanced Mechatronics Laboratory-Carnegie Mellon University), Atron (Adaptronics Group-University of southern Denmark), Titech (Tokyo Institute of Technology). Also modular reconfigurable robots are studied in [4], [5] and [6].

Reconfigurable tripod – based robots are introduced in [7] and [8]. Some of the robot's links are adjustable, hence the structure's reconfigurability.

A new family of parallel reconfigurable robots is proposed by Gogu in [9]. The structure, called Isogliden-TaRb can have up tu 5 degrees of freedom witch are a combination of maximum 3 independent translations and 2 rotations. The reconfiguration is obtained by blocking several actuators without any change in the architecture of the robot.

Yang has focused on the design and kinematics analysis of 3-leged modular reconfigurable robots [10].

Another parallel reconfigurable machine was developed by Negri in [11].

Choi studied in [12] a reconfigurable planar parallel robot that has the capacity of being reconfigured into various types of planar parallel robots. The robot's workspace can be changed by coupling or decoupling the platform and two or more 2R chains, each having a passive joint on level 2.

Other similar works on parallel reconfigurable robots have been made in [13], [14], [15], [16], [17]. The robot under study is contained in the patent [18], and continues the work presented in [19].

2. RECONFIGURATION OF RECROB

Recrob, the robot studied in this paper can be static reconfigured. The base model has 6 degrees of freedom, but can be reconfigured to a 5, 4, 3, or 2 degree of freedom robot, by blocking one ore more motors and introducing some constraint links. Figure 1 presents the structure of Recrob.

Recrob's reconfiguration possibilities are presented below:

a) 5 d.o.f Recrob

The drive are $q_6 = q_5, q_4, q_3, q_2, q_1$. The end-effector performs translations along OX, OY and OZ axes and two rotations (precession an d nutation angles). The coordinates of the end-effector are: $X_E, Y_E, Z_E, \psi, \theta, \varphi = 0$.

b) 4 d.o.f Recrob

The drive coordinates are: $q_6 = q_5 = q_4, q_3, q_2, q_1$. The mobile platform performs one rotation and 3 translations. The coordinates of the end-effector are: $X_E, Y_E, Z_E, \psi, \ \theta = 0, \ \varphi = 0$

c) 3 d.o.f planar Recrob

The drive coordinates are: $q_6 = q_5 = q_4 = ct, q_3, q_2, q_1$.The end-effector can make translations along OX and OY axes and a rotation around OZ axis. The coordinates of the end-effector are: $X_E, Y_E, Z_E = ct, \psi, \ \theta = 0, \ \varphi = 0$

d) 3 d.o.f spatial Recrob

The drive coordinates are: $q_6 = q_5 = q_4, q_3 = q_2, q_1$. This time, the mobile platform is able to move along the OX, OY and OZ. The end-effector coordinates are: $X_E, Y_E, Z_E, \psi = 0, \ \theta = 0, \ \varphi = 0$

e) 2 d.o.f Recrob

The drive coordinates are: $q_6 = q_5 = q_4 = ct, q_3 = q_2, q_1$. The end-effector performs translations along Ox and OZ axes and it's coordinates are: $X_E, Y_E, Z_E = ct, \psi = 0, \ \theta = 0, \ \varphi = 0$.

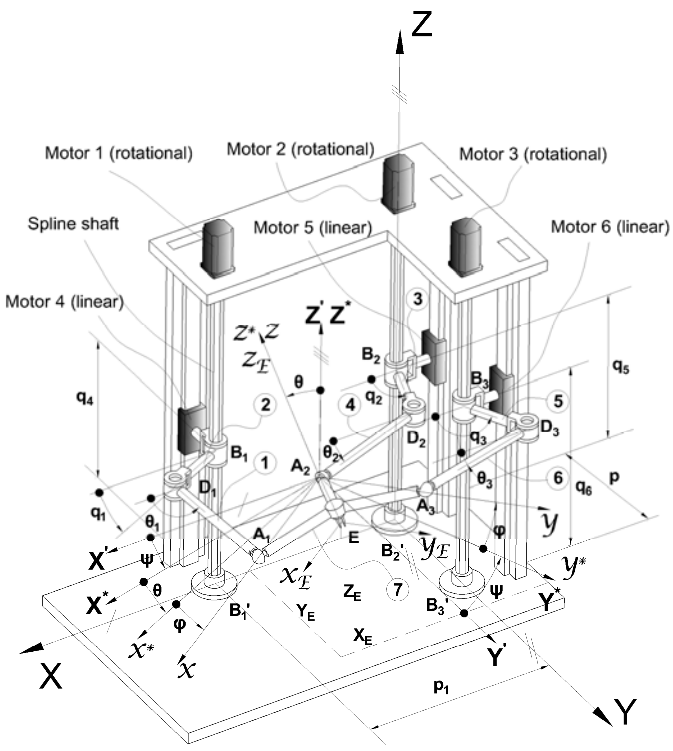

Fig. 1. Recrob – Kinematic Scheme

3. THE GEOMETRIC MODEL OF RECROB

In order to solve the kinematic model of Recrob, the main equation of the inverse and direct geometric model will be first presented.

3.1 Inverse geometric model

In the inverse geometric model [18], the generalized coordinates of the end-effector

$X_E, Y_E, Z_E, \psi, \theta, \varphi$ are given and the driven coordinates $q_1, q_2, ..., q_6$ of the robot are computed. The resulting equations for computing the driving coordinates are presented below, but are not proved here because they are not the objective of this paper.

$$q_{i+3} = Z_{Ai}, i = 1,2,3 \quad (1)$$

$$q_i = \arctan 2(c_i, \pm\sqrt{a_i^2 + b_i^2 - c_i^2}) - a\tan 2(a_i, b_i) \quad (2)$$
$$i = 1,2,3$$

Where:

$$a_i = X_{Ai} - X_{Bi} \quad (3)$$

$$b_i = Y_{Ai} - Y_{Bi} \quad (4)$$

$$c_i = \frac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^2 - e_i^2}{2 \cdot d_i} \quad (5)$$

3.2 Direct geometric model

In the direct geometric model, the driving coordinates q_1, q_2, ..., q_6 are given, and the end-effector's coordinates $X_E, Y_E, Z_E, \psi, \theta, \varphi$ are computed.

The implicit system of functions for solving the direct geometric model is a nonlinear system of functions (6), thus a numerical approach is best suited for solving it.

$$\begin{cases} F_i \equiv [X_E + (x_{Ai} - x_E)c\alpha' + (y_{Ai} - y_E)c\alpha'' + \\ (z_{Ai} - z_E)c\alpha''' - X_{Di}]^2 + [Y_E + (x_{Ai} - x_E)c\beta' + \\ (y_{Ai} - y_E)c\beta'' + (z_{Ai} - z_E)c\beta''' - Y_{Di}]^2 - e_i^2 = 0 \\ F_{i+3} \equiv Z_E + (x_{Ai} - x_E)c\gamma' + (y_{Ai} - y_E)c\gamma'' + \\ (z_{Ai} - z_E)c\gamma''' - q_{i+3} = 0 \\ \qquad i = 1,2,3 \end{cases} \quad (6)$$

where:

$c\alpha' = c\psi c\theta c\varphi - s\psi s\varphi$	$c\alpha'' = -c\psi c\theta c\varphi - s\psi c\varphi$	$c\alpha''' = c\psi s\theta$
$c\beta' = s\psi c\theta c\varphi + c\psi s\varphi$	$c\beta'' = -s\psi c\theta s\varphi + c\psi c\varphi$	$c\beta''' = s\psi s\theta$
$c\gamma' = -s\theta c\varphi$	$c\gamma'' = s\theta s\varphi$	$c\gamma''' = c\theta$

(7)

4. KINEMATICS

4.1 The inverse kinematic model of Recrob

Knowing the end-effector's velocities, $\dot{X}_E, \dot{Y}_E, \dot{Z}_E, \dot{\psi}, \dot{\theta}, \dot{\varphi}$, the driving velocities $\dot{q}_1$, $\dot{q}_2$, ..., $\dot{q}_6$ are computed using equation (9).

$$A \cdot \dot{X} + B \cdot \dot{q} = 0 \quad (8)$$

$$\dot{q} = -B^{-1} \cdot A \cdot \dot{X} \quad (9)$$

where:

$$\dot{X} = \begin{bmatrix} \dot{X}_E & \dot{Y}_E & \dot{Z}_E & \dot{\psi} & \dot{\theta} & \dot{\varphi} \end{bmatrix}^T \quad (10)$$

$$\dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix}^T \quad (11)$$

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} & \frac{\partial F_1}{\partial q_6} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} & \frac{\partial F_2}{\partial q_6} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} & \frac{\partial F_3}{\partial q_6} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} & \frac{\partial F_4}{\partial q_6} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} & \frac{\partial F_5}{\partial q_6} \\ \frac{\partial F_6}{\partial q_1} & \frac{\partial F_6}{\partial q_2} & \frac{\partial F_6}{\partial q_3} & \frac{\partial F_6}{\partial q_4} & \frac{\partial F_6}{\partial q_5} & \frac{\partial F_6}{\partial q_6} \end{bmatrix} \quad (13)$$

The driving accelerations $\ddot{q}_1, \ddot{q}_2, ..., \ddot{q}_6$ will be computed, knowing the end-effector accelerations $\ddot{X}_E, \ddot{Y}_E, \ddot{Z}_E, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$.

By deriving equation (8) it results:

$$\mathrm{A} \cdot \ddot{\mathrm{X}} + \dot{\mathrm{A}} \cdot \dot{\mathrm{X}} + \mathrm{B} \cdot \ddot{\mathrm{q}} + \dot{\mathrm{B}} \cdot \dot{\mathrm{q}} = 0 \qquad (14)$$

$$\ddot{\mathrm{q}} = -\mathrm{B}^{-1} \cdot (\dot{\mathrm{A}} \cdot \dot{\mathrm{X}} + \mathrm{A} \cdot \ddot{\mathrm{X}} + \dot{\mathrm{B}} \cdot \dot{\mathrm{q}}) \qquad (15)$$

4.1 The direct kinematic model of Recrob

In the direct kinematic model the input data are the velocities of the actuators $\dot{q}_1$, $\dot{q}_2$, ..., $\dot{q}_6$ and the output data are the velocities of the end-effector $\dot{X}_E, \dot{Y}_E, \dot{Z}_E, \dot{\psi}, \dot{\theta}, \dot{\phi}$.

Having equation (8), it results:

$$\dot{X} = -A^{-1} \cdot B \cdot \dot{q} \qquad (16)$$

Also, having as input data the driving accelerations $\ddot{q}_1$, $\ddot{q}_2$, ..., $\ddot{q}_6$, the end-effector's accelerations result from equation (14):

$$\ddot{X} = -A^{-1} \cdot (\dot{A} \cdot \dot{X} + B \cdot \ddot{q} + \dot{B} \cdot \dot{q}) \qquad (17)$$

The partial derivates of the Jacobi matrix B are presented next using the system (6):

$$\frac{\partial F_1}{\partial q_1} = 2 \cdot (X_E + \frac{2}{3} p(c\psi c\theta c\varphi - s\psi s\varphi) - \frac{1}{3} p(-c\psi c\theta s\varphi - s\psi c\varphi) - z_E c\psi s\theta - X_{B_1} - dcq_1) dsq_1 - 2(Y_E + \frac{2}{3} p(s\psi c\theta c\varphi + c\psi s\varphi) - \frac{1}{3} p(-s\psi c\theta s\varphi + c\psi c\varphi) - z_E s\psi s\theta - dsq_1) dcq_1 \qquad (18)$$

$$\frac{\partial F_2}{\partial q_2} = 2\left(X_E - \frac{1}{3} p(c\psi c\theta c\varphi - s\psi s\varphi) - \frac{1}{3} p(-c\psi c\theta s\varphi - s\psi c\varphi) - z_E c\psi s\theta - dcq_2 \right) dsq_2 - 2\left(Y_E - \frac{1}{3} p(s\psi c\theta c\varphi + c\psi s\varphi) - \frac{1}{3} p(-s\psi c\theta s\varphi + c\psi c\varphi) - z_E s\psi s\theta - dsq_2 \right) dcq_2 \qquad (19)$$

$$\frac{\partial F_3}{\partial q_3} = 2\left(X_E - \frac{1}{3} p(c\psi c\theta c\varphi - s\psi s\varphi) + \frac{2}{3} p(-c\psi c\theta s\varphi - s\psi c\varphi) - z_E c\psi s\theta - dcq_3 \right) dsq_3 - 2\left(Y_E - \frac{1}{3} p(s\psi c\theta c\varphi + c\psi s\varphi) + \frac{2}{3} p(-s\psi c\theta s\varphi + c\psi c\varphi) - z_E s\psi s\theta - p - dsq_3 \right) dcq_3 \qquad (20)$$

$$\frac{\partial F_4}{\partial q_4} = \frac{\partial F_5}{\partial q_5} = \frac{\partial F_6}{\partial q_6} - 1 \qquad (21)$$

The rest of the partial derivates contained in the Jacobi matrix B are equal to zero, thus the Jacobi matrix B is a diagonal matrix.

Next, the partial derivates of the Jacobi matrix A are computed:

$$\frac{\partial F_1}{\partial X_E} = 2X_E + \frac{4}{3} p(c\psi c\theta c\varphi - s\psi s\varphi) - \frac{2}{3} p(-c\psi c\theta s\varphi - s\psi c\varphi) - 2z_E c\psi s\theta - 2X_{B_1} - 2dcq_1 \qquad (22)$$

$$\frac{\partial F_1}{\partial Y_E} = 2X_E + \frac{4}{3} p(c\psi c\theta c\varphi + c\psi s\varphi) - \frac{2}{3} p(-s\psi c\theta s\varphi + c\psi c\varphi) - 2z_E s\psi s\theta - 2dsq_1 \qquad (23)$$

$$\frac{\partial F_1}{\partial Z_E} = 0 \qquad (24)$$

$$\frac{\partial F_1}{\partial \psi} = 2(X_E + \frac{2}{3} p(c\psi c\theta c\varphi - s\psi s\varphi) - \frac{1}{3} p(-c\psi c\theta s\varphi - s\psi c\varphi) - z_E c\psi s\theta - X_{B_1} - dcq_1)\left(\frac{2}{3} p(-s\psi c\theta c\varphi - c\psi s\varphi) - \frac{1}{3} p \cdot \cdot (s\psi c\theta s\varphi - c\psi c\varphi) + z_E s\psi s\theta \right) + 2(Y_E + \frac{2}{3} p(s\psi c\theta c\varphi + c\psi s\varphi) - z_E s\psi s\varphi - dsq_1)\left(\frac{2}{3} p(c\psi c\theta c\varphi - s\psi s\varphi) - \frac{1}{3} p \cdot \cdot (-c\psi c\theta s\varphi - s\psi c\varphi) - z_E c\psi s\theta \right) \qquad (25)$$

$$
\begin{aligned}
&\frac{\partial F_1}{\partial \theta}=2\left(X_E+\frac{2}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\right.\\
&\frac{1}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-z_E c\psi s\theta-X_{B_1}-\\
&\left.dcq_1\right)\left(-\frac{2}{3}pc\psi s\theta c\varphi-\frac{1}{3}p\,c\psi s\theta s\varphi-z_E c\psi c\theta\right)+\\
&2\left(Y_E+\frac{2}{3}p\left(s\psi c\theta c\varphi+c\psi s\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\cdot\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-z_E s\psi s\theta-\\
&\left.dsq_1\right)\left(-\frac{2}{3}ps\psi s\theta c\varphi-\frac{1}{3}ps\psi s\theta s\varphi-z_E s\psi c\theta\right)
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\frac{\partial F_1}{\partial \varphi}=2\left(X_E+\frac{2}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\right.\\
&\frac{1}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-z_E c\psi s\theta-X_{B_1}-\\
&\left.dcq_1\right)\left(\frac{2}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(-c\psi c\theta c\varphi+s\psi s\varphi\right)\right)+2\left(Y_E+\frac{2}{3}p\cdot\right.\\
&\cdot\left(s\psi c\theta c\varphi+c\psi s\varphi\right)-\frac{1}{3}p\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-\\
&\left.z_E s\psi s\theta-dsq_1\right)\left(\frac{2}{3}p\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(-s\psi c\theta c\varphi-c\psi s\varphi\right)\right)
\end{aligned}
\tag{27}
$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial X_E}=2X_E-\frac{2}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\\
&\frac{2}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-2z_E c\psi s\theta-\\
&2dcq_2
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial Y_E}=2Y_E-\frac{2}{3}p\left(s\psi c\theta c\varphi+c\psi s\varphi\right)-\\
&\frac{2}{3}p\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-2z_E s\psi s\theta-\\
&2dsq_2
\end{aligned}
\tag{29}
$$

$$
\frac{\partial F_2}{\partial Z_E}=0
\tag{30}
$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial \psi}=2\left(X_E-\frac{1}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\right.\\
&\frac{1}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-z_E c\psi s\theta-\\
&\left.dcq_2\right)\left(-\frac{1}{3}p\left(-s\psi c\theta c\varphi-c\psi s\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(s\psi c\theta s\varphi-c\psi c\varphi\right)+z_E s\psi s\theta\right)+\\
&2\left(Y_E-\frac{1}{3}p\left(s\psi c\theta c\varphi+c\psi s\varphi\right)-\right.\\
&-\frac{1}{3}p\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-z_E s\psi s\varphi-\\
&\left.dsq_2\right)\left(-\frac{1}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-z_E c\psi s\theta\right)
\end{aligned}
\tag{31}
$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial \theta}=2\left(X_E-\frac{1}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\right.\\
&\left.\frac{1}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-z_E c\psi s\theta-dcq_2\right)\cdot\\
&\cdot\left(\frac{1}{3}pc\psi s\theta c\varphi-\frac{1}{3}p\,c\psi s\theta s\varphi-z_E c\psi c\theta\right)+\\
&2\left(Y_E-\frac{1}{3}p\left(s\psi c\theta c\varphi+c\psi s\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-z_E s\psi s\theta-dsq_2\right)\cdot\\
&\cdot\left(\frac{1}{3}ps\psi s\theta c\varphi-\frac{1}{3}ps\psi s\theta s\varphi-z_E s\psi c\theta\right)
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial \varphi}=2\left(X_E-\frac{1}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)-\right.\\
&\left.\frac{1}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-z_E c\psi s\theta-dcq_2\right)\cdot\\
&\cdot\left(-\frac{1}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(-c\psi c\theta c\varphi+s\psi s\varphi\right)\right)+2\left(Y_E-\frac{1}{3}p\cdot\right.\\
&\cdot\left(s\psi c\theta c\varphi+c\psi s\varphi\right)-\frac{1}{3}p\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-\\
&\left.z_E s\psi s\theta-dsq_2\right)\left(-\frac{1}{3}p\left(-s\psi c\theta s\varphi+c\psi c\varphi\right)-\frac{1}{3}p\cdot\right.\\
&\left.\cdot\left(-s\psi c\theta c\varphi-c\psi s\varphi\right)\right)
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
&\frac{\partial F_2}{\partial X_E}=2X_E-\frac{2}{3}p\left(c\psi c\theta c\varphi-s\psi s\varphi\right)+\\
&\frac{4}{3}p\left(-c\psi c\theta s\varphi-s\psi c\varphi\right)-2z_E c\psi s\theta-\\
&2dcq_3
\end{aligned}
\tag{34}
$$

$$\frac{\partial F_3}{\partial Y_E} = 2Y_E - \frac{2}{3}p\left(s\psi c\theta c\varphi + c\psi s\varphi\right) + \frac{4}{3}p\left(-s\psi c\theta s\varphi + c\psi c\varphi\right) - 2z_E s\psi s\theta - 2p - 2dsq_3 \quad (35)$$

$$\frac{\partial F_3}{\partial Z_E} = 0 \quad (36)$$

$$\frac{\partial F_3}{\partial \psi} = 2\left(X_E - \frac{1}{3}p\left(c\psi c\theta c\varphi - s\psi s\varphi\right) + \frac{2}{3}p\left(-c\psi c\theta s\varphi - s\psi c\varphi\right) - z_E c\psi s\theta - dcq_3\right)\left(-\frac{1}{3}p\left(-s\psi c\theta c\varphi - c\psi s\varphi\right) + \frac{2}{3}p \cdot \cdot\left(s\psi c\theta s\varphi - c\psi c\varphi\right) + z_E s\psi s\theta\right) + 2\left(Y_E - \frac{1}{3}p\left(s\psi c\theta c\varphi + c\psi s\varphi\right) + \frac{2}{3}p\left(-s\psi c\theta s\varphi + c\psi c\varphi\right) - z_E s\psi s\varphi - p\, dsq_3\right)\left(-\frac{1}{3}p\left(c\psi c\theta c\varphi - s\psi s\varphi\right) + \frac{2}{3}p \cdot \cdot\left(-c\psi c\theta s\varphi - s\psi c\varphi\right) - z_E c\psi s\theta\right) \quad (37)$$

$$\frac{\partial F_3}{\partial \theta} = 2\left(X_E - \frac{1}{3}p\left(c\psi c\theta c\varphi - s\psi s\varphi\right) + \frac{2}{3}p\left(-c\psi c\theta s\varphi - s\psi c\varphi\right) - z_E c\psi s\theta - dcq_3\right) \cdot \left(\frac{1}{3}pc\psi s\theta c\varphi + \frac{2}{3}p\,c\psi s\theta s\varphi - z_E c\psi c\theta\right) + 2\left(Y_E - \frac{1}{3}p\left(s\psi c\theta c\varphi + c\psi s\varphi\right) + \frac{2}{3}p \cdot \cdot\left(-s\psi c\theta s\varphi + c\psi c\varphi\right) - z_E s\psi s\theta - dsq_3\right) \cdot \left(\frac{1}{3}ps\psi s\theta c\varphi + \frac{2}{3}ps\psi s\theta s\varphi - z_E s\psi c\theta\right) \quad (38)$$

$$\frac{\partial F_3}{\partial \varphi} = 2\left(X_E - \frac{1}{3}p\left(c\psi c\theta c\varphi - s\psi s\varphi\right) + \frac{2}{3}p\left(-c\psi c\theta s\varphi - s\psi c\varphi\right) - z_E c\psi s\theta - dcq_3\right) \cdot \left(-\frac{1}{3}p\left(-c\psi c\theta s\varphi - s\psi c\varphi\right) + \frac{2}{3}p \cdot \cdot\left(-c\psi c\theta c\varphi + s\psi s\varphi\right)\right) + 2\left(Y_E - \frac{1}{3}p \cdot \cdot\left(s\psi c\theta c\varphi + c\psi s\varphi\right) + \frac{2}{3}p\left(-s\psi c\theta s\varphi + c\psi c\varphi\right) - z_E s\psi s\theta - p - dsq_3\right)\left(-\frac{1}{3}p\left(-s\psi c\theta s\varphi + c\psi c\varphi\right) + \frac{2}{3}p \cdot \cdot\left(-s\psi c\theta c\varphi - c\psi s\varphi\right)\right) \quad (39)$$

$$\frac{\partial F_4}{\partial X_E} = \frac{\partial F_4}{\partial Y_E} = \frac{\partial F_4}{\partial \psi} = 0 \quad (40)$$

$$\frac{\partial F_4}{\partial Z_E} = 1 \quad (41)$$

$$\frac{\partial F_4}{\partial \theta} = -\frac{2}{3}pc\theta c\varphi - \frac{1}{3}pc\theta s\varphi + z_E s\theta \quad (42)$$

$$\frac{\partial F_4}{\partial \varphi} = \frac{2}{3}ps\theta s\varphi - \frac{1}{3}ps\theta c\varphi \quad (43)$$

$$\frac{\partial F_5}{\partial X_E} = \frac{\partial F_5}{\partial Y_E} = \frac{\partial F_5}{\partial \psi} = 0 \quad (44)$$

$$\frac{\partial F_5}{\partial Z_E} = 1 \quad (45)$$

$$\frac{\partial F_5}{\partial \theta} = \frac{1}{3}pc\theta c\varphi - \frac{1}{3}pc\theta s\varphi + z_E s\theta \quad (46)$$

$$\frac{\partial F_4}{\partial \varphi} = \frac{2}{3}ps\theta s\varphi - \frac{1}{3}ps\theta c\varphi \quad (47)$$

$$\frac{\partial F_6}{\partial X_E} = \frac{\partial F_6}{\partial Y_E} = \frac{\partial F_6}{\partial \psi} = 0 \quad (48)$$

$$\frac{\partial F_6}{\partial Z_E} = 1 \quad (49)$$

$$\frac{\partial F_6}{\partial \theta} = \frac{1}{3}pc\theta c\varphi + \frac{2}{3}pc\theta s\varphi + z_E s\theta \quad (50)$$

$$\frac{\partial F_6}{\partial \varphi} = -\frac{1}{3}ps\theta s\varphi + \frac{2}{3}ps\theta c\varphi \quad (51)$$

5. RESULTS OF THE KINEMATIC SIMULATION

The graphical data from the next figures was obtained using the software package Matlab.

For the simulation, the following geometrical parameters were chosen:

$$\begin{cases} d_1 = d_2 = d_3 = d = 0.058[m] \\ e_1 = e_2 = e_3 = e = 0.109[m] \\ x_{A_1} = p, y_{A_1} = 0, z_{A_1} = 0 \\ x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0 \\ x_{A_3} = 0, y_{A_3} = p, z_{A_3} = 0 \\ x_E = \frac{p}{3}, y_E = \frac{p}{3}, z_E = -h = -0.04[m] \\ X_{B_i} = X_{B_i^{'}} = p_1, Y_{B_i} = Y_{B_i^{'}}, Z_{B_i} = Z_{B_i^{'}} + q_{i+3} \\ p = 0.01[m] \end{cases} \quad (52)$$

Also it needs to be mentioned that the maximum speed was chosen to be 0.01 [m/s] and the maximum acceleration was chosen 0.005 [m/s^2].

The results for a linear trajectory of the end-effector with the followind data is presented in figure 2:

- the starting point is A(0.120, 0.140, 0.150)
- the end point is B(0.140, 0.130, 0.130)

Where A and B are two arbitrary points contained in the workspace.

Figure 2 presents the movements, velocities and accelerations profiles for the end-effector, respectively active joints.

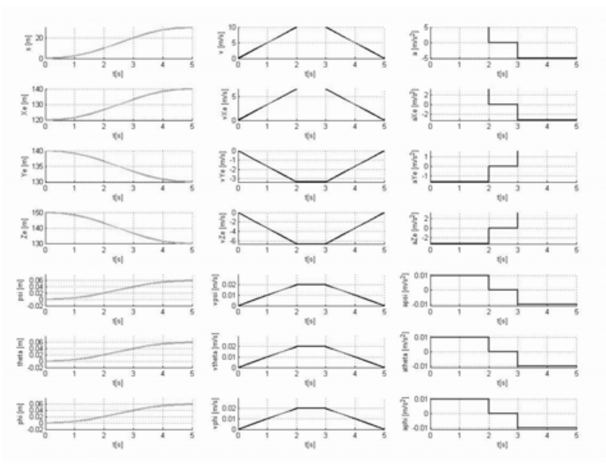

Fig. 2. Recrob – Linear trajectory in space

6. WORKSPACE OF RECROB

As the reachable locations of an end-effector reference point are dependent on it's orientation, a complete representation of the workspace should be embedded in a six-dimensional space, of witch graphical illustration for an intuitive interpretation is not possible [20].

Out of the many types of workspaces, the constant orientation workspace was chosen. The constant orientation workspace represents the set of all possible locations that can be reached by the end-effector reference point with a specific orientation.

Figure 3 presents the workspace of Recrob with the orientation: $\psi = 5^\circ, \theta = 10^\circ, \Psi = 4^\circ$.

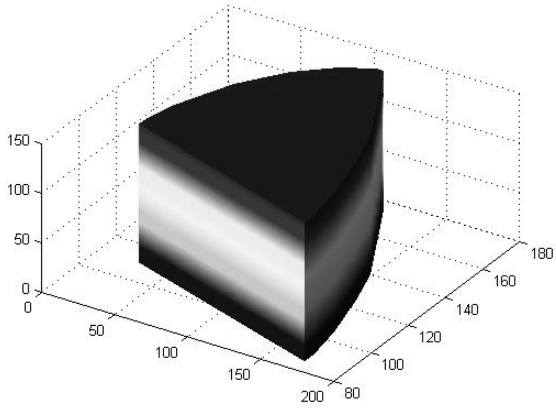

Fig. 3. Recrob – Constant orientation workspace

In figure 4 it is represented a section in the constant orientation workspace; the section is parallel with the Z plane.

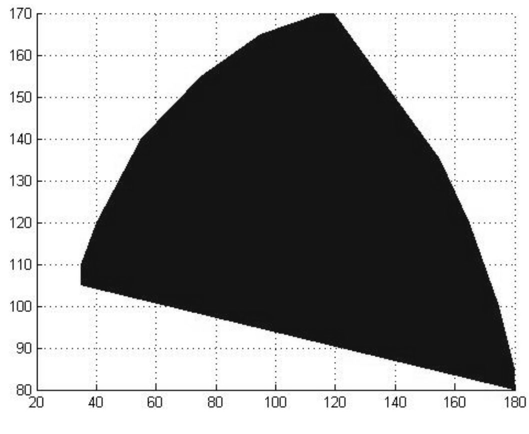

Fig. 4. Recrob – Constant orientation workspace section parallel with the Z plane

The workspace presented in figure 3 and 4 was programmed using the package software Matlab.

5. SINGUARITY ANALYSIS OF RECROB

5.1 Singularity points type I

This type of singularity occurs when the B matrix (13) looses rank. In this case the robot blocks loosing one or more degrees of freedom.

For the command of the robot this point need to be eliminated from the working area.

$$\det(B) = 0 \tag{53}$$

The B matrix takes the following form:

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial F_6}{\partial q_6} \end{bmatrix} \tag{54}$$

The determinant of B can be written as:

$$\det(B) = \prod_{i=1}^{6} \frac{\partial F_i}{\partial q_i},\ i = 1,\ldots,6 \tag{55}$$

Equation (55) leads to:

$$(X_{Ai} - X_{Bi}) \cdot \mathrm{s}(q_i) - (Y_{Ai} - Y_{Bi}) \cdot \mathrm{c}(q_i) = 0 \quad i = 1,2,3 \tag{56}$$

If relation (56) is divided by cq_i:

$$\mathrm{tg}(q_i) = \frac{Y_{Ai} - Y_{Bi}}{X_{Ai} - X_{Bi}},\quad i = 1,2,3 \tag{57}$$

If in equation (57) $X_{Ai} = X_{Bi}$ (witch means that the segment B_iD_i is collinear with the segment D_iA_i), then the robot is in a singular point type I. In figure 1, it can be seen that this occurs when:

$$\theta_i = \pi,\ i = 1,2,3 \tag{58}$$

5.2 Singularity points type II

Singularity points type II appear when the determinant of A matrix (12) looses rank. In this case, the mobile platform can move, even if the actuators are fixed – the robot gains one ore more degrees of freedom.

$$\det(A) = 0 \tag{59}$$

The A matrix takes the following form, after computing the partial derivates in it:

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & 0 & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & 0 & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ 0 & 0 & 1 & 0 & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ 0 & 0 & 1 & 0 & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ 0 & 0 & 1 & 0 & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \tag{60}$$

Having some zeroes and ones, the determinant of A matrix can be further decomposed:

$$\det(A) = \begin{vmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi} \end{vmatrix} \cdot \left[\left(\frac{\partial F_5}{\partial \theta} - \frac{\partial F_6}{\partial \theta} \right)\left(\frac{\partial F_4}{\partial \varphi} - \frac{\partial F_6}{\partial \varphi} \right) - \left(\frac{\partial F_5}{\partial \varphi} - \frac{\partial F_6}{\partial \varphi} \right)\left(\frac{\partial F_4}{\partial \theta} - \frac{\partial F_6}{\partial \theta} \right) \right] \tag{61}$$

The next notations are made:

$$M = \begin{vmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi} \end{vmatrix} \tag{62}$$

$$N = \left(\frac{\partial F_5}{\partial\theta} - \frac{\partial F_6}{\partial\theta}\right)\left(\frac{\partial F_4}{\partial\varphi} - \frac{\partial F_6}{\partial\varphi}\right) - \left(\frac{\partial F_5}{\partial\varphi} - \frac{\partial F_6}{\partial\varphi}\right)\left(\frac{\partial F_4}{\partial\theta} - \frac{\partial F_6}{\partial\theta}\right) \quad (63)$$

To see if A looses rank or not, either M=0, or N=0.

Case 1.

$$N = 0 \quad (64)$$

In this case, due to the long expressions of the partial derivates, a program was made to determine the zeroes of the equation. This program was developed in Maple 12 Package Software.

After the reduction of terms it results:

$$-p^2 c\theta s\theta = 0 \quad (65)$$

Equation (30) equals to zero when:

$$\theta = 0, \pi, \frac{\pi}{2} \quad (66)$$

Case 2.

$$M = 0 \quad (67)$$

For the same reasoning that was used in case 1, the long expression containing the partial derivates was reduced using Maple, yet no solutions were found.

The values (66) represent the only values for witch matrix A looses rank.

It is very important to mention that θ (the rotation around the OY' axis and one of the Euler angles) from equation (66) is different from θ_i witch represents the angle between the B_iD_i segment and D_iA_i segment.

5.3 Singularity points type III

The last type of singularity that can occur is when both the determinants of the two matrixes are null.

$$\det(A) = 0 \ \ and \ \ \det(B) = 0 \quad (68)$$

Equation (68) can never be true since the two determinants have different roots.

6. CONCLUSIONS

In this paper there was presented Recrob, as part of the family of robots called reconfigurable robots. The kinematics, workspace and singular points of this robot were also presented.

As future works there can be mentioned the modeling of the reconfiguration stages of this robot and the dynamic model.

7. ACKNOWLEDGEMENTS

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Cinematica, spatiul de lucru si analiza singularitatilor unui nou robot parallel reconfigurabil

Robotii reconfigurabili sunt mecanisme capabile sa realizeze diferite aplicatii industriale, folosind aceleasi elemente ale robotului. Diferite configuratii ale acestor roboti implica diferite grade de libertate si diferite spatii de lucru. Lucrarea de fata incepe prin a prezenta stadiul actual el cercetarilor in acest domeniu si continua prin a prezenta Recrob – robotul present in aceasta lucrare, cinematica, spatial de lucru si analiza singularitatilor ale acestuia. Modelele matematice prezentate aici sunt facute pentru cazul general in care robotul are 6 grade de libertate. Posibilitatile de reconfigurare sunt de asemenea prezentate.

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