TECHNICAL UNIVERSITY OF CLUJ-NAPOCA

ACTA TECHNICA NAPOCENSIS

Series: Applied Mathematics and Mechanics
Vol. 54, Issue I, 2011

MODELING OF A PARALLEL RECONFIGURABLE ROBOT WITH SIX DEGREES OF MOBILITY AND ITS CONFIGURATIONS WITH FIVE, FOUR, THREE AND TWO DEGREES OF MOBILITY

Dorin-Bogdan LEŞE, Nicolae PLITEA

Abstract: *Reconfigurable robots are more and more studied, due to the flexibility they offer to the user. This flexibility exists because reconfigurable robots may be configured to have different geometries in order to cope with different industrial tasks. This work presents Recrob – an innovative reconfigurable robot and it's reconfiguration possibilities.*
Key words: *parallel robot, reconfiguration, reconfigurability, geometric modeling, design.*

1. INTRODUCTION

A new trend in robotics is "reconfigurable robotics". Reconfigurable robots are robots that can change their form. Such a system can provide flexibility to the user, enabling him to accomplish a variety of tasks through proper selection and reconfiguration of the parallel robot.

A parallel reconfigurable robot can be rapidly modified by changing leg positions, joint types, the link lengths or by adding different constraint elements.

This robotic structure is the objective of a pending patent presented in [1].This paper continues the work presented in [2] and details the reconfiguration possibilities of Recrob.

Reconfigurability is, as mentioned in [3], a change of the characteristics of the robot during operation. Stechert proposes in his paper a classification of reconfiguration: static and dynamic reconfiguration.

Static reconfiguration assumes a manual reconstruction of the robot. For example the orientation of actuators can be changed, thus the workspace is changed.

There are two types of dynamic reconfiguration: The first type uses the transition of singularities of type I and II in order to create an additional area of the workspace. The second uses a change of kinematic characteristics in operation. In order to do this, variable length links and multiple degree joints that can block one degree of freedom are used.

Reconfigurable robot prototypes have been developed in research institutes worldwide, the most important are presented below.

Chen and Dash presents in [4] and [5] a modular reconfigurable robot that can be assembled into a serial or a parallel structure depending on the desire of the user. Other modular reconfigurable robots are studied in [6], [7] and [8].

Reconfigurable tripod – based robots are introduced in [9] and [10]. Some of the robot's links are adjustable, hence the structure's reconfigurability.

A new family of parallel reconfigurable robots is proposed by Gogu in [11]. The structure, called Isogliden-TaRb can have up tu 5 degrees of freedom witch are a combination of maximum 3 independent translations and 2 rotations. The reconfiguration is obtained by blocking several actuators without any change in the architecture of the robot.

Yang presented in [12] the design and kinematics of 3-legged modular reconfigurable robots.

Another parallel reconfigurable machine was developed by Negri in [13].

Choi studied in [14] a reconfigurable planar parallel robot that has the capacity of being reconfigured into various types of planar parallel robots.

Other similar works on parallel reconfigurable robots have been made in [15], [16], [17], [18], [19].

2. STRUCTURAL CONSIDERATIONS

Recrob, the robot studied in this paper can be static reconfigured. The base model presented in figure 1 has 6 degrees of freedom, but can be reconfigured to have 5, 4, 3 or 2 dof. This configuration can be obtained by blocking one ore more motors, thus constraining the desired degrees of freedom.

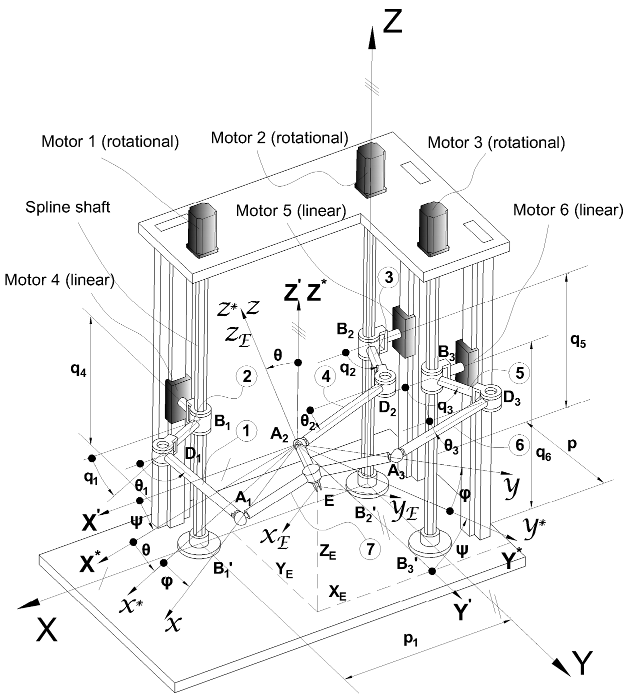

Fig. 1. Recrob-6dof-kinematic scheme

Next, the degree of freedom of the general structure is computed with the following equation [20]:

$$M=(6-F)N-(5-F)C_5-(4-F)C_4-(3-F)C_3-(2-F)C_2-(1-F)C_1 \quad (1)$$

where:

- M – mobility degree of the mechanism;
- F – Mechanism family = the number of common constraints for all mechanism elements;
- N – number of mobile elements;
- C_i – the number of joints of i – class;

Observation: In equation (1) the coefficients (1-F), … , (6-F) cannot take negative values.

$$F=0, N=7, C_5=3, C_4=3, C_3=3 \quad (2)$$

$$M=6N-5C_5-4C_4-3C_3 \quad (3)$$

$$M=6 \quad (4)$$

2.1 Inverse geometric model for M=6 dof*

In the general case the end-effector's coordinates $X_E, Y_E, Z_E, \psi, \theta, \varphi$ are given and the driving coordinates $q_1, q_2, q_3, q_4, q_5, q_6$ are computed (ψ, θ, φ are Z'Y*z* - Euler angles).

$$X_E, Y_E, Z_E, \psi, \theta, \varphi \Rightarrow q_1, q_2, q_3, q_4, q_5, q_6 \quad (5)$$

The coordinates of points $A_i, i=1,2,3$ corresponding to the moving platform can be expressed with the next equations:

$$\begin{cases} X_{A_i} = X_{B_i} + d_i c q_i + e_i c\theta_i \\ Y_{A_i} = Y_{B_i} + d_i s q_i + e_i s\theta_i \quad, i=1,2,3 \\ Z_{A_i} = q_{i+3} \end{cases} \quad (6)$$

The angles between the axes of the mobile system A_2xyz attached to the mobile platform, and the fixed system $OXYZ$ are expressed in (7):

	A_2x	A_2y	A_2z
OX	$c\alpha'$	$c\alpha''$	$c\alpha'''$
OY	$c\beta'$	$c\beta''$	$c\beta'''$
OZ	$c\gamma'$	$c\gamma''$	$c\gamma'''$

(7)

The coordinates A_i, $i=1,2,3$ can be written as follows:

$$\begin{bmatrix} X_{A_i}-X_E \\ Y_{A_i}-Y_E \\ Z_{A_i}-Z_E \end{bmatrix} = \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i}-x_E \\ y_{A_i}-y_E \\ z_{A_i}-z_E \end{bmatrix} \quad (8)$$

$$i=1,2,3$$

The direction cosine matrix for $Z'Y^*z^*$ variant is:

---

*In all the equations were used the following notations: s=sin, c=cos; dof=degree of freedom;

$$\begin{array}{c|c|c} c\alpha' = c\psi c\theta c\varphi - & c\alpha'' = -c\psi c\theta c\varphi - & \multirow{2}{*}{$c\alpha''' = c\psi s\theta$} \\ -s\psi s\varphi & -s\psi c\varphi & \\ \hline c\beta' = s\psi c\theta c\varphi + & c\beta'' = -s\psi c\theta s\varphi + & \multirow{2}{*}{$c\beta''' = s\psi s\theta$} \\ +c\psi s\varphi & +c\psi c\varphi & \\ \hline c\gamma' = -s\theta c\varphi & c\gamma'' = s\theta s\varphi & c\gamma''' = c\theta \end{array} \quad (9)$$

The rotation matrix expressed in (9) resulted from:

$$\begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c\theta & 0 & s\theta \\ 0 & 1 & 0 \\ -s\theta & 0 & c\theta \end{bmatrix} \cdot \begin{bmatrix} c\varphi & -s\varphi & 0 \\ s\varphi & c\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Where ψ (the precesion angle) represents the rotation around OZ' axis, θ (nutation angle) represents the rotation around OY^* axis, φ represents the rotation around Oz^* axis.

The coordinates of $A_i,\ i = 1,2,3$ points resulted from equation (8) are:

$$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix} \quad (11)$$
$$i = 1,2,3$$

The driving coordinates $q_{i+3}, i = 1,2,3$ are computed with the equation (12):

$$q_{i+3} = Z_{A_i},\ \ i = 1,2,3 \quad (12)$$

For computing the driving coordinates $q_i, i = 1,2,3$ the algorithm starts from the next equation:

$$(X_{A_i} - X_{D_i})^2 + (Y_{A_i} - Y_{D_i})^2 + (Z_{A_i} - Z_{D_i})^2 - e_i^{\,2} = 0,\ \ i = 1,2,3 \quad (13)$$

Where the coordinates $D_i, i = 1,2,3$ can be determined as follows:

$$\begin{cases} X_{D_i} = X_{B_i} + d_i \cdot c(q_i) \\ Y_{D_i} = Y_{B_i} + d_i \cdot s(q_i) \\ Z_{D_i} = q_{i+3} \end{cases} \quad (14)$$

Using equation (14), the equation (13) can be written:

$$\left(X_{A_i} - X_{B_i} - d_i c q_i\right)^2 + \left(Y_{A_i} - Y_{B_i} - d_i s q_i\right)^2 - e_i^2 = 0, \quad (15)$$
$$i = 1,2,3$$

After the reduction of terms, equation (15) becomes:

$$\left(X_{A_i} - X_{B_i}\right) c q_i + \left(Y_{A_i} - Y_{B_i}\right) s q_i = = \frac{1}{2d_i}\left[\left(X_{A_i} - X_{B_i}\right)^2 + \left(Y_{A_i} - Y_{B_i}\right)^2 + d_i^2 - e_i^2\right]$$
$$i = 1,2,3 \quad (16)$$

For ease in computations, the following notations were made:

$$\begin{cases} a_i = X_{Ai} - X_{Bi} \\ b_i = Y_{Ai} - Y_{Bi} \\ c_i = \dfrac{1}{2d_i}\left[\left(X_{Ai} - X_{Bi}\right)^2 + \left(Y_{Ai} - Y_{Bi}\right)^2 + d_i^2 - e_i^2\right] \\ \quad i = 1,2,3 \end{cases} \quad (17)$$

Considering the notations (17), equation (16) becomes:

$$a_i c q_i + b_i s q_i = c_i,\ i = 1,2,3 \quad (18)$$

Equation (18) has the following solution:

$$q_i = \mathrm{atan2}(c_i, \pm\sqrt{a_i^{\,2} + b_i^{\,2} - c_i^{\,2}}) - \mathrm{atan2}(a_i, b_i)$$
$$i = 1,2,3 \quad (19)$$

The algorithm for the inverse geometric model is presented in table 1:

Table 1

Inverse geometric algorithm for M=6 dof

Given	$X_E, Y_E, Z_E, \psi, \theta, \varphi$
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	$q_1, q_2, q_3, q_4, q_5, q_6$
Variables	**Solving equations**

$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$c\alpha' = c\psi c\theta c\varphi - s\psi s\varphi$; $c\alpha'' = -c\psi c\theta c\varphi - s\psi c\varphi$; $c\alpha''' = c\psi s\theta$; $c\beta' = s\psi c\theta c\varphi + c\psi s\varphi$; $c\beta'' = -s\psi c\theta s\varphi + c\psi c\varphi$; $c\beta''' = s\psi s\theta$; $c\gamma' = -s\theta c\varphi$; $c\gamma'' = s\theta s\varphi$; $c\gamma''' = c\theta$
X_{Ai}, Y_{Ai}, Z_{Ai} $i = 1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$ $i = 1,2,3$
q_{i+3} $i = 1,2,3$	$q_{i+3} = Z_{Ai},\ i = 1,2,3$
$a_i, b_i, c_i,$ $i = 1,2,3$	$a_i = X_{Ai} - X_{Bi}$ $b_i = Y_{Ai} - Y_{Bi}$ $c_i = \frac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^2 - e_i^2}{2 \cdot d_i}$ $i = 1,2,3$
q_i $i = 1,2,3$	$q_i = \operatorname{atan2}(c_i, \pm\sqrt{a_i^2 + b_i^2 - c_i^2}) - \operatorname{atan2}(a_i, b_i)$ $i = 1,2,3$

2.2 Direct geometric model for M=6 dof

In the general case of direct modeling the driving coordinates $q_1, q_2, \dots, q_6$ are given, and the end-effector's coordinates (ψ, θ, φ are $Z' Y^* z^*$ - Euler angles) are computed.

$$q_1, q_2, q_3, q_4, q_5 \Rightarrow X_E, Y_E, Z_E, \psi, \theta, \varphi \quad (20)$$

For solving the direct geometric model, equation (6) can be written as follows:

$$\begin{cases} X_{A_i} - X_{B_i} - d_i c q_i = e_i c\theta_i \\ Y_{A_i} - Y_{B_i} - d_i s q_i = e_i s\theta_i \quad, i = 1,2,3 \\ Z_{A_i} = q_{i+3} \end{cases} \quad (21)$$

After squaring and adding the first two equations from (21) results:

$$\left[\left(X_{A_i} - X_{B_i}\right) - d_i c q_i\right]^2 + \left[\left(Y_{A_i} - Y_{B_i}\right) - d_i s q_i\right]^2 = e_i^2$$
$$i = 1,2,3 \quad (22)$$

The coordinates $A_i,\ i = 1,2,3$ can be computed with equation (11), and the direction cosine matrix (9).

From the equations (11) and (22) results:

$$\begin{cases} F_i \equiv [X_E + (x_{Ai} - x_E)c\alpha' + (y_{Ai} - y_E)c\alpha'' + \\ (z_{Ai} - z_E)c\alpha''' - X_{Di}]^2 + [Y_E + (x_{Ai} - x_E)c\beta' + \\ (y_{Ai} - y_E)c\beta'' + (z_{Ai} - z_E)c\beta''' - Y_{Di}]^2 - e_i^2 = 0 \\ F_{i+3} \equiv Z_E + (x_{Ai} - x_E)c\gamma' + (y_{Ai} - y_E)c\gamma'' + \\ (z_{Ai} - z_E)c\gamma''' - q_{i+3} = 0 \\ i = 1,2,3 \end{cases} \quad (23)$$

The implicit system of equations (23) raises problems when solved analytically because it is a nonlinear system of equations, so a numerical approach (for example Newton-Raphson method, Newton method or secant method) is best suited for the job. For this article it was used the Newton-Raphson method which was programmed using the Matlab – package software.

The approach would be to solve all six equations using Newton-Raphson Method. The algorithm for solving all six coordinates is shortly described in the following paragraphs.

The iterative formula for solving the end-effector coordinates using Newton-Raphson is:

$$X^{(p+1)} = X^{(p)} - W^{-1}\left(X^{(p)}\right) f\left(X^{(p)}\right)$$
$$p = 0,1,2,3,\dots \quad (24)$$

Where:
p represents the number of iterations;

$$X = \begin{bmatrix} X_E & Y_E & Z_E & \psi & \theta & \varphi \end{bmatrix}^T \quad (25)$$

W(X) represents the Jacobian matrix of the system (23);

$$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \quad (26)$$

For solving both the inverse and direct geometric model the next constants were used:

$$\begin{cases} d_1 = d_2 = d_3 = d = 0.058[m] \\ e_1 = e_2 = e_3 = e = 0.109[m] \\ x_{A_1} = p, y_{A_1} = 0, z_{A_1} = 0 \\ x_{A_2} = 0, y_{A_2} = 0, z_{A_2} = 0 \\ x_{A_3} = 0, y_{A_3} = p, z_{A_3} = 0 \\ x_E = \frac{p}{3}, y_E = \frac{p}{3}, z_E = -h = -0.04[m] \\ X_{B_i} = X_{B_i^{'}} = p_1, Y_{B_i} = Y_{B_i^{'}}, Z_{B_i} = Z_{B_i^{'}} + q_{i+3} \\ p = 0.01[m] \end{cases} \qquad (27)$$

The algorithm for solving the direct geometrical model with the second method is presented in table 2:

Table 2

Direct geometric algorithm for M=6 dof

Given	$q_1, q_2, q_3, q_4, q_5, q_6$
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1, 2, 3$
Unknowns	$X_E, Y_E, Z_E, \psi, \theta, \varphi$
Variables	**Solving equations**
System of implicit functions	$F_1 \equiv [X_E + (x_{A_1} - x_E) c\alpha' + (y_{A_1} - y_E) c\alpha'' + (z_{A_1} - z_E) c\alpha''' - X_{D_1}]^2 + [Y_E + (x_{A_1} - x_E) c\beta' + (y_{A_1} - y_E) c\beta'' + (z_{A_1} - z_E) c\beta''' - Y_{D_1}]^2 - e_1^2 = 0$ $F_2 \equiv [X_E + (x_{A_2} - x_E) c\alpha' + (y_{A_2} - y_E) c\alpha'' + (z_{A_2} - z_E) c\alpha''' - X_{D_2}]^2 + [Y_E + (x_{A_2} - x_E) c\beta' + (y_{A_2} - y_E) c\beta'' + (z_{A_2} - z_E) c\beta''' - Y_{D_2}]^2 - e_2^2 = 0$ $F_3 \equiv [X_E + (x_{A_3} - x_E) c\alpha' + (y_{A_3} - y_E) c\alpha'' + (z_{A_3} - z_E) c\alpha''' - X_{D_3}]^2 + [Y_E + (x_{A_3} - x_E) c\beta' + (y_{A_3} - y_E) c\beta'' + (z_{A_3} - z_E) c\beta''' - Y_{D_3}]^2 - e_3^2 = 0$ $F_4 \equiv Z_E + (x_{A_1} - x_E) c\gamma' + (y_{A_1} - y_E) c\gamma'' + (z_{A_1} - z_E) c\gamma''' - q_4 = 0$ $F_5 \equiv Z_E + (x_{A_2} - x_E) c\gamma' + (y_{A_2} - y_E) c\gamma'' + (z_{A_2} - z_E) c\gamma''' - q_5 = 0$ $F_6 \equiv Z_E + (x_{A_3} - x_E) c\gamma' + (y_{A_3} - y_E) c\gamma'' + (z_{A_3} - z_E) c\gamma''' - q_6 = 0$
Vector X - E point coordinates Vector F(X)	$X = \begin{bmatrix} X_E \\ Y_E \\ Z_E \\ \psi \\ \theta \\ \varphi \end{bmatrix}, \; F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \\ F_4(X) \\ F_5(X) \\ F_6(X) \end{bmatrix}$
Jacobian W(X)	$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix}$
The end-effector coordinates	$X^{(p+1)} = X^{(p)} - W^{-1}\left(X^{(p)}\right) F\left(X^{(p)}\right)$ $p = 0, 1, 2, 3, \ldots$

3. CONFIGURATION POSSIBLITIES

In the following paragraphs the 5 cases of reconfiguration of Recrob are described.

3.1 Inverse and direct geometric model for M=5 dof

In this case, as seen in figure 2 the constraint $q_6 = q_5$ was added and the actuator q_6 was turned into a passive joint. As a consequence of this, $\varphi = 0$, and the E point coordinates are: $X_E, Y_E, Z_E, \psi, \theta$.

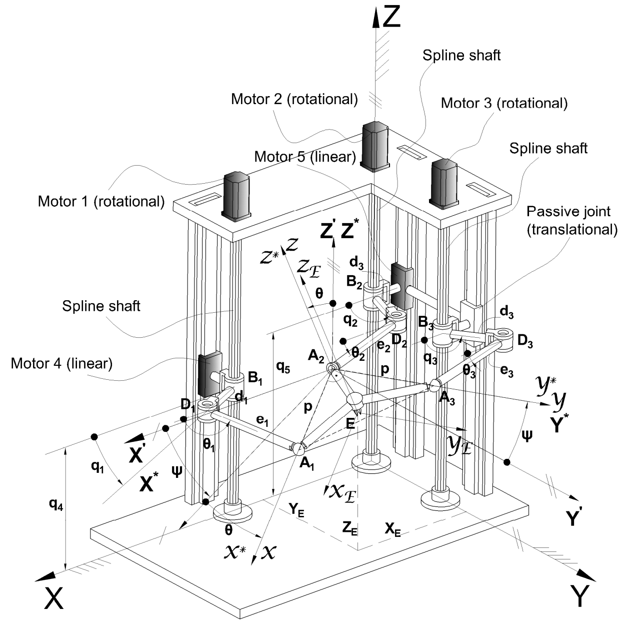

Fig. 2. Recrob-5dof-kinematic scheme

The end-effector performs translations along OX, OY and OZ axes and two rotations (precession and nutation angles).

Table 3

Inverse geometric algorithm for M=5 dof

Given	$X_E, Y_E, Z_E, \psi, \theta$
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i=1,2,3$
Unknowns	q_1, q_2, q_3, q_4, q_5
Variables	**Solving equations**
$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$c\alpha' = c\psi c\theta$; $c\alpha'' = -s\psi$; $c\alpha''' = c\psi s\theta$ $c\beta' = s\psi c\theta$; $c\beta'' = c\psi$; $c\beta''' = s\psi s\theta$ $c\gamma' = -s\theta$; $c\gamma'' = 0$; $c\gamma''' = c\theta$
X_{Ai}, Y_{Ai}, Z_{Ai} $i=1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$ $i=1,2,3$
q_{i+3} $i=1,2,3$	$q_{i+3} = Z_{Ai},\ i=1,2$
$a_i, b_i, c_i,$ $i=1,2,3$	$a_i = X_{Ai} - X_{Bi}$ $b_i = Y_{Ai} - Y_{Bi}$ $c_i = \dfrac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^{\,2} - e_i^{\,2}}{2 \cdot d_i}$ $i=1,2,3$
q_i $i=1,2,3$	$q_i = \operatorname{atan2}(c_i, \pm\sqrt{a_i^{\,2} + b_i^{\,2} - c_i^{\,2}}) -$ $-\operatorname{atan2}(a_i, b_i)$ $i=1,2,3$

Tables 3 and 4 present the algorithm containing the inverse and direct geometric model in the case the degree of mobility of Recrob equals to five.

Table 4

Direct geometric algorithm for M=5 dof

Given	q_1, q_2, q_3, q_4, q_5
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i=1,2,3$
Unknowns	$X_E, Y_E, Z_E, \psi, \theta$
Variables	**Solving equations**
System of implicit functions	$F_1 \equiv [X_E + (x_{A_1} - x_E)c\alpha' + (y_{A_1} - y_E)c\alpha'' + (z_{A_1} - z_E)c\alpha''' - X_{D_1}]^2 + [Y_E + (x_{A_1} - x_E)c\beta' + (y_{A_1} - y_E)c\beta'' + (z_{A_1} - z_E)c\beta''' - Y_{D_1}]^2 - e_1^2 = 0$ $F_2 \equiv [X_E + (x_{A_2} - x_E)c\alpha' + (y_{A_2} - y_E)c\alpha'' + (z_{A_2} - z_E)c\alpha''' - X_{D_2}]^2 + [Y_E + (x_{A_2} - x_E)c\beta' + (y_{A_2} - y_E)c\beta'' + (z_{A_2} - z_E)c\beta''' - Y_{D_2}]^2 - e_2^2 = 0$ $F_3 \equiv [X_E + (x_{A_3} - x_E)c\alpha' + (y_{A_3} - y_E)c\alpha'' + (z_{A_3} - z_E)c\alpha''' - X_{D_3}]^2 + [Y_E + (x_{A_3} - x_E)c\beta' + (y_{A_3} - y_E)c\beta'' + (z_{A_3} - z_E)c\beta''' - Y_{D_3}]^2 - e_3^2 = 0$ $F_4 \equiv Z_E + (x_{A_1} - x_E)c\gamma' + (y_{A_1} - y_E)c\gamma'' + (z_{A_1} - z_E)c\gamma''' - q_4 = 0$ $F_5 \equiv Z_E + (x_{A_2} - x_E)c\gamma' + (y_{A_2} - y_E)c\gamma'' + (z_{A_2} - z_E)c\gamma''' - q_5 = 0$
Vector X - E point coordinates Vector F(X)	$X = \begin{bmatrix} X_E \\ Y_E \\ Z_E \\ \psi \\ \theta \end{bmatrix},\ F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \\ F_4(X) \\ F_5(X) \end{bmatrix}$

Jacobian W(X)	$W(X)=\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} \end{bmatrix}$
The end-effector coordinates	$X^{(p+1)} = X^{(p)} - W^{-1}\left(X^{(p)}\right)F\left(X^{(p)}\right)$ $p = 0,1,2,3,...$

3.2 Inverse and direct geometric model for M=4 dof

In this case the imposed constrained is $q_6 = q_5 = q_4$ (figure 3), as a consequence q_4 linear motor remains and $\varphi = \theta = 0$.The mobile platform makes three translations: along OX, OY and Oz axes and one rotation around OZ axis.

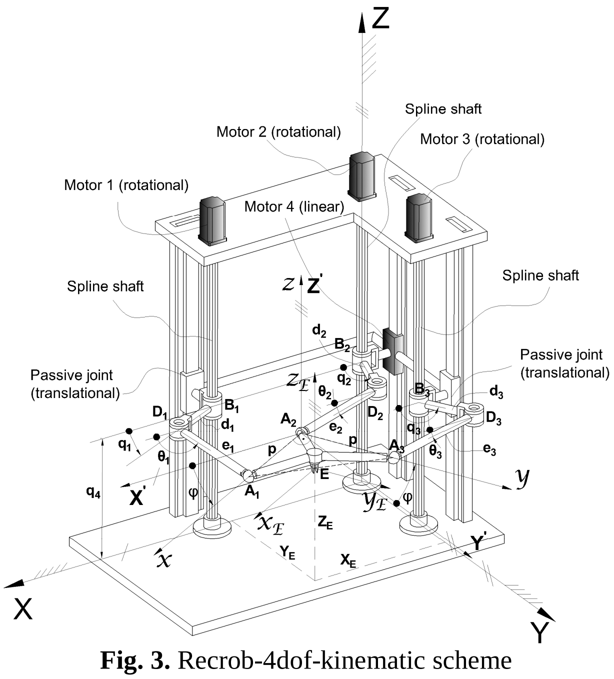

Fig. 3. Recrob-4dof-kinematic scheme

Table 5

Inverse geometric algorithm for M=4 dof

Given	X_E, Y_E, Z_E, ψ
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	q_1, q_2, q_3, q_4
Variables	**Solving equations**
$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$c\alpha' = c\psi$, $c\alpha'' = -s\psi$, $c\alpha''' = 0$; $c\beta' = s\psi$, $c\beta'' = c\psi$, $c\beta''' = 0$; $c\gamma' = 0$, $c\gamma'' = 0$, $c\gamma''' = 1$
X_{Ai}, Y_{Ai}, Z_{Ai} $i = 1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$ $i = 1,2,3$
q_4	$q_4 = Z_{A_2},\ i = 1,2$
$a_i, b_i, c_i,$ $i = 1,2,3$	$a_i = X_{Ai} - X_{Bi}$ $b_i = Y_{Ai} - Y_{Bi}$ $c_i = \frac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^2 - e_i^2}{2 \cdot d_i}$ $i = 1,2,3$
q_i $i = 1,2,3$	$q_i = \operatorname{atan2}(c_i, \pm\sqrt{a_i^2 + b_i^2 - c_i^2}) - \operatorname{atan2}(a_i, b_i)$ $i = 1,2,3$

Tables 5 and 6 present the algorithm containing the inverse and direct geometric model in the case the degree of mobility of Recrob equals to four.

Table 6

Direct geometric algorithm for M=4 dof

Given	q_1, q_2, q_3, q_4
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	X_E, Y_E, Z_E, ψ
Variables	**Solving equations**
System of implicit functions	$\begin{cases} F_1 \equiv [X_E + (x_{A_1} - x_E)c\alpha' + (y_{A_1} - y_E)c\alpha'' + (z_{A_1} - z_E)c\alpha''' - X_{D_1}]^2 + [Y_E + (x_{A_1} - x_E)c\beta' + (y_{A_1} - y_E)c\beta'' + (z_{A_1} - z_E)c\beta''' - Y_{D_1}]^2 - e_1^2 = 0 \\ F_2 \equiv [X_E + (x_{A_2} - x_E)c\alpha' + (y_{A_2} - y_E)c\alpha'' + (z_{A_2} - z_E)c\alpha''' - X_{D_2}]^2 + [Y_E + (x_{A_2} - x_E)c\beta' + (y_{A_2} - y_E)c\beta'' + (z_{A_2} - z_E)c\beta''' - Y_{D_2}]^2 - e_2^2 = 0 \\ F_3 \equiv [X_E + (x_{A_3} - x_E)c\alpha' + (y_{A_3} - y_E)c\alpha'' + (z_{A_3} - z_E)c\alpha''' - X_{D_3}]^2 + [Y_E + (x_{A_3} - x_E)c\beta' + (y_{A_3} - y_E)c\beta'' + (z_{A_3} - z_E)c\beta''' - Y_{D_3}]^2 - e_3^2 = 0 \end{cases}$

	$\begin{cases} F_4 \equiv Z_E + \left(x_{A_1} - x_E\right)c\gamma' + \left(y_{A_1} - y_E\right)c\gamma'' + \\ \left(z_{A_1} - z_E\right)c\gamma''' - q_4 = 0 \end{cases}$
Vector X - E point coordinates Vector F(X)	$X = \begin{bmatrix} X_E \\ Y_E \\ Z_E \\ \psi \end{bmatrix}, \; F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \\ F_4(X) \end{bmatrix}$
Jacobian W(X)	$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} \end{bmatrix}$
The end-effector coordinates	$X^{(p+1)} = X^{(p)} - W^{-1}\left(X^{(p)}\right)F\left(X^{(p)}\right)$ $p = 0,1,2,3,...$

3.3 Inverse and direct geometric model for M=3 dof(spatial translation)

Figure 4 presents the configuration of Recrob with 3 degrees of freedom in space. There are two constraints made: the three linear motors move simultaneously and two rotational motors move simultaneously. The end-effector performs only translations along OX,OY,OZ axes. The coordinates of the E point are X_E, Y_E, Z_E. The motor q_3 is eliminated and replaced with a rotational joint.

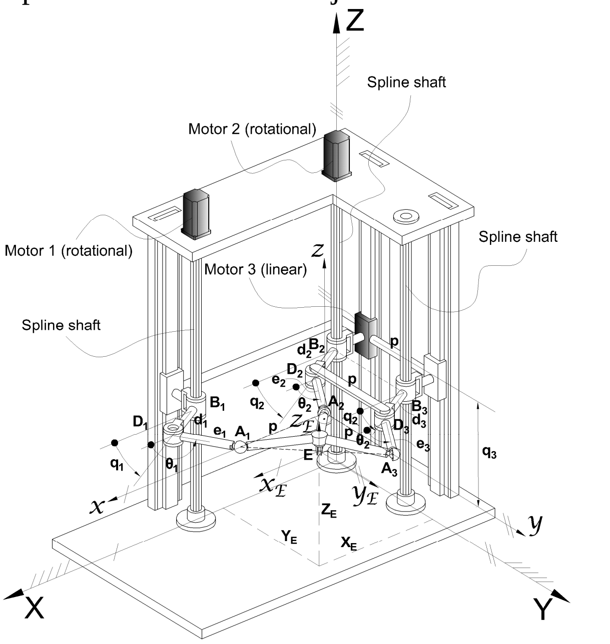

Fig. 4. Recrob-3dof spatial translation-kinematic scheme

Table 7

Inverse geometric algorithm for M=3 dof (spatial translation)

Given	X_E, Y_E, Z_E
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	q_1, q_2, q_3
Variables	**Solving equations**
$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$c\alpha' = 1$, $c\alpha'' = 0$, $c\alpha''' = 0$ $c\beta' = 0$, $c\beta'' = 1$, $c\beta''' = 0$ $c\gamma' = 0$, $c\gamma'' = 0$, $c\gamma''' = 1$
X_{Ai}, Y_{Ai}, Z_{Ai} $i = 1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$ $i = 1,2,3$
q_3	$q_3 = Z_{A_2}, \; i = 1,2$
$a_i, b_i, c_i,$ $i = 1,2,3$	$a_i = X_{Ai} - X_{Bi}$ $b_i = Y_{Ai} - Y_{Bi}$ $c_i = \frac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^2 - e_i^2}{2 \cdot d_i}$ $i = 1,2,3$
q_i $i = 1,2$	$q_i = \text{atan}\,2(c_i, \pm\sqrt{a_i^2 + b_i^2 - c_i^2}) -$ $-\text{atan}2(a_i, b_i)$ $i = 1,2$

Table 8

Direct geometric algorithm for M=3 dof (spatial translation)

Given	q_1, q_2, q_3
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	X_E, Y_E, Z_E
Variables	**Solving equations**
System of implicit functions	$\begin{cases} F_1 \equiv [X_E + \left(x_{A_1} - x_E\right)c\alpha' + \left(y_{A_1} - y_E\right)c\alpha'' + \\ \left(z_{A_1} - z_E\right)c\alpha''' - X_{D_1}]^2 + [Y_E + \left(x_{A_1} - x_E\right)c\beta' + \\ \left(y_{A_1} - y_E\right)c\beta'' + \left(z_{A_1} - z_E\right)c\beta''' - Y_{D_1}]^2 - e_1^2 = 0 \\ \\ F_2 \equiv [X_E + \left(x_{A_2} - x_E\right)c\alpha' + \left(y_{A_2} - y_E\right)c\alpha'' + \\ \left(z_{A_2} - z_E\right)c\alpha''' - X_{D_2}]^2 + [Y_E + \left(x_{A_2} - x_E\right)c\beta' + \\ \left(y_{A_2} - y_E\right)c\beta'' + \left(z_{A_2} - z_E\right)c\beta''' - Y_{D_2}]^2 - e_2^2 = 0 \end{cases}$

	$\begin{cases} F_3 \equiv Z_E + (x_{A_1} - x_E)c\gamma' + (y_{A_1} - y_E)c\gamma'' + \\ (z_{A_1} - z_E)c\gamma''' - q_3 = 0 \end{cases}$
Vector X - E point coordinates Vector F(X)	$X = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix}, \; F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \end{bmatrix}$
Jacobian W(X)	$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} \end{bmatrix}$
The end-effector coordinates	$X^{(p+1)} = X^{(p)} - W^{-1}\left(X^{(p)}\right)F\left(X^{(p)}\right)$ $p = 0,1,2,3,...$

3.4 Inverse and direct geometric model for M=3 dof (planar movement)

In this case there was imposed that all the linar motors are blocked, the resulting structure presented in figure 4 realizes a planar movement: two translations along OX and OY and one rotation around OZ axis.

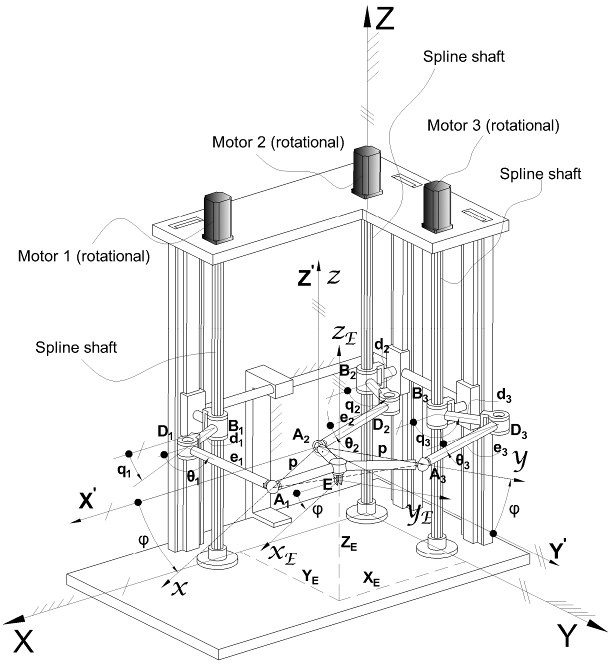

Fig. 4. Recrob-3dof planar movement-kinematic scheme

Tables 9 and 10 present the algorithm for solving the inverse and direct geometric model.

In this case, all three linear actuators from the base are eliminated, and the coordinates of the E point corresponding to the mobile platform has 3 coordinates: X_E, Y_E, ψ.

Table 9

Inverse geometric algorithm M=3 dof (planar movement)

Given	X_E, Y_E, ψ
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1, 2, 3$
Unknowns	q_1, q_2, q_3
Variables	**Solving equations**
$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$c\alpha' = c\psi$ ¦ $c\alpha'' = -s\psi$ ¦ $c\alpha''' = 0$ $c\beta' = s\psi$ ¦ $c\beta'' = c\psi$ ¦ $c\beta''' = 0$ $c\gamma' = 0$ ¦ $c\gamma'' = 0$ ¦ $c\gamma''' = 1$
X_{Ai}, Y_{Ai}, Z_{Ai} $i = 1, 2, 3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$ $i = 1, 2, 3$
$a_i, b_i, c_i,$ $i = 1, 2, 3$	$a_i = X_{Ai} - X_{Bi}$ $b_i = Y_{Ai} - Y_{Bi}$ $c_i = \frac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^2 - e_i^2}{2 \cdot d_i}$ $i = 1,2,3$
q_i $i = 1, 2, 3$	$q_i = \text{atan}\,2(c_i, \pm\sqrt{a_i^2 + b_i^2 - c_i^2}) -$ $-\text{atan2}(a_i, b_i)$ $i = 1,2,3$

Table 10

Direct geometric algorithm for M=3 dof (planar movement)

Given	q_1, q_2, q_3
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1, 2, 3$
Unknowns	X_E, Y_E, ψ
Variables	**Solving equations**
System of implicit functions	$\begin{cases} F_1 \equiv [X_E + (x_{A_1} - x_E)c\alpha' + (y_{A_1} - y_E)c\alpha'' + \\ (z_{A_1} - z_E)c\alpha''' - X_{D_1}]^2 + [Y_E + (x_{A_1} - x_E)c\beta' + \\ (y_{A_1} - y_E)c\beta'' + (z_{A_1} - z_E)c\beta''' - Y_{D_1}]^2 - e_1^2 = 0 \\ \\ F_2 \equiv [X_E + (x_{A_2} - x_E)c\alpha' + (y_{A_2} - y_E)c\alpha'' + \\ (z_{A_2} - z_E)c\alpha''' - X_{D_2}]^2 + [Y_E + (x_{A_2} - x_E)c\beta' + \\ (y_{A_2} - y_E)c\beta'' + (z_{A_2} - z_E)c\beta''' - Y_{D_2}]^2 - e_2^2 = 0 \end{cases}$

	$\begin{cases} F_3 \equiv [X_E + (x_{A_3} - x_E)c\alpha' + (y_{A_3} - y_E)c\alpha'' + \\ (z_{A_3} - z_E)c\alpha''' - X_{D_3}]^2 + [Y_E + (x_{A_3} - x_E)c\beta' + \\ (y_{A_3} - y_E)c\beta'' + (z_{A_3} - z_E)c\beta''' - Y_{D_3}]^2 - e_3^2 = 0 \end{cases}$
Vector X - E point coordinates Vector F(X)	$X = \begin{bmatrix} X_E \\ Y_E \\ \psi \end{bmatrix}, \; F(X) = \begin{bmatrix} F_1(X) \\ F_2(X) \\ F_3(X) \end{bmatrix}$
Jacobian W(X)	$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial \psi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial \psi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial \psi} \end{bmatrix}$
The end-effector coordinates	$X^{(p+1)} = X^{(p)} - W^{-1}(X^{(p)})F(X^{(p)})$ $p = 0,1,2,3,...$

3.5 Inverse and direct geometric model for M=2 dof

The constraints imposed in this case (figure 6) are: all linear motors are blocked and two rotational motors are made to rotate simultaneously. The mobile platform makes a planar movement in a plane parallel with OZ plane. The remaining two coordinates of the E point are X_E, Y_E.

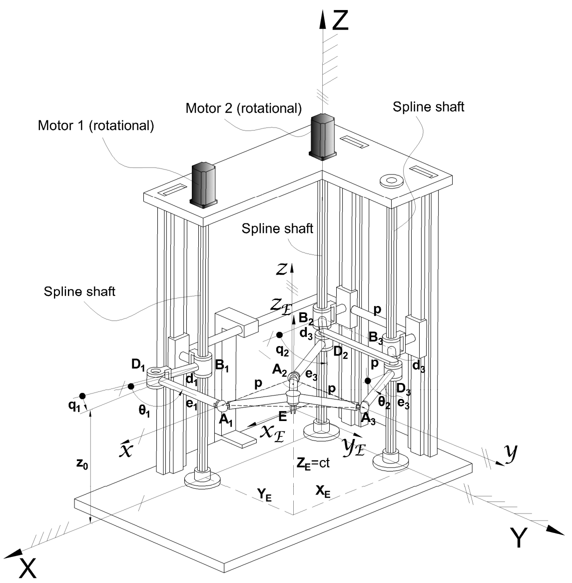

Fig. 6. Recrob-2dof planar movement-kinematic scheme

Table 11

Inverse geometric algorithm for M=2 dof movement

Given	X_E, Y_E
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	q_1, q_2
Variables	**Solving equations**
$c\alpha', c\alpha'', c\alpha'''$ $c\beta', c\beta'', c\beta'''$ $c\gamma', c\gamma'', c\gamma'''$	$c\alpha' = 1 \mid c\alpha'' = 0 \mid c\alpha''' = 0$ $c\beta' = 0 \mid c\beta'' = 1 \mid c\beta''' = 0$ $c\gamma' = 0 \mid c\gamma'' = 0 \mid c\gamma''' = 1$
X_{Ai}, Y_{Ai}, Z_{Ai} $i = 1,2,3$	$\begin{bmatrix} X_{A_i} \\ Y_{A_i} \\ Z_{A_i} \end{bmatrix} = \begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} + \begin{bmatrix} c\alpha' & c\alpha'' & c\alpha''' \\ c\beta' & c\beta'' & c\beta''' \\ c\gamma' & c\gamma'' & c\gamma''' \end{bmatrix} \begin{bmatrix} x_{A_i} - x_E \\ y_{A_i} - y_E \\ z_{A_i} - z_E \end{bmatrix}$ $i = 1,2,3$
$a_i, b_i, c_i,$ $i = 1,2,3$	$a_i = X_{Ai} - X_{Bi}$ $b_i = Y_{Ai} - Y_{Bi}$ $c_i = \frac{(X_{Ai} - X_{Bi})^2 + (Y_{Ai} - Y_{Bi})^2 + d_i^2 - e_i^2}{2 \cdot d_i}$ $i = 1,2,3$
q_i $i = 1,2$	$q_i = \text{atan}\,2(c_i, \pm\sqrt{a_i^2 + b_i^2 - c_i^2}) -$ $-\text{atan}2(a_i, b_i)$ $i = 1,2$

Table 12

Direct geometric algorithm for M=2 dof movement

Given	q_1, q_2
Constants	$x_{Ai}, y_{Ai}, z_{Ai}, x_E, y_E, z_E, X_{Bi}, Y_{Bi}$ $i = 1,2,3$
Unknowns	X_E, Y_E
Variables	**Solving equations**
System of implicit functions	$\begin{cases} F_1 \equiv [X_E + (x_{A_1} - x_E)c\alpha' + (y_{A_1} - y_E)c\alpha'' + \\ (z_{A_1} - z_E)c\alpha''' - X_{D_1}]^2 + [Y_E + (x_{A_1} - x_E)c\beta' + \\ (y_{A_1} - y_E)c\beta'' + (z_{A_1} - z_E)c\beta''' - Y_{D_1}]^2 - e_1^2 = 0 \\ \\ F_2 \equiv [X_E + (x_{A_2} - x_E)c\alpha' + (y_{A_2} - y_E)c\alpha'' + \\ (z_{A_2} - z_E)c\alpha''' - X_{D_2}]^2 + [Y_E + (x_{A_2} - x_E)c\beta' + \\ (y_{A_2} - y_E)c\beta'' + (z_{A_2} - z_E)c\beta''' - Y_{D_2}]^2 - e_2^2 = 0 \end{cases}$

Vector X - E point coordinates Vector F(X)	$X = \begin{bmatrix} X_E \\ Y_E \end{bmatrix},\ F(X) = \begin{bmatrix} F_1(X) \\ F_1(X) \end{bmatrix}$
Jacobian W(X)	$W(X) = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} \end{bmatrix}$
The end-effector coordinates	$X^{(p+1)} = X^{(p)} - W^{-1}\left(X^{(p)}\right)F\left(X^{(p)}\right)$ $p = 0,1,2,3,...$

Table 11 and 12 presents the algorithm for solving the direct and inverse geometric model.

4. CONCLUSION

In this paper was presented Recrob – a parallel reconfigurable robot and it's configurations for M=6,5,4,3, 2 degrees of mobility. The geometric model algorithm for each of these configurations is also presented.

5. ACKNOWLEDGEMENTS

This paper was supported by the project "Proiect de dezvoltare a studiilor de doctorat in tehnologii avansate" – "PRODOC" contract no. POSDRU/88/1.5/S/61178, project co-funded by the European Social Fund through the Sectorial Operational Program Human Resources 2007-2013.

11. REFERENCES

[1]Plitea, N., Pisla, D., Vaida, C., Lese, D-B., Konya, B., Dadarlat, R., Scurtu, I., Sabou, C., *Familie de roboti paraleli cu sase grade de mobilitate*, Patent nr. A/10013/2011.

[2] Lese, D-B., Pisla, D., Vaida, C., Scurtu, I., Plitea, N., *Inverse and direct geometrical model of a new reconfigurable parallel robot*, Proceedings of the 1st International Conference on Innovation in Engineering and Management, pp.307-313, Cluj-Napoca, Romania, 2011.

[3] Stechert, C., Franke, H.-J., *Oriented configuration of Parallel Robotic Systems*, The Future of Product Development, Proceedings of the 17th CIRP Design Conference, pp.259-268, Berlin, 2007.

[4] Chen, I-M., Yang, G., Yeo, S. H., *Automatic Modelling for Modular Reconfigurable Robotic Systems – Theory and Practice*, Industrial Robotics: Theory, Modelling and Control, Germany, 2006.

[5] Dash, A. K., Chen, I.-M., Yeo, S.-H., Yang, G., *Task – Oriented Configuration for Reconfigurable Parallel Manipulator Systems*, International Journal of Computer Integrated Manufacturing, Vol. 18, No. 7, pp. 615-634(20), 2005.

[6] Ping, X., Zhu, X.-J., Fei, Y. Q., *Mechanical Design and Locomotion Control of a Homogenous Lattice Modular Self-Reconfigurable Robot*, Journal of Zhejian University – Science A, Vol. 7, pp. 368-373, 2006.

[7] Behnam, S., Moll, M., Shen, W.-M., *A Deployable, Multi-Functional, and Modular Self-Reconfigurable Robotic System*, Proceedings of the 2006 IEEE/RSJ International Conference on Intelligent Robots and Systemns, China, 2006.

[8] Zhang, Y., Eldershaw, C., Yim, M., Roufas, K., Duff, D., *A platform for Studying Locomotion systems: Modular reconfigurable Robots*, NIST Workshop on Performance Metrics for Intelligent Systems, 2002.

[9] Bi, Z.M., Wang L., *Optimal design of Reconfigurable Parallel Machining Systems*, Robotics and Computer – Integrated Manufacturing, Vol.25, pp. 951 -961, 2009.

[10] Finistauri, A. D., Xi, F., Petz, B., *Architecture design and Optimization of an On-the-Fly Reconfigurable Parallel Robot*, Parallel Manipulators, Towards New Applications, pp. 379-404, Austria, (2008).

[11] Gogu, G., *Isogliden TaRb: a Family of up to Five Axes Reconfigurable and Maximally Regular Parallel Kinematic Machines*, International Conference on Smart Machining Systems, 2007.

[12] Yang, G., Chen, I.-M., Lim, W. K., Yeo, S.H., *Design and Kinematic Analysis of Modular Reonfugurable Parallel Robots*,

Proceedings of the 1999 IEEE International Conference on Robotics & Automation Vol. 4, pp.2501-2506, 1999.
[13] Negri, S., et. al., *Improvement and optimization of a reconfigurable parallel kinematic machine*, 2002.
[14] Choi, J., et. al., *Dynamic and stable reconfiguration of self-reconfigurable planar parallel robots*, Advanced Robotics, Vol. 18, pp. 565-582, 2004.
[15] Krefft, M., Last, P., Hesselbach, J., *New Concepts to Adapt the PKM Performance to Application Requirements,* Parallel Kinematic Machines in Researh and Practice, The 5th Chemnitz Parallel Kinematics Seminar, pp. 115-133, Chemnitz, 2006.
[16] Borras, J., Thoms, F., Ottaviano, E., Ceccarelli, M., *A Reconfigurable 5-Dof 5-SPU Parallel Platform,* Proceedings of the 2009 ASME/IFTOMM International Conferece of Reconfigurable Mechanisms and Robots, pp. 617-623, 2009.
[17] Pisla, D., Plitea, N., Vidrean, A., Prodan, B., Lese, D., *Kinematics and Design of Two Variants of a Reconfigurable Parallel Robot,* Reconfigurable Mechanisms ans Robots, REMAR, Vol. 22, pp. 524-631, 2009.
[18] Plitea, N., Hesselbach, J., Pisla, D., Simnofske, M., Prodan, B., Burish, A., Vidrean, A., *Modelling of Parallel Microrobots With Two To Six Degrees Of Freedom*, Acta Technica Napocensis, 2007.
[19] Plitea, N., Pisla, D., Vidrean, A., Vaida, C., Gyurka, B., *Workspace and Singularity Analysis for a Reconfigurable Parallel Robot*, Proceedings of SYROM 2009, the 10th IFTOMM International Symposium on Science of Mechanisms and Machines, 2009.
[20]Plitea, N., *Noi Structuri de Mecanisme Paralele pentru Roboti Industriali (Manipulatoare).* Sistematica si Dezvoltare, Universitatea Tehnica Cluj-napoca, (1992).

MODELAREA UNUI ROBOT PARALLEL RECONFIGURABIL CU SASE GRADE DE MOBILITATE SI CONFIGURATIILE SALE CU CINCI, PATRU, TREI SI DOUA GRADE DE MOBILITATE

Rezumat: Robotii reconfigurabili sunt din ce in ce mai mult studiati, datorita flexibilitatii pe care o ofera utilizatorului. Aceasta flexibilitate exista datorata faptului ca robotii reconfigurabili pot fi astfel configurati incat sa aiba geometrii diferite care sa fie potrivite diferitor procese din sistemul de productie. Acesta lucrare prezinta un robot parallel reconfigurabil numit Recrob, precum si posibilitatile sale de reconfigurare.

Dorin-Bogdan LEȘE, Phd. Student, Technical University of Cluj-Napoca, Department of Mechanical Engineering and Computer Programming, dorinlese@yahoo.com, 0264401684, Memorandumului 28, RO-400114

Nicolae PLITEA, Corresponding Author – Prof. Dr. Ing., Technical University of Cluj-Napoca, Department of Mechanical Engineering and Computer Programming, plitea@rdslink.ro, 0264401684, Memorandumului 28, RO-400114