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THE KINEMATICS OF A NEW PARALLEL ROBOT WITH SIX DEGREES OF FREEDOM AND THREE GUIDING KINEMATIC CHAINS OF THE PLATFORM

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Abstract: *The paper presents a new parallel robot with six degrees of freedom and three guiding kinematic chains of the platform. The principal characteristic of the parallel robot is given by the position on the fixed base of all the six linear actuators. The inverse and direct kinematic problem, the workspace generated through a geometrical method, and the relationships for determining the singularities of the new parallel structure are presented. The robot can be used in assembly tasks or as a module for minimally invasive surgical systems.*

Key words: *6-DOF parallel robot, design, kinematics, workspace, singularities.*

1. INTRODUCTION

Due to their structure, the traditional serial chain robots have less load carrying capacity. Also the serial robots tend to bend at high load and vibrate at high speed. Though they possess a large workspace, the positioning capability is rather poor, one of the drawbacks of serial manipulators being the lack of accuracy in the positioning of the terminal tool because of the error accumulations in each link. By comparison with serial architectures, parallel architectures present also a greater compactness and stiffness, along with an interesting ratio between moved load and the robot's mass.

So, where high load carrying capacity and precise positioning are prime concerns, an alternative is provided by parallel actuated and closed loops robots which have attracted tremendous research interest. The most celebrated parallel robot manipulator is a six-degrees-of-freedom (6-DOF) mechanism composed of two bodies connected together by six extensible legs. This manipulator is obtained from a generalization of the mechanism originally proposed by Stewart [22] to be used as a flight simulator.

However, the increase in stiffness of the parallel robots is obtained to the detriment of the workspace. Thus, the main disadvantage of parallel manipulators is their limited workspace [2, 4, 9], difficulties in trajectory planning and their control due to singular configurations [24]. The determination of the topological parameters of a parallel manipulator leading to an appropriate workspace and a minimum number of singular configurations is a challenging problem and has motivated many researchers in the past 2 decades to study various techniques in these areas [3, 6, 8, 10-13, 15, 16, 21, 23, 25, 26].

In parallel robot research area, the problem of Jacobian matrix determination is an open and interesting problem. Indeed, the inverse Jacobian matrix computation is currently known and mastered [14, 7, 5] but its analytical expression remains relatively complex. Thus analytical formulation of the Jacobian matrix, by symbolic inversion or even by using some formal computing tools, is difficult [14].

The organization of this paper is as follows. The geometrical description of the 6-DOF parallel robot is given in Section 2. The inverse and direct kinematic problems are the subject of Section 3. The workspace of the robot, obtained

through a geometrical volume intersection method is presented in section 4. The relationship for determining the singularities of the MOTBAT6 parallel robot are presented in section 5. In the last section are presented the conclusions.

2. DESIGN CONSIDERATIONS

A certain advantage of parallel structures is the fact that the actuators can be placed on the fixed platform, imposing so a smaller actuator size, lower power dissipation and a higher payload capacity, which makes it possible for parallel robots to achieve higher speed and accelerations. Also, by using three limbs instead of six the possibility of interference between limbs decreases.

As a conclusion of the advantages mentioned above in Figure 1 is presented a parallel structure with 6-DOF and three guiding kinematic chains of the mobile platform called MOTBAT6 [19].

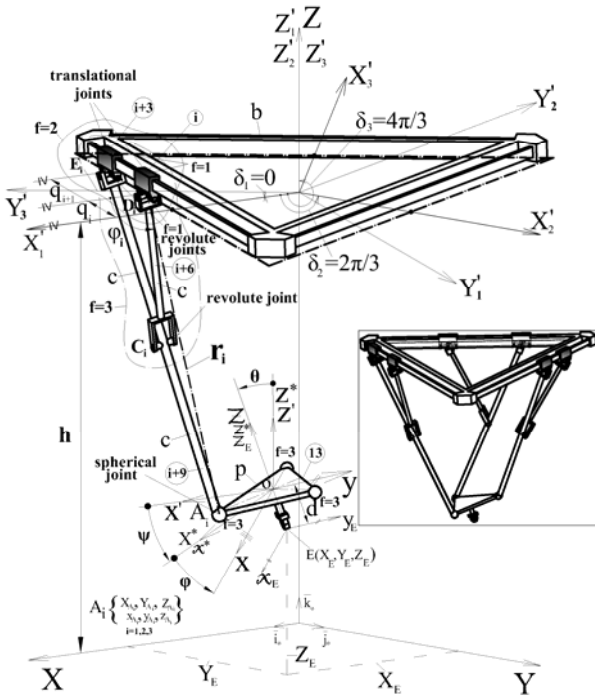


Fig. 1. MOTBAT6 parallel robot

The moving platform and the fixed base have an equilateral form. The platforms are connected through three 3-DOF modules, each module being formed by two active linear joints positioned at the base and a revolute joint that connects the rods in points C_i , and with the mobile platform through a spherical joint in

point A_i . The structure present the following passive joint: six revolute joints between the linear actuators and each rod of the limb, and three revolute joints that connects the rods (revolute joints from the 3-DOF modules) [18].

3. KINEMATICS

Starting from the six positional equations bellow obtained through the geometrical model [18], expressed in function by the coordinates of the end-effector $X_E, Y_E, Z_E, \psi, \theta, \varphi$:

$$\begin{aligned}
 & \left\{ \begin{aligned}
 & F_i(X_E, Y_E, Z_E, \psi, \theta, \varphi) \equiv -[X_E + (x_{A_i} - x_E) \\
 & (c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_i} - y_E) \cdot \\
 & (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_i} - z_E) s\psi s\theta] s\delta_i + \\
 & + [Y_E + (x_{A_i} - x_E)(s\psi c\varphi + c\psi c\theta s\varphi) + \\
 & \cdot (y_{A_i} - y_E)(-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 & - (z_{A_i} - z_E) c\psi s\theta] c\delta_i + q_i = 0 \\
 & F_{i+3}(X_E, Y_E, Z_E, \psi, \beta, \varphi) \equiv (q_{i+3} + \{[X_E + \\
 & + (x_{A_i} - x_E)(c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_i} - y_E) \cdot \\
 & \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_i} - z_E) s\psi s\theta] s\delta_i - \\
 & - [Y_E + (x_{A_i} - x_E)(s\psi c\varphi + c\psi c\theta s\varphi) + \\
 & + (y_{A_i} - y_E) \cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 & - (z_{A_i} - z_E) c\psi s\theta] c\delta_i \})^2 - (2c)^2 + \{[X_E + \\
 & + (x_{A_i} - x_E) \cdot (c\psi c\varphi - s\psi c\theta s\varphi) \cdot (y_{A_i} - y_E) \cdot \\
 & \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_i} - z_E) s\psi s\theta] c\delta_i + \\
 & + [Y_E + (x_{A_i} - x_E) \cdot (s\psi c\varphi + c\psi c\theta s\varphi) + \\
 & + (y_{A_i} - y_E) \cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 & - (z_{A_i} - z_E) c\psi s\theta] s\delta_i - h_i \}^2 + [Z_E + \\
 & + (x_{A_i} - x_E) s\theta s\varphi + (y_{A_i} - y_E) s\theta c\varphi + \\
 & + (z_{A_i} - z_E) c\theta - h_i]^2 = 0
 \end{aligned} \right.
 \end{aligned}
 \tag{1}$$

Where $\delta_1, \delta_2, \delta_3$ are the angles between de system of axis attached to the fixed base; ψ, θ, φ the Euler's angles; $x_{A_i}, y_{A_i}, z_{A_i}$ the coordinates of point A_i with respect to the system of axis attached to mobile platform and x_E, y_E, z_E the coordinates of the manipulated

object with respect to the system of axis attached also to the mobile platform.

The relationships for the six positional equations are:

$$\begin{aligned}
 F_1(X_E, Y_E, Z_E, \psi, \theta, \varphi) &\equiv -[X_E + (x_{A_1} - x_E) \cdot \\
 &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_1} - y_E) \cdot \\
 &\cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_1} - z_E) s\psi s\theta] s\delta_1 + \\
 &+ [Y_E + (x_{A_1} - x_E) (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_1} - y_E) \cdot \\
 &\cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - (z_{A_1} - z_E) c\psi s\theta] c\delta_1 + q_1 = 0 \\
 F_2(X_E, Y_E, Z_E, \psi, \theta, \varphi) &\equiv -[X_E + (x_{A_2} - x_E) \cdot \\
 &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_2} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &+ (z_{A_2} - z_E) s\psi s\theta] s\delta_2 + [Y_E + (x_{A_2} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_2} - y_E) (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 &- (z_{A_2} - z_E) c\psi s\theta] c\delta_2 + q_2 = 0 \\
 F_3(X_E, Y_E, Z_E, \psi, \theta, \varphi) &\equiv -[X_E + (x_{A_3} - x_E) \cdot \\
 &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_3} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &+ (z_{A_3} - z_E) s\psi s\theta] s\delta_3 + [Y_E + (x_{A_3} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_3} - y_E) (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 &- (z_{A_3} - z_E) c\psi s\theta] c\delta_3 + q_3 = 0 \\
 F_4(X_E, Y_E, Z_E, \psi, \beta, \varphi) &\equiv \{q_4 + \{[X_E + (x_{A_1} - x_E) \cdot \\
 &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_1} - y_E) \cdot \\
 &\cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + (z_{A_1} - z_E) s\psi s\theta] s\delta_1 - [Y_E + \\
 &(x_{A_1} - x_E) (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_1} - y_E) \cdot \\
 &\cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - (z_{A_1} - z_E) c\psi s\theta] c\delta_1\}^2 - \\
 &- (2c)^2 + \{[X_E + (x_{A_2} - x_E) \cdot (c\psi c\varphi - s\psi c\theta s\varphi) \cdot \\
 &\cdot (y_{A_4} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &+ (z_{A_4} - z_E) s\psi s\theta] c\delta_4 + [Y_E + (x_{A_4} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_4} - y_E) \cdot \\
 &\cdot (-s\psi s\varphi + c\psi c\theta c\varphi) - (z_{A_4} - z_E) c\psi s\theta] s\delta_1 - h_1\}^2 + \\
 &+ [Z_E + (x_{A_1} - x_E) s\theta s\varphi + (y_{A_1} - y_E) \cdot s\theta c\varphi + \\
 &(z_{A_1} - z_E) c\theta - h_1]^2 = 0 \\
 F_5(X_E, Y_E, Z_E, \psi, \beta, \varphi) &\equiv \{q_5 + \{[X_E + (x_{A_2} - x_E) \\
 &(c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_2} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &(z_{A_2} - z_E) \cdot s\psi s\theta] s\delta_2 - [Y_E + (x_{A_2} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_2} - y_E) (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 &- (z_{A_2} - z_E) c\psi s\theta] c\delta_2\}^2 - (2c)^2 + \{[X_E + (x_{A_2} - x_E) \cdot \\
 &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) \cdot (y_{A_2} - y_E) (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &+ (z_{A_2} - z_E) s\psi s\theta] c\delta_2 + [Y_E + (x_{A_2} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_2} - y_E) (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 &- (z_{A_2} - z_E) c\psi s\theta] s\delta_2 - h_2\}^2 + [Z_E + (x_{A_2} - x_E) s\theta s\varphi + \\
 &(y_{A_2} - y_E) s\theta c\varphi + (z_{A_2} - z_E) c\theta - h_2]^2 = 0 \\
 F_6(X_E, Y_E, Z_E, \psi, \beta, \varphi) &\equiv \{q_6 + \{[X_E + (x_{A_3} - x_E) \\
 &(c\psi c\varphi - s\psi c\theta s\varphi) + (y_{A_3} - y_E) \cdot (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &+ (z_{A_3} - z_E) s\psi s\theta] s\delta_3 - [Y_E + (x_{A_3} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_3} - y_E) (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 &- (z_{A_3} - z_E) c\psi s\theta] c\delta_3\}^2 - (2c)^2 + \{[X_E + (x_{A_3} - x_E) \cdot \\
 &\cdot (c\psi c\varphi - s\psi c\theta s\varphi) \cdot (y_{A_3} - y_E) (-c\psi s\varphi - s\psi c\theta c\varphi) + \\
 &(z_{A_3} - z_E) s\psi s\theta] c\delta_3 + [Y_E + (x_{A_3} - x_E) \cdot \\
 &\cdot (s\psi c\varphi + c\psi c\theta s\varphi) + (y_{A_3} - y_E) (-s\psi s\varphi + c\psi c\theta c\varphi) - \\
 &- (z_{A_3} - z_E) c\psi s\theta] s\delta_3 - h_3\}^2 + [Z_E + (x_{A_3} - x_E) s\theta s\varphi + \\
 &+ (y_{A_3} - y_E) s\theta c\varphi + (z_{A_3} - z_E) c\theta - h_3]^2 = 0
 \end{aligned} \tag{2}$$

3.1 The velocities problem

Starting from the six positional equations:

$$\begin{cases}
 F_1(X_E, Y_E, Z_E, \psi, \theta, \varphi, q_1, q_2, q_3, q_4, q_5, q_6) = 0 \\
 F_2(X_E, Y_E, Z_E, \psi, \theta, \varphi, q_1, q_2, q_3, q_4, q_5, q_6) = 0 \\
 F_3(X_E, Y_E, Z_E, \psi, \theta, \varphi, q_1, q_2, q_3, q_4, q_5, q_6) = 0 \\
 F_4(X_E, Y_E, Z_E, \psi, \theta, \varphi, q_1, q_2, q_3, q_4, q_5, q_6) = 0 \\
 F_5(X_E, Y_E, Z_E, \psi, \theta, \varphi, q_1, q_2, q_3, q_4, q_5, q_6) = 0 \\
 F_6(X_E, Y_E, Z_E, \psi, \theta, \varphi, q_1, q_2, q_3, q_4, q_5, q_6) = 0
 \end{cases} \tag{3}$$

With the following notations:

$$\dot{X} = \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix}, \quad \dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}, \tag{4}$$

Where $\dot{X}_E, \dot{Y}_E, \dot{Z}_E, \dot{\psi}, \dot{\theta}, \dot{\varphi}$ are the generalized velocities of the end-effector and

$\dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5, \dot{q}_6$ the generalized velocities of the robot.

By deriving in respect to time is obtained:

$$A\dot{X} + B\dot{q} = 0 \tag{5}$$

where the matrices A and B are defined as follows:

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & \frac{\partial F_1}{\partial Z_E} & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & \frac{\partial F_2}{\partial Z_E} & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & \frac{\partial F_3}{\partial Z_E} & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & \frac{\partial F_1}{\partial q_2} & \frac{\partial F_1}{\partial q_3} & \frac{\partial F_1}{\partial q_4} & \frac{\partial F_1}{\partial q_5} & \frac{\partial F_1}{\partial q_6} \\ \frac{\partial F_2}{\partial q_1} & \frac{\partial F_2}{\partial q_2} & \frac{\partial F_2}{\partial q_3} & \frac{\partial F_2}{\partial q_4} & \frac{\partial F_2}{\partial q_5} & \frac{\partial F_2}{\partial q_6} \\ \frac{\partial F_3}{\partial q_1} & \frac{\partial F_3}{\partial q_2} & \frac{\partial F_3}{\partial q_3} & \frac{\partial F_3}{\partial q_4} & \frac{\partial F_3}{\partial q_5} & \frac{\partial F_3}{\partial q_6} \\ \frac{\partial F_4}{\partial q_1} & \frac{\partial F_4}{\partial q_2} & \frac{\partial F_4}{\partial q_3} & \frac{\partial F_4}{\partial q_4} & \frac{\partial F_4}{\partial q_5} & \frac{\partial F_4}{\partial q_6} \\ \frac{\partial F_5}{\partial q_1} & \frac{\partial F_5}{\partial q_2} & \frac{\partial F_5}{\partial q_3} & \frac{\partial F_5}{\partial q_4} & \frac{\partial F_5}{\partial q_5} & \frac{\partial F_5}{\partial q_6} \\ \frac{\partial F_6}{\partial q_1} & \frac{\partial F_6}{\partial q_2} & \frac{\partial F_6}{\partial q_3} & \frac{\partial F_6}{\partial q_4} & \frac{\partial F_6}{\partial q_5} & \frac{\partial F_6}{\partial q_6} \end{bmatrix} \tag{7}$$

In the matrices (6) and (7) $F_1, F_2, F_3, F_4, F_5, F_6$, represent the six positional nonlinear equations depending on the generalized coordinates of the end-effector $X_E, Y_E, Z_E, \psi, \theta, \varphi$ and the generalized coordinates of the robot $q_1, q_2, q_3, q_4, q_5, q_6$.

Some partial derivates of matrices are zero, resulting so for equations (6) and (7) the following:

$$A = \begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & 0 & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & 0 & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial F_6}{\partial q_6} \end{bmatrix} \tag{9}$$

Using the relations (8) and (9) is obtained from (5):

$$\begin{bmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \varphi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & 0 & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \varphi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & 0 & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \varphi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \varphi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \varphi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \varphi} \end{bmatrix} \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ \dot{Z}_E \\ \dot{\psi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} +$$

$$\begin{aligned}
 & \left[\begin{array}{cccccc}
 \frac{d}{dt} \left(\frac{\partial F_1}{\partial X_E} \right) & \frac{d}{dt} \left(\frac{\partial F_1}{\partial Y_E} \right) & 0 & \frac{d}{dt} \left(\frac{\partial F_1}{\partial \psi} \right) & \frac{d}{dt} \left(\frac{\partial F_1}{\partial \theta} \right) & \frac{d}{dt} \left(\frac{\partial F_1}{\partial \phi} \right) \\
 \frac{d}{dt} \left(\frac{\partial F_2}{\partial X_E} \right) & \frac{d}{dt} \left(\frac{\partial F_2}{\partial Y_E} \right) & 0 & \frac{d}{dt} \left(\frac{\partial F_2}{\partial \psi} \right) & \frac{d}{dt} \left(\frac{\partial F_2}{\partial \theta} \right) & \frac{d}{dt} \left(\frac{\partial F_2}{\partial \phi} \right) \\
 \frac{d}{dt} \left(\frac{\partial F_3}{\partial X_E} \right) & \frac{d}{dt} \left(\frac{\partial F_3}{\partial Y_E} \right) & 0 & \frac{d}{dt} \left(\frac{\partial F_3}{\partial \psi} \right) & \frac{d}{dt} \left(\frac{\partial F_3}{\partial \theta} \right) & \frac{d}{dt} \left(\frac{\partial F_3}{\partial \phi} \right) \\
 \frac{d}{dt} \left(\frac{\partial F_4}{\partial X_E} \right) & \frac{d}{dt} \left(\frac{\partial F_4}{\partial Y_E} \right) & \frac{d}{dt} \left(\frac{\partial F_4}{\partial Z_E} \right) & \frac{d}{dt} \left(\frac{\partial F_4}{\partial \psi} \right) & \frac{d}{dt} \left(\frac{\partial F_4}{\partial \theta} \right) & \frac{d}{dt} \left(\frac{\partial F_4}{\partial \phi} \right) \\
 \frac{d}{dt} \left(\frac{\partial F_5}{\partial X_E} \right) & \frac{d}{dt} \left(\frac{\partial F_5}{\partial Y_E} \right) & \frac{d}{dt} \left(\frac{\partial F_5}{\partial Z_E} \right) & \frac{d}{dt} \left(\frac{\partial F_5}{\partial \psi} \right) & \frac{d}{dt} \left(\frac{\partial F_5}{\partial \theta} \right) & \frac{d}{dt} \left(\frac{\partial F_5}{\partial \phi} \right) \\
 \frac{d}{dt} \left(\frac{\partial F_6}{\partial X_E} \right) & \frac{d}{dt} \left(\frac{\partial F_6}{\partial Y_E} \right) & \frac{d}{dt} \left(\frac{\partial F_6}{\partial Z_E} \right) & \frac{d}{dt} \left(\frac{\partial F_6}{\partial \psi} \right) & \frac{d}{dt} \left(\frac{\partial F_6}{\partial \theta} \right) & \frac{d}{dt} \left(\frac{\partial F_6}{\partial \phi} \right)
 \end{array} \right] \begin{bmatrix} \ddot{X}_E \\ \ddot{Y}_E \\ \ddot{Z}_E \\ \ddot{\psi} \\ \ddot{\theta} \\ \ddot{\phi} \end{bmatrix} + \\
 & \left[\begin{array}{cccccc}
 \frac{d}{dt} \left(\frac{\partial F_1}{\partial q_1} \right) & 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{d}{dt} \left(\frac{\partial F_2}{\partial q_2} \right) & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{d}{dt} \left(\frac{\partial F_3}{\partial q_3} \right) & 0 & 0 & 0 \\
 0 & 0 & 0 & \frac{d}{dt} \left(\frac{\partial F_4}{\partial q_4} \right) & 0 & 0 \\
 0 & 0 & 0 & 0 & \frac{d}{dt} \left(\frac{\partial F_5}{\partial q_5} \right) & 0 \\
 0 & 0 & 0 & 0 & 0 & \frac{d}{dt} \left(\frac{\partial F_6}{\partial q_6} \right)
 \end{array} \right] \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \\ \ddot{q}_6 \end{bmatrix} \quad (23)
 \end{aligned}$$

From (20) results the solving equations for the direct kinematical problem for accelerations when are given the generalized accelerations of the robot $\ddot{q}_1, \ddot{q}_2, \ddot{q}_3, \ddot{q}_4, \ddot{q}_5, \ddot{q}_6$ and the unknowns are the generalized accelerations of the end-effector $\ddot{X}_E, \ddot{Y}_E, \ddot{Z}_E, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$, respectively from relation (21) results the generalized accelerations of the robot $\ddot{q}_1, \ddot{q}_2, \ddot{q}_3, \ddot{q}_4, \ddot{q}_5, \ddot{q}_6$ when are given the position, the generalized velocities of the robot and the accelerations of the end-effector $\ddot{X}_E, \ddot{Y}_E, \ddot{Z}_E, \ddot{\psi}, \ddot{\theta}, \ddot{\phi}$.

4. GEOMETRICAL WORKSPACE GENERATION

Exact computation of the workspace presents a considerable interest due to its impact on robot design, robot placement in an environment or in a efficient operation. The robot workspace is a region that can be reached by a reference point, which is usually considered as coincident with a center point of the robot extremity as the operation point. Also, the orientation workspace can be defined as the set of orientations that can be reached.

Besides of the general definition of the workspace, it can be divided in subspaces of the workspace: the reachable workspace – the set of locations that may be reached by the end-

effector reference point with at least one orientation; the total orientation workspace – all the locations of end-effector reference point that may be reached with all the orientations among a set defined by ranges on the orientation angles; dexterous workspace – all the locations of end-effector reference point for which all orientations are possible; orientation workspace – all the possible orientations that can be reached while the end-effector reference point is in a fixed location and, the constant orientation workspace – the set of all possible locations that can be reached by the end-effector reference point with a specified orientation [16].

The workspace could be determined by using the inverse geometrical model, the direct geometrical model or by a geometrical method which implies the intersection of the workspaces (the 3 cylinders section) generated by each kinematic guiding chain [17].

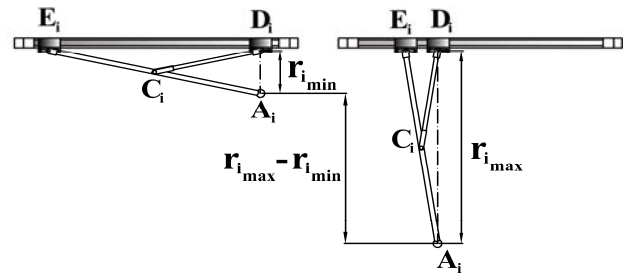


Fig. 2. The minimum and maximum radii of one guiding kinematic chain

The workspace generated by each kinematic chain is a cylinder determined by the minimum and maximum radii (Fig. 2) that rotates about a vertical axis and translates along the sides of the fixed base. In figure bellow (Fig. 3) is presented the workspace generated by the point A_i corresponding to the spherical joint.

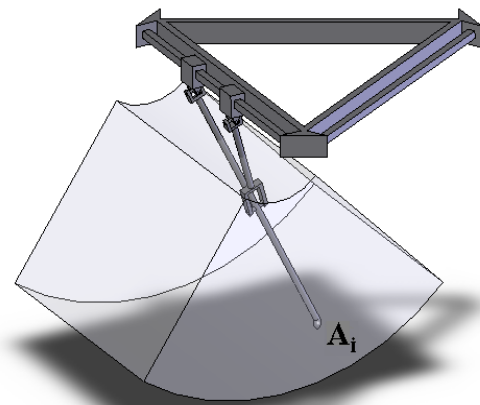


Fig. 3. MOTBAT6 Workspace generated by one guiding chain

In figures bellow are represented the geometrical intersection workspace generated by the three limbs of the robot (Figure 4):

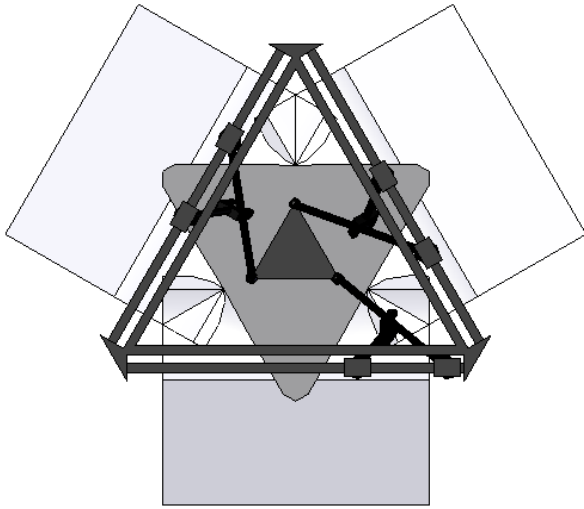


Fig. 4. The intersection of workspaces generated by the three limbs. Top view

and the total geometrical workspace of the robot (Figure 5):

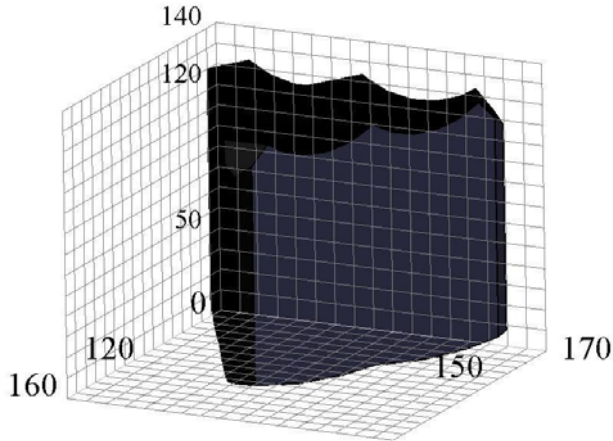


Fig. 5. Total workspace of the structure. Isometric view

5. SYNGULARITY ANALYSIS

Singular configurations of a mechanism are undesirable due to the fact that the motion freedom of the mechanism changes instantaneously. While the singular positions of serially connected manipulators result in the loss of one or more motion freedoms [11, 12, 25, 26] those of parallel manipulators result in either a gain or a loss of one or more motion freedoms [1, 3, 6, 13, 15, 16]. At singular configurations, the determinant of the manipulator Jacobian becomes zero. Usually, the determinants are large in size and are

difficult to handle mathematically. The singularity of Jacobian matrix corresponds to loss or gain of motion freedom and singularity of both occurs only when the mechanism is architecturally singular.

The algorithm used for the singularity analysis is based on deriving the determinants for the two Jacobian matrices obtained from the inverse and direct geometrical model [17].

Type I singularities. First type of singularities occurs when the determinant of the Jacobian matrix B is 0, case in which the robot loses one or more degrees of freedom [17].

$$\begin{vmatrix} \frac{\partial F_1}{\partial q_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial F_2}{\partial q_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial F_3}{\partial q_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial F_4}{\partial q_4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\partial F_5}{\partial q_5} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial F_6}{\partial q_6} \end{vmatrix} = 0 \tag{24}$$

Type II singularities. Second type of singularities occurs when the determinant of the Jacobian matrix A is 0.

$$\begin{vmatrix} \frac{\partial F_1}{\partial X_E} & \frac{\partial F_1}{\partial Y_E} & 0 & \frac{\partial F_1}{\partial \psi} & \frac{\partial F_1}{\partial \theta} & \frac{\partial F_1}{\partial \phi} \\ \frac{\partial F_2}{\partial X_E} & \frac{\partial F_2}{\partial Y_E} & 0 & \frac{\partial F_2}{\partial \psi} & \frac{\partial F_2}{\partial \theta} & \frac{\partial F_2}{\partial \phi} \\ \frac{\partial F_3}{\partial X_E} & \frac{\partial F_3}{\partial Y_E} & 0 & \frac{\partial F_3}{\partial \psi} & \frac{\partial F_3}{\partial \theta} & \frac{\partial F_3}{\partial \phi} \\ \frac{\partial F_4}{\partial X_E} & \frac{\partial F_4}{\partial Y_E} & \frac{\partial F_4}{\partial Z_E} & \frac{\partial F_4}{\partial \psi} & \frac{\partial F_4}{\partial \theta} & \frac{\partial F_4}{\partial \phi} \\ \frac{\partial F_5}{\partial X_E} & \frac{\partial F_5}{\partial Y_E} & \frac{\partial F_5}{\partial Z_E} & \frac{\partial F_5}{\partial \psi} & \frac{\partial F_5}{\partial \theta} & \frac{\partial F_5}{\partial \phi} \\ \frac{\partial F_6}{\partial X_E} & \frac{\partial F_6}{\partial Y_E} & \frac{\partial F_6}{\partial Z_E} & \frac{\partial F_6}{\partial \psi} & \frac{\partial F_6}{\partial \theta} & \frac{\partial F_6}{\partial \phi} \end{vmatrix} = 0 \tag{25}$$

Type III singularities. So-called architectural singularities appear when both Jacobian determinants are 0.

6. CONCLUSION

The contribution of this paper consists in the development of a new parallel structure with 6 DOF and three guiding kinematic chains of the platform. The principal characteristic of this structure is that it has all the six actuators mounted on the fixed base thus imposing a limited weight to the moving parts, a lower power dissipation, higher payload capacity and three guiding kinematic chains of platform decreasing so the mechanical interference between the limbs. The kinematical model, and also the workspace and singularities defining relationships are presented. Future work includes the study of the robot dynamics.

The parallel structure is patented [19].

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CINEMATICA UNUI NOU ROBOT PARALEL CU ȘASE GRADE DE MOBILITATE ȘI TREI LANȚURI CINEMATICE DE GHIDARE A PLATFORMEI

Rezumat: Lucrarea prezintă un nou robot paralel cu șase grade de mobilitate și trei lanțuri cinematice de ghidare a platformei. Principala caracteristică a robotului este faptul că cuplele active sunt montate pe batiu. Sunt prezentate problema cinematică directă și inversă, spațiul de lucru, precum și relațiile pentru determinarea singularităților structurii. Robotul poate fi utilizat în operațiile (liniile) de asamblare sau ca și modul în sistemele chirurgicale minim invazive.

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